Exercise 1 (7 points)

Show that the resolution calculus is consistent, i.e. show that $\nvdash_{\text{Res}} \square$.

(a) Without using soundness, i.e. without using $\Gamma \vdash_{\text{Res}} S \implies \Gamma \vDash S$. (6 points)

   Hint: Prove the following statement by an induction on the structure of derivations:

   If $\underset{X \vdash Y}{\overset{\mathscr{D}}{}}$ is a derivation with $Hyp\left(\underset{X \vdash Y}{\overset{\mathscr{D}}{}}\right) = \emptyset$, then $X \neq \emptyset$ and $Y \neq \emptyset$.

(b) By using soundness. (1 point)

Exercise 2 (3 points)

Is the rule

$$\frac{X_1 \vdash Y_1, A, B \quad A, B, X_2 \vdash Y_2}{X_1, X_2 \vdash Y_1, Y_2}$$

sound? Explain your answer.