Activation of Silver and Indium with Slow Neutrons

Physikalisches Institut der Universität Tübingen

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Contents

1 Introduction

In this lab course, natural samples of silver and indium are activated using thermal neutrons. The decay curves of the unstable isotopes created through neutron capture are measured.

A moderated AmBe source serves as a source of neutrons. The half lives and the initial activities of the samples are determined by detecting the decay radiation with two Geiger-Müller counters.

Experimentally this is a relatively simple experiment. However, this method of neutron activation analysis (NAA) is of great practical importance and widely used in non-destructive materials analysis.

2 Theoretical Foundations of Neutron Activation

2.1 Radioactivity

Atoms are called unstable or radioactive, when they are transformed spontaneously into a different kind of atom by emitting particles or through spontaneous fission, or if they transition from an excited state to a lower energy level by emitting a γ-ray. When radioactivity was first discovered, one distinguished between three types of radiation:

- α radiation
- β^- radiation
- γ radiation

Later, additional types of radiation were discovered:

- proton radiation
- β^+ radiation
- neutron radiation

The common feature of all these types of radiation is their ability to ionize matter.

2.1.1 α decay

In α decay, an α particle is emitted from a (heavy) nuclei. The α particle is a helium nucleus consisting of two protons and two neutrons:

$$ {}^{A}_{Z}X_N \rightarrow {}^{A-4}_{Z-2}X'_{N-2} + {}^{4}_{2}\mathrm{He}_2 \tag{1} $$

Quantum mechanically, two protons and two neutrons in a nucleus ${}^{A}_{Z}X$, whose energy levels are occupied according to the Pauli principle up to an energy of $E_{\max}$,

can, with a certain probability, form an α particle inside the nuclear potential. Because of its exceptionally high binding energy, a large amount of energy is released in the formation of the α particle. It will thus be excited to a higher energy level, which, however, still is below the maximum energy of the potential. But, in quantum mechanics, it is possible to tunnel through this potential barrier with probability T. This type of decay mainly occurs in heavy nuclei (heavier than lead, with a few exceptions like ^{146}Sm oder ^{144}Nd) with a surplus of protons.

α particles lose their energy in matter mainly through ionization, i.e. through scattering with shell electrons, rarely also through direct impacts with nuclei, and through bremsstrahlung. Their range in air is usually only a few centimeters. They can be shielded effectively by a sheet of paper.

2.1.2 β radiation

With β decays, one has to differentiate between two subtypes: β^-- and β^+-decay. Electron capture EC (sometimes also ϵ) can be considered a form of the latter. These decays are a result of the weak interaction. In β^- decays, a neutron is converted into a proton and an electron. In addition, an electron anti-neutrino is emitted in the decay, which ensures that momentum and energy as well as lepton number are conserved. β^- decays happen in nuclei with a surplus of neutrons. In β^+ decays, the opposite reaction takes place, i.e. a proton is converted into a neutron, a positron and an electron neutrino. It therefore occurs in nuclei with a surplus of protons.

$$\begin{aligned} {}^{A}_{Z}X_N &\rightarrow {}^{A}_{Z+1}X'_{N-1} + e^- + \overline{\nu}_e \\ {}^{A}_{Z}X_N &\rightarrow {}^{A}_{Z-1}X'_{N+1} + e^+ + \nu_e \end{aligned} \tag{2}$$

β particles interact with matter mainly through scattering with the shell electrons and has a typical range in air of a few meters (depending on their energy!). They can be effectively shielded with a few centimeters of metal.

2.1.3 γ emission

γ rays are photons emitted from nuclei in transitions from an excitated state to a lower energy level - analogous to transitions between discrete energy levels in the atomic shell. The energies of γ ray photons emitted in nuclei are on the order of a few MeV. γ ray emission typically accompanies most of the other types of radioactive decay, as the nuclei often remain in an excited state after such decays. [1]

$$ {}^{A}_{Z}X^*_N \rightarrow {}^{A}_{Z}X_N + \gamma \tag{3}$$

These high energy photons interact with matter mostly in three ways: photo effect, Compton effect and pair production. Photo effect means, that the γ ray is absorbed by a shell electron, transferring all of its energy to the electron, which in turn is emitted from the atom. If the photon only transfers part of its energy to a

[1] One also has to consider the momentum, that is carried away by the emission of γ rays. This leads to a recoil on the nucleus (cf. Mössbauer effect).

shell electron in a scattering process, this is called Compton effect. If the γ energy is sufficiently large ($> 2m_e$), an electron-positron pair can be created, which in turn continues to interacts with matter.

2.1.4 Spontaneous emission of protons and neutrons

Protons can be spontaneously emitted from nuclei with a large deficiency of neutrons, neutrons from nuclei with a large surplus of neutrons. Neutrons have a very long range, as they hardly interact electromagnetically. They mainly lose their energy by scattering with other particles directly. In the energy range relevant for this experiment, they mainly scatter with nuclei. This energy transfer is most efficient, when the two involved particles have a similar mass. Neutrons can thus be shielded most efficiently by materials with a large hydrogen content (e.g. water, paraffin, ...).

2.1.5 Decay law and activity

Consider a number N of radioactive particles. The probability $\lambda = \frac{dP}{dt}$ that one of these particles decays in a given time frame is the same for all particles of the same type. The total number of decays per time unit is therefore:

$$\frac{dN}{dt} = -\lambda N = -A\left(t\right) \tag{4}$$

The time-dependent quantity A is called *activity* of the sample. Its SI unit is *Becquerel* $\left([A] = 1\,\mathrm{Bq} = 1\frac{\mathrm{Zerfall}}{\mathrm{s}}\right)$. An older (but sometimes still used) unit is 1 Curie ($1\,\mathrm{Ci} = 3,7 \times 10^{10}\,\mathrm{Bq}$), which is the activity of 1 g of radium.

Integrating this function yields

$$N\left(t\right) = N_0 e^{-\lambda t}$$

and for the activity

$$A\left(t\right) = A_0 e^{-\lambda t}$$

with $A_0 = \lambda N_0$. The mean lifetime is given by the average value of the actual lifetimes of the individual particles and equals

$$\tau = \frac{1}{\lambda}$$

The half-life, i.e. the time after which exactly half of the initial activity remains, is given by

$$t_{\frac{1}{2}} = \tau \ln 2 = \frac{\ln 2}{\lambda} \tag{5}$$

2.1.6 Activation

Now let's consider a source, that is constantly adding new particles to the above mentioned sample of particles with the production rate P. In this case, this production rate is given by $P = \sigma\Phi n$, where Φ is the neutron flux, σ the neutron capture cross section of the corresponding isotope and n the number of atoms in the irradiated sample.

The modified differential equation can now be written as

$$\frac{dN}{dt} = -\lambda N + P \tag{6}$$

integrating this yields the activity

$$A\left(t_a\right) = A_S\left(1 - e^{-\lambda t}\right) \tag{7}$$

where $A_S = P = \sigma\Phi n$ is the saturation activity.

2.2 Neutrons

2.2.1 Neutron sources

Creating neutron radiation for the use in an experiment like this is not trivial. Free neutrons decay after a mean lifetime of 885.7±0.8 s and thus occur very rarely in nature. They therefore have to be created in special sources. The underlying mechanisms, which can be exploited to create a neutron source are:

Fission denotes the process where a nucleus is split into two or more fragments, which is usually accompanied by the emission of neutrons. In some isotopes the fission occurs spontaneously (e.g. ^{252}Cf), in other isotopes (like ^{235}U), the fission process needs to be triggered by bombardment with neutrons. To sustain the fission process with such isotopes, a nuclear reactor is needed.

Spallation is a process, where nuclei are smashed by other high energy particles. Some of the created fragments are neutrons. Spallation naturally occurs in the atmosphere, triggered by cosmic rays, but it can also be artificially induced using a beam from a particle accelerator.

Nuclear reactions occur in a myriad of forms. The AmBe source used in this experiment is a so-called (α, n) source. This means, that a nucleus absorbs α particles (either from an accelerator or from a strong radioactive source) and gets excited to a meta-stable state, which de-excites under the emission of a neutron. There is also the analogous (p,n) process. In addition to these capture reactions, there are also fusion reaction, like e.g. (d,d) fusion. Here, mostly light elements, like hydrogen isotopes, coalesce into a new nucleus. These processes are usually accompanied by the emission of a neutron.

2.2.2 AmBe source used in the experiment

The thermal neutrons used to activate the sample in this experiment come from a AmBe source, which is shielded by lead. The activity of the ^{241}Am ($t_{1/2} = 432\,\mathrm{a}$) contained in the source is $\approx 2\,\mathrm{Ci}$ (70 GBq). The source is located in a cube with an edge length of 150 cm made of paraffin. The actual source is located in the center and is shielded by 1 cm of lead. The inner part is a single cube cast from paraffin. The outer layers consist of bricks made of borated paraffin. The boron in these bricks has an enormously large cross section for the interaction with thermal neutrons ($\sigma = 3800\,\mathrm{b}$ for a (n,α) reaction), efficiently absorbing the neutrons from the source. The entire source is wrapped in foil and the edges are secured with metal bars. On the front, one can see a red plastic knob. This extends all the way to the lead shield and the actual source is mounted on the end of it. On the back, there are two slits, where the sample holders can be inserted. Since these slits have a direct line of sight to the source, nobody should stand in front of these openings!

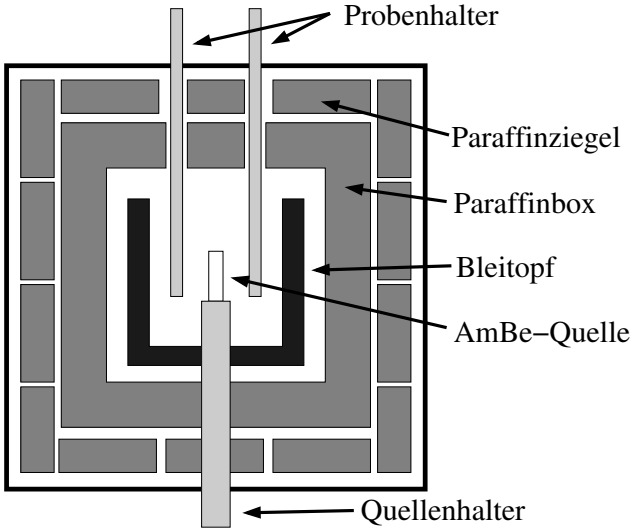

Figure 1: Schematic setup of the AmBe source used in the experiment

2.2.3 Neutron capture

The physical process leading to the conversion of stable isotopes into unstable isotopes is called neutron capture. Since neutrons are uncharged, there is no electric repulsion due to the electron shell or the nucleus. Because of this they can reach the nucleus quite easily and interact with it. In our case we are interested in (n,γ) reactions, i.e. capturing of the neutron with subsequent de-excitation of the nucleus via γ emission. The resulting isotope can of course be radioactive and decay itself.

The neutron capture process can in this case be explained by the compound nucleus model of Niels Bohr. Here one assumes, that there is a unstable intermediate state between initial and final state of the system. This state is reached by capturing

a particle (here: a neutron), but it immediately decays again (here: γ decay). The basic reaction therefore looks like this:

$$X + n \rightarrow Y^* \rightarrow Y + \gamma \tag{8}$$

Since Y^* has quasi infinite lifetime in comparison to nuclear physics processes ($\tau_{\text{Comp}} \approx 10^{-13}$ compared to $\tau_{(n,\alpha)} \approx 10^{-21}$), this new state can be considered a new nucleus, before it decays on its part. Hence, the decay of the compound nucleus is independent of the kinematics of the capture process.

The total interaction cross section, i.e. the geometrically and energetically weighted probability of a reaction, is defined as follows:

$$\sigma_{\text{tot}} = \frac{\text{reactions per unit time}}{\text{beam particles per unit time} \times \text{scattering centers per unit area}} \tag{9}$$

From this follows, that σ_{tot} is proportional to $E^{-\frac{1}{2}}$, because the flux (*beam particles per unit time*) contains the velocity of the particles. However, this is only part of the truth, as these kind of reactions can also contain resonances. These resonances originate from the fact, that neutrons can form a bound system in the nucleus resulting in quantized energy levels, like in the atomic shell. Fig. 2 shows a curve with such resonances. The maximum cross section for neutron capture can usually be found in one of the resonances or at very low energies. The narrow resonances, however, are difficult to hit and if one misses the resonance, the cross section quickly becomes very small again and for very low energies the range of the neutrons becomes very small.

Therefore the most practical neutron energy for these kinds of experiments is on the order of 1/40 eV (thermal neutrons). Neutrons of this energy can be obtained through multiple scattering of higher energy neutrons (e.g. in paraffin) and scattering them back towards the sample. Although the neutrons are also scattering on the lead around the source to shield the γ radiation, they barely lose any energy in these collisions.

2.3 Radiation Protection

Radiation from radioactivity can cause radiation damage when absorbed in the human or animal body. The ions and free electrons created in the living cells can induce chemical reactions in the complicated organic molecules, that can lead to serious consequences for the organism, e.g. genetic damage, cancer o, with very high radiation doses, acute signs of poisoning, in extreme cases with a deadly outcome. Most of the time, however, these damaging effects do not surface immediately after the exposure, but on longer timescales (days to years, genetic damage sometimes only in the next generation). One differentiates between deterministic and statistical radiation damage. Deterministic damage (macroscopic damage of the tissue, e.g. burns) only occurs with very high doses and is of course accompanied by statistical damage (change of cell functions, e.g. changes in DNA). One can distinguish the following forms of statistical damage:

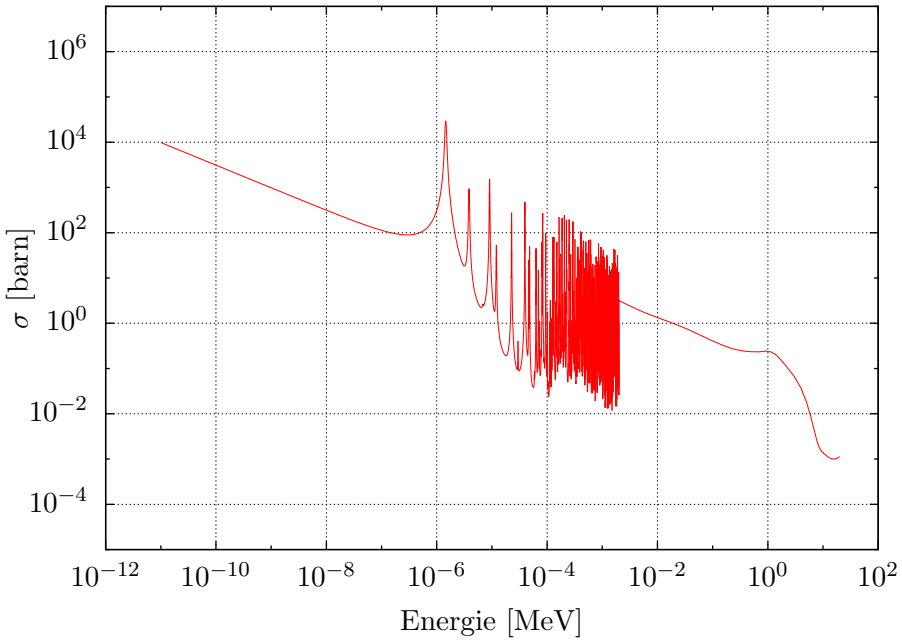

Figure 2: Radiative (n,γ) reaction in ^{115}In. Source: [NNDC]

Somatic: Acute radiation sickness in the organism including radiation disease or leukemia.

Genetic: Damages occuring only in the subsequent generations.

Teratogenic: Damages of the unborn child.

Therefore, one should exercise extreme caution when dealing with radioactive substances or x-rays. Incorporation of radioactive substances by ingestion with foods or inhalation via the respiratory tract are particularly dangerous. This results in a permanent radiation exposure due to the radionuclides, which last for the entire time the corresponding isotope or its radioactive daughter nuclei spends inside the body.

The human body is constantly exposed to radiation. As long as a certain dose level, which can differ for different organs, is not exceeded and the exposure remains within the natural range of ambient radiation levels, one doesn't expect a significant increase in radiation damages. Nonetheless, every exposure to radiation should be kept *as low as reasonably achievable* (*ALARA* principle), as the probability for the occurrence of radiation damage can accumulate over time.

To quantify the exposure to radiation and account for its effects, one uses different units of dose. The absorbed dose:

$$D = \frac{dE}{dm}$$

Table 1: Weighting factor for different types of radiation

radiation type	**q**
α radiation	20
β radiation	1-1,7
γ radiation	1
thermal neutrons	3
protons, fast neutrons	10

$$[D] = 1\,\text{Gray} = 1\,\text{Gy} = 1\frac{J}{kg} = 100\,\text{Rad} = 100\,\text{rd}$$

The equivalent dose is relevant for radiation protection. It accounts for the biological effect of different types of radiation by multiplying the absorbed dose with a weighting factor q:

$$D_q = q * D$$

$$[D_q] = 1\,\text{Sievert} = 1\,\text{Sv} = 100\,\text{rem} = 100\,\text{rd}$$

The mean effective yearly dose in Germany is roughly 4 mSv. It consists to 50% from natural and 50% from artificial radiation exposure (see Tab. 2).

The natural radiation exposure consists of the cosmic radiation, which permanently hits the earth, a certain level of radiation coming from radioactive elements in the environment, e.g. in building materials or uranium and thorium and their decay products (especially gaseous radon which is inhaled) in the soil (the levels of which can be significantly different from region to region) and naturally occurring radioactive isotopes ingested with food (especially ^{40}K). The exposure to cosmic radiation depends on the altitude, a single intercontinental flight adds a dose of about 0,2 mSv.

The artificial radiation exposure is dominated by medical diagnostics (x-ray imaging, CAT scans), which contributes 95% to the average artificial exposure (this of course varies strongly person by person). The remaining 5% are made up from the remnants of above ground nuclear weapons tests (and the use of nuclear weapons) and of the reactor catastrophe in Tchernobyl as well as running nuclear reactors and research facilities. Acute medical interventions can surpass the mean effective dose many times over. A radiotherapy for cancer e.g. adds a dose of 30-70 mSv.

3 Particle Detectors

3.1 Detection of Particles

The following sections give an overview about the different detectors used in nuclear, particle and astro physics. In this experiment, only Geiger counters will be used, however, it pays off to have a rough overview about the different detection methods.

Table 2: Mean effective yearly dose due to ionizing radiation per person in 2016. Source: [BFS]

natural exposure	**dose [mSv]**
cosmic radiation	0.3 (0.2 - 0.4)
direct terrestrial radiation	0.4 (0.1 - 0.8)
food	0.3
radon and its decay products	1.1 (1 - 6)
sum	2.1
artificial exposure	
nuclear facilities	< 0.01
fallout from nuclear weapons	< 0.01
Tchernobyl	< 0.01
science & technology	< 0.01
x-ray diagnostics	1.6
nuclear medicine	0,1
sum	< 2
total sum	< 4

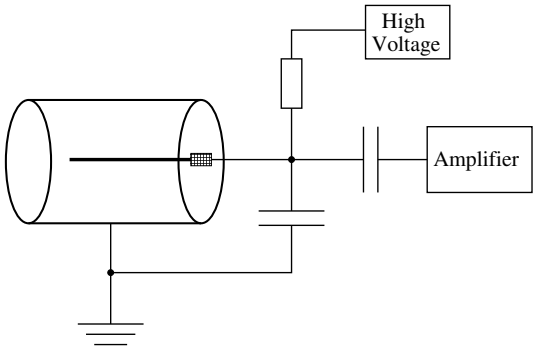

Figure 3: Working principle of a Geiger-Müller counter

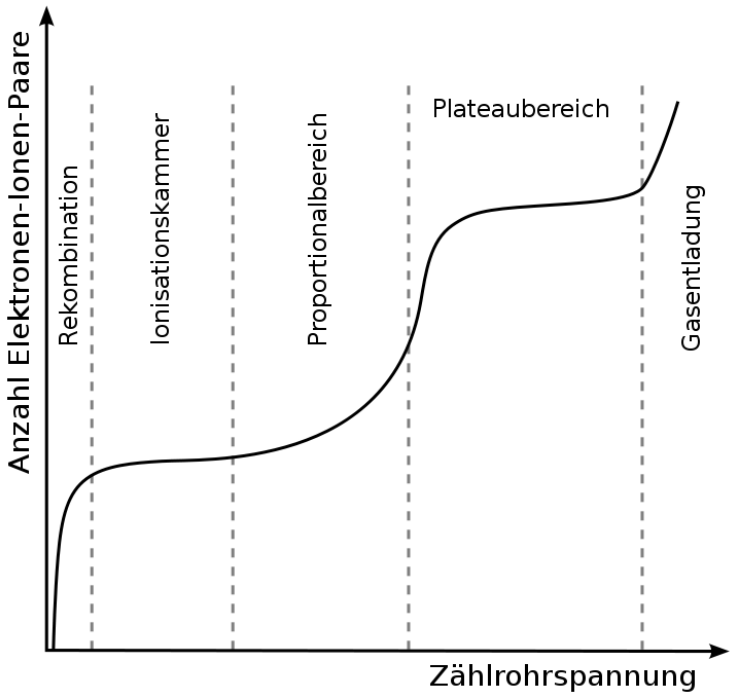

Figure 4: Current-voltage characteristic of a counting tube. Source: [WIKI]

3.1.1 Geiger counters

Fig. 3 shows a schematic, which is generally valid for all kinds of gaseous ionization detectors, e.g. ionization chambers, proportional or Geiger-Müller counters. In general gaseous ionization detectors are plate or cylinder capacitors filled with gas. The incoming radiation creates ion pairs in the gas volume, that are collected by the electrodes of the capacitor. One distinguishes two types of these detectors: ionization chambers and counting tubes. The former work with ambient pressure and rather low voltages of about 100 V, the latter are filled with noble gases at reduced pressure (≈ 100 mbar); in counting tubes, amplification occurs due to secondary ionization. Fig. 4 shows the I-V curve of a counting tube, it can be separated in 5 consecutive regions.

Recombination region: The circuit behaves like an ohmic resistor. The created ions travel very slowly towards the electrodes and the probability for them to recombine is quite high. This region is not useable for the detection of particles.

Ionization region: In this region, all created ions reach the electrodes before they can recombine. Different types of particles produce different currents.

Proportional counting region: The voltage now is large enough, that the primary charges can create additional ion pairs through secondary ionization. In

this region, the amplitude of the current pulse caused by the primary particle is proportional to its deposited energy on a logarithmic scale.

Geiger-Müller region: For even higher voltages, each ionizing particle triggers an avalanche of secondary electrons, independently of its energy. Hence, only the number of particles hitting the detector can be counted. However, one needs to take care that the avalanches don't lead to a permanent discharge of the tube. One therefore adds a quenching gas to the tube, whose molecules limit the number of electrons in the avalanche by impact dissociation.

A further increase in voltage finally leads to a permanent discharge of the counting tube, which can't even be stopped with the quench gas. In this region the tube again can't be used as a particle detector and can even be permanently damaged through continuous operation.

3.1.2 Cloud and bubble chambers

A cloud chamber contains a supersaturated vapor of water or alcohol. An ionizing particle, traversing the chamber, leaves behind a trail of ion pairs. These ion pairs act as condensation centers around which the liquid condensates in small droplets, resulting in a visible track along the path of the particle.

Bubble chambers are especially suitable for the detection of high energy particles with high ionizing power causing long tracks, that are easy to observe. The bubble chamber is filled with a liquid that is brought in a superheated state. Energy depositions of the particle along their trajectory cause a phase transition to the vapor phase, leading to bubbles in the detector. Neutrons only create bubbles when they cause a nuclear recoil.

After a particle passes through such a chamber, it will be illuminated and either a photo plate or a CCD camera will be exposed. The advantage of a bubble chamber compared to a cloud chamber is the significantly higher density increasing the energy loss per unit length, the density of the tracks and thus the energy resolution. The chambers are usually put in a magnetic field in order to determine the momentum of the particles from the curvature of the track.

The main advantage of these chambers consists of the ability to detect entire particle tracks, so momentum and energy as well as particle type can be determined with quite high precision. The essential disadvantage is the large dead time, since the chambers have to be regenerated first after each event. In addition triggering and analysis is quite difficult.

3.1.3 Scintillation detectors

Scintillators are materials, that emit light when a particle deposits energy. The particles to be detected excite the atoms or molecules through ionization. The excited atoms or molecules then de-excite via the emission of light, that can for example be detected with photomultipliers. There are solid, liquid and gaseous scintillators. Often used are inorganic crystals, doped with so-called activation centers, to enhance

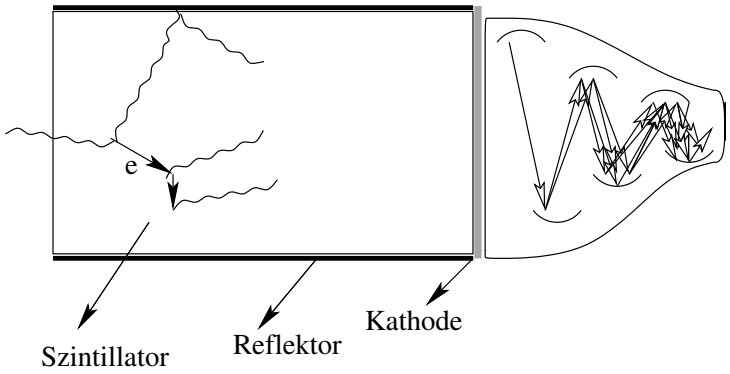

Figure 5: Scintillator with photomultiplier

the light yield. For each type of particle, the amount of emitted light is proportional to the deposited energy. Hence one can obtain an energy spectrum by histogramming the amplitudes of the signals, detected by the photomultipliers. A suitable scintillator can not only measure heavy charged particles, but also light or neutral particles.

3.1.4 Semiconductor detectors

A semiconductor detector is essentially a reverse biased p-n diode. By applying a voltage, one creates a region around the p-n junction, that is depleted of charge carriers. If an ionizing particle passes through this region, it creates a large amount of electron-hole pairs, that are separated by the electric field and collected at the electrodes. The working principle thus is analogous to the gaseous ionization detectors discussed above. However, since the density is much higher, particles with higher energy can be stopped in a much smaller volume. In addition, one only needs about 1 eV to create an electron-hole pair. This leads to a much better energy resolution. Finally, the dead time is much lower, on the order of 10 to 100 ns.

3.1.5 Cherenkov detectors

Cherenkov radiation was discovered in 1934 by the Russian physicist Pawel Alexeyewvich Cherenkov. It arises when highly energetic charged particles travel through a dielectric medium at a speed higher than the speed of light in that medium. It propagates as a conical wave front, similar to the sonic boom of a supersonic aircraft. The spectrum of Cherenkov radiation, which is coherent and linearly polarized, is partly visible.

From the opening angle of the light cone, one can determine the velocity of the particle:

$$\cos\theta = \frac{\frac{c}{n}}{v} = \frac{c}{n \cdot v} \tag{10}$$

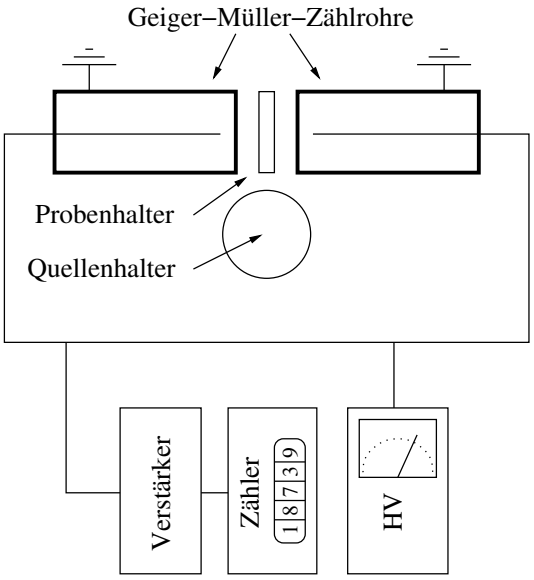

Figure 6: Schematic setup.

where θ is half the opening angle and n is the refractive index of the medium. Cherenkov detectors are very fast and have low dead time.

4 Experimental Procedure and Lab Report

4.1 Experimental Procedure

To prepare for this experiment and the lab report, you should prepare answers to the questions in this instruction manual and study the literature.

On the day of the experiment, the activation measurements using silver and indium samples are performed and the background radiation, one needs to consider when analysing the decay curves, is determined. The measurements and analysis are performed with the help of a computer.

The analysis can be performed either on the same day on the spot using the prepared programs or later at home. The used software packages are available for free. If the participants have an activated version of `Cassylab` as well as `gnuplot` installed on a laptop, this can also be used to acquire the data.

4.1.1 Calibrating the Geiger-Müller counters

The Geiger-Müller counters register the radiation emitted in the radioactive decays. A mica window protects the entry hole of the counting tubes. The counting tubes are mounted such, that the activated samples can be fixed in a holder between the entry windows. **Caution**, the mica windows are sensitive!

To choose a suitable working voltage for the following activity measurements, the location of the plateau in the I-V curve has to be determined for both Geiger-

Müller counters individually. The working voltage should lie in the first third of the plateau. To determine the plateau, the counting rate N is plotted versus the applied voltage U (including statistical error). To get a higher counting rate for the calibration measurements, a weak ^{60}Co source is mounted in the circular holder next to the sample holder.

The two counting tubes can be turned on and off separately with two switches. The voltage source for the tubes is a stabilized high voltage generator, that can deliver a continuously adjustable positive voltage between 0-500 V. The voltage, at which the tubes begin to work, is around 325 V, their plateaus extend over roughly 200 V having a slope of 5% per 100 V. In the event, that the count rates increase sharply at the end of the plateau, the voltage has to be turned down immediately!

Caution! When switching between the two counting tubes make sure that the voltage is turned off!

The count rate is determined by feeding the preamplified signals from the counting tubes to the scaler. We measure the number of registered decays N in a time interval with fixed length Δt as a function of the applied voltage U. For each tube, you should measure ten data points with a time interval of $\Delta t = 60s$.

The voltage interval ΔU, where the transition to the plateau region takes place, is rather narrow (≈ 2 V). Nevertheless, you should try to take a data point also in this region, where the count rate goes from zero to the plateau value.

4.1.2 Measurement of the background

To determine the correct half-lives and initial activities from the measured decay curves of the activated silver and indium samples, the background radiation level in the room has to be measured and subtracted from the measured count rates. The determination of the background level will be done with the help of the computer. For this, the preamplified signals are fed into the `Cassylab` with a BNC cable and read out by the PC, which is connected to the `Cassylab` via USB. The background level is determined in a 60 minute measurement using time intervals of $\Delta t = 60s$.

4.1.3 Handling the samples

The silver and indium samples consist of the naturally occurring metal, respectively. When handling the indium sample, one therefore has to be careful not to destroy the very soft metal (even softer than lead). There are two aluminum rods, that can hold the samples, to insert the samples safely into the neutron source. The samples themselves are only handled with pliers, especially when they are activated. The rods must not be touched on the end that was inserted into the source, particularly after long activation times. The rods too are significantly activated by the neutron source! Aluminum activated by a single capture decays to silicon with a half-life of $t_{1/2} = 2.25$ min, and therefore poses no real danger, at least in the beginning of the experiment.

4.1.4 Measuring the decay curves of silver

To determine the activity curves (activity as a function of time) for the activated silver isotopes ^{108}Ag and ^{110}Ag, the silver sample is irradiated consecutively with irradiation times t_a =5 s, 10 s, 20 s, 30 s, 45 s, 60 s, 90 s, 2 min, 5 min, 10 min and 15 min. This measurement is also performed using the computer. The silver sample is put into the neutron source for the time t_a. Afterwards, the activated sample has to be put in the holder in between the two Geiger-Müller counters and the measurement started as quickly as possible. The offset between the end of the irradiation and the start of the measurement has to be determined carefully using a stopwatch. The offset can be taken into account in the analysis to obtain the true initial activity. The total duration of all measurements should be 400 s, the interval should be $\Delta t = 5$ s.

4.1.5 Measuring the decay curves of indium

Finally, we will measure (again with the help of the computer) the decay curve of the activated indium sample to determine the half-life and initial activity. You have to determine the time, at which the sample is taken out of the neutron source and the activation time t_a. The decay curve of the indium sample is acquired with the same program and the same parameters as for the silver sample.

4.2 Lab Report

The lab report should - at least - cover the following points:

Theory

1. Radioactivity: α, β, γ decay
2. Activity
3. Differential equations for decay and activation
4. Saturation activity
5. Creation of free neutrons

Experiments

1. Description of the experimental procedure
2. Sketch and description of the neutron source
3. Sketch and working principle of the detector
4. Capture and decay reaction of all relevant isotopes, branching ratios for the different decay channels

Analysis

1. Graph of the plateau curve, justification for the chosen operating voltage
2. Formula and explanation for the determination of the half-lives
3. Graph and fit of the measured decay curves
4. Graph and fit of the determined half-lives as a function of activation time
5. Graph and fit of the initial activity as a function of activation time for each isotope (for silver also for the mixture of isotopes)
6. Discussion of the results

5 Questions for Preparation

- How does one "navigate" the table of nuclides and what do the different colors represent? What is the natural isotopic composition of silver and indium? Which of these isotopes are relevant for us?
- Which reactions happen in the AmBe-source, i.e. how are the free neutrons created? Why are neutrons with an energy of 1/40 eV called *thermal* neutrons? What is the energy of unmoderated neutrons?
- How does a quartz fiber dosimeter work? Why is it strictly necessary to shield the source with lead and paraffin?
- How large is the capture cross section σ for the (n,γ) reaction of ^{109}Ag in comparison to ^{107}Ag. The relevant numbers can be obtained from the website of the [NNDC]: One therefore chooses the `Nuclear Reaction Database` and here specifically the `ENDF` database. As "Target" one enters the two silver isotopes (separated by a semi-colon) and as "Reaction" `n,g` (i.e. radiative neutron capture). In the following window, one can choose between different data sets. One should use select one for each isotope coming from the same database (e.g. `ENDF/B-VII.0`). By clicking on `Plot` at the top, a graph will be created. What does this mean for the production of the different radioactive silver isotopes?
- How much energy is needed to create an electron-ion-pair in a usual Geiger-Müller counter? What kinds of radiation can be measured with the counter? What kind of radiation is emitted by the ^{60}Co source? What makes up the background radiation, measured with the counting tubes?
- What is the necessary counting rate resulting in a statistical error of only 1 %? Why does it make sense to measure in 5 s intervals instead of 1 s intervals, especially for short activation times? `Cassylab` automatically calculates rates in [1/s] from the individual 5 s measurements. What is the error on such a measurement, knowing only the rate?

6 Techniques and Programs

6.1 Statistics

Here, a short "overview" about relevant statistical methods in physics will be given. The measured data and fit results are expected to be quoted with statistical errors and error bars, including goodness of fit (χ^2/ndf, explained below).

Statistics deals with random processes. The physical processes observed in this experiment - radioactive decays - are random processes, hence they have to be described with statistical methods.

Consider a random variable X, e.g. the number of decays of a radioactive sample in a fixed time interval Δt. Then $P(x)$ is the probability to measure the value x (i.e. x decays). The expectation value of a measurement is given by

$$\begin{aligned} E[x] &= \int xP(x)\,dx \\ E[x] &= \sum x_i P(x_i) \\ E[f(x)] &= \int f(x)\,P(x)\,dx \end{aligned}$$

for continuously or discretely distributed random variables, respectively. The last line describes the expectation value of a function. The expectation value is sometimes also called the mean μ. The variance σ^2 is calculated by

$$\sigma^2 = E\left[(x_i - \mu)^2\right]$$

the standard deviation σ is obtained by taking the square root of the variance. Also relevant is the weighted mean:

$$\hat{\mu} = \frac{\sum \frac{x_i}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2}}$$

with error

$$\sigma = \sqrt{\frac{1}{\sum \frac{1}{\sigma_i^2}}}$$

From the multitude of possible probability distributions, mainly three distributions are important in physics: the binomial, the Poisson and the Gaussian distribution. The binomial distribution describes the outcome of repeated events with two distinct possible outcomes (e.g. yes/no questions), like the repeated flip of a two-sided coin. The probability to obtain a given result p exactly r times for N trials is given by

$$P(r) = \frac{N!}{r!\,(N-r)!} p^r\,(1-p)^{N-r}$$

The mean and variance are given by:

$$\begin{aligned} \mu &= \sum rP(r) = Np \\ \sigma^2 &= Np\,(1-p) \end{aligned}$$

The Poisson distribution follows from the binomial distribution in the limit $N \to \infty$ and $p \to 0$:

$$P(r) = \frac{\mu^r e^{-\mu}}{r!}$$

Like the binomial distribution, the Poisson distribution is discrete. It describes the probability for a certain number of events being observed in a given interval if these events occur with a fixed rate and the individual events are independent of each other. This is exactly the case for radioactive decays (and other particle reactions). The variance is given by:

$$\sigma^2 = \mu$$

Correspondingly, the standard deviation of Poisson distributed count rates is

$$\sigma = \sqrt{x}$$

.

The Gaussian (or normal) distribution is used, for example, to treat the systematical uncertainties of measured quantities. It is very useful and widely used in physics because of the central limit theorem, which states that for a sufficiently large number of observations, the sample mean is normally distributed, more or less independently of the underlying distributions. E.g. both the binomial and the Poisson distributions converge to the normal distribution for large N. It is given by:

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ and σ^2 correspond to the mean and variance, respectively.

Another relevant distribution is the χ^2 distribution, which can be used to judge the quality of a fit of a theoretical model to experimental data.

$$\chi^2 = \sum \left(\frac{x_i - \mu_i}{\sigma_i} \right)^2$$

where x_i is the experimental value, μ_i the corresponding value calculated from the model and σ_i the uncertainty of the measurement. If the model contains m parameters and the measurement consists of n data points, the number of degrees of freedom result in

$$f = n - m$$

For a good fit (i.e., if the model is a good description of the data), the χ^2 value should approximately equal the number of degrees of freedom (assuming the uncertainty of the measured data points is modeled correctly and is normally distributed), or the so-called reduced χ^2 - $\chi^2_{red} = \frac{\chi^2}{f}$ - should be close to 1.

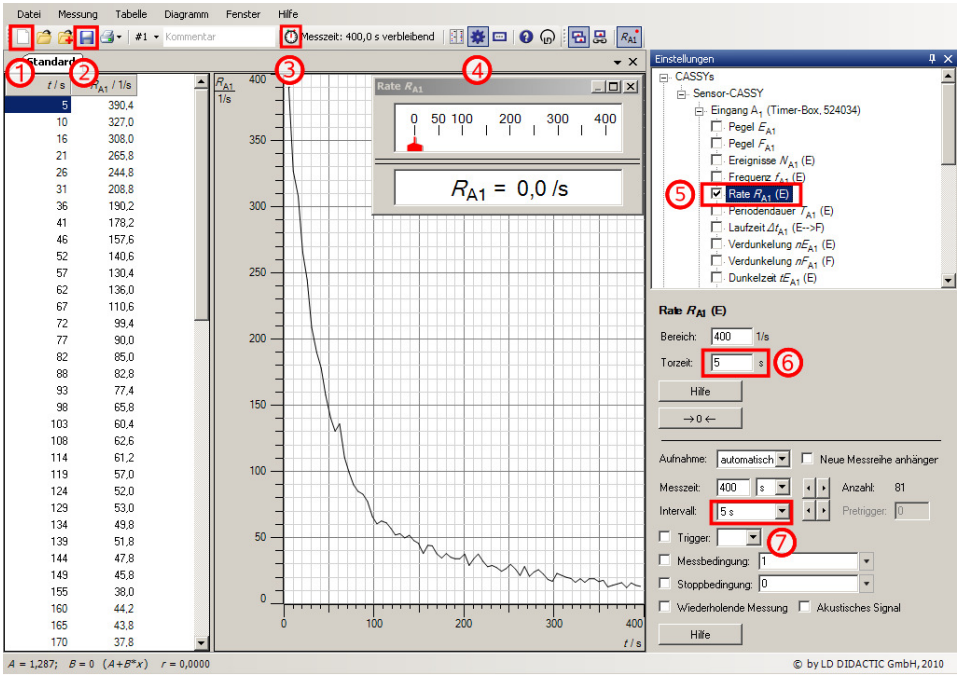

Figure 7: `Cassylab` window

6.2 Cassylab

`Cassylab` is a product of LD Didactic, which allows to process the signals from the counting tubes digitally. This happens with the program of the same name `CASSY Lab`. The program can be started through the link on the desktop. After starting the program, it is a good idea to familiarize oneself with the settings, which can be accessed by clicking on the cog wheel symbol ④ in the toolbar. Make sure that a rate measurement ⑤ is selected and the dead time ⑥ and measurement interval ⑦ are both set to 5 seconds. It is also helpful to set the unit of the x-axis to time.

A measurement is started or stopped by pressing the `F9` key or clicking on the stopwatch ③ in the toolbar. The measured data can be saved by clicking on the disk symbol ②. When saving the data for the use in Gnuplot, one should save the data as `.txt` files in addition to the `CASSY Lab` format `.labx`. A new measurement is created by clicking on the empty paper symbol ① in the toolbar.

`CASSY Lab` also allows the fitting of functions to the measured data. For this, one performs a right-click in the graph window an selects the option “Anpassung durchführen” (perform a fit) and further “Freie Anpassung” (free fit). Alternatively one can use the shortcut `Alt-f`. A window appears, where an arbitrary function can be defined. In our case this will be mostly `A*exp(B*x)+C*exp(D*X)+U`, where `A`, `B`, `C` and `D` are the free parameters of the fit function, in this case a sum of two exponential functions, and `U` is the background radiation level, which was determined in a separate measurement. The parameters can be fixed to a certain value or constrained to an interval, which can be set by clicking and drawing with the

mouse after clicking on "Weiter mit Bereich markieren" (continue to mark interval). Finally, the resulting fit parameters are displayed in the lower bar.

6.3 Gnuplot

Gnuplot is an application to plot data and mathematical functions. It can be operated with a command prompt or with scripts and offers a good mix between a rich feature set and user-friendliness. Compared to some other (sometimes more advanced) programs (like MATLAB, Origin, IDL, ...) it has the advantage of being light-weight and free to use. You can plot and fit all data from this lab course with `gnuplot`, however, you are of course free to use different software packages, as long as the results fulfill the requirements.

Gnuplot can be downloaded from www.gnuplot.info. On the web page there is also a detailed manual. The most important features will be presented in the following.

6.3.1 Using gnuplot

Gnuplot can be used in two ways:

Command prompt: After starting `gnuplot`, there is a text window, where one can enter the different commands one after the other. This mode is useful to quickly plot something. By entering `cd 'c:\data'` one can navigate to the respective folder. Gnuplot can be terminated with the command `quit`.

Script: A text file can be created, containing the series of `gnuplot` commands needed to perform a certain task. This file can then be opened and executed with `gnuplot`, e.g. by entering `gnuplot macro-file` in the shell. On a Windows system, one can choose a specific file ending (e.g. `.gp` or `.gnu`) and specify in the context menu, that files with this ending should always be opened with `gnuplot`. Then one can execute the script by double-clicking on the corresponding file. Alternatively, one can navigate to the folder containing the script using the command prompt and simply enter the name of the file containing the script.

6.3.2 Creating a plot

To draw a function or a data set, one needs the following command:
`plot {ranges} ⟨function⟩ | {'datafile' {using ⟨columns⟩}}`

{ranges} are given in square brackets, e.g. `[-pi:pi]`

⟨**function**⟩ mathematical function, e.g. $abs(x)$, $\cos(x)$, $\exp(x)$, `x*2` ($2x$), `x**2` (x^2), etc.

{style} drawing style, e.g. `lines`, `points`, `linespoints`, `impulses`, `dots`, `errorbars`, etc. The option `errorbars` calls for a corresponding specification of ⟨columns⟩, e.g. x:y:ydelta or x:y:ylow:yhigh

⟨**columns**⟩ specifies, which data columns should be used for what, e.g. ⟨ycol⟩ or ⟨xcol⟩:⟨ycol⟩:⟨ydelta⟩. The columns are indexed by numbers starting from 1, except if a function is applied to the values e.g. 1:(exp($2))

Examples:

- `plot [-2:2] 2*x**2`
- `plot 'datafile.txt' using 1:2:(sqrt($2)) title 'Activation of Ag' with errorbars`

6.3.3 Fitting a function

Data are nice, but a function fitted to the data is even nicer. When fitting, the parameters of a function are changed such, that the residuals of the function with respect to the data (or the χ^2) is minimized. If a `fit` command is issued in `gnuplot`, a file `fit.log` containing the best-fit values and their statistical uncertainties is created automatically. The best-fit values can be found under "Final set of parameters". Directly above that, there should be the values for χ^2, number of degrees of freedom, etc. After each new fit, the new set of parameters is appended to the `fit.log` file. In many cases, `gnuplot` requires starting values for the parameters of the fit. They need to be guessed from the experimental data and should at least be in the correct order of magnitude.
`fit {[xrange] {[yrange]}} ⟨function⟩ 'datafile' via ⟨var1⟩{,⟨var2⟩,...}`

range range of values, where the function should be fitted to the data

⟨**function**⟩ function to be fit to the data (as above), can be pre-defined

via starting values for the parameters of the fit function

Example:

- ```
  f(x)=a*x+b
  a=10
  b=0.1
  fit f(x) 'datafile.txt' using 1:2 via a,b
  ```

6.3.4 set-show commands

With these commands on can specify (`set`) or display (`show`) specific plotting options. Here is a short list of useful options:

grid draws a grid in the plot

logscale ⟨**axis**⟩ switches the specified axis to a logarithmic scale

⟨**axis**⟩**label 'text'** labels the specified axis with "text"

⟨**axis**⟩**range {range}** display range of the specified axis (in square brackets)
  ```

term ⟨**terminal**⟩ specifies an output terminal. If none is specified, the plot will be shown on the display. One can, however, also save the plot to a file, e.g. using the command `set term postscript landscape color`. If one changes the terminal, one has to draw the graph again using `replot`. Other possible terminals are e.g. `jpeg`, `gif`

output 'filename' specifies the filename, under which the plot should be saved

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