Profile functions the adaptive way

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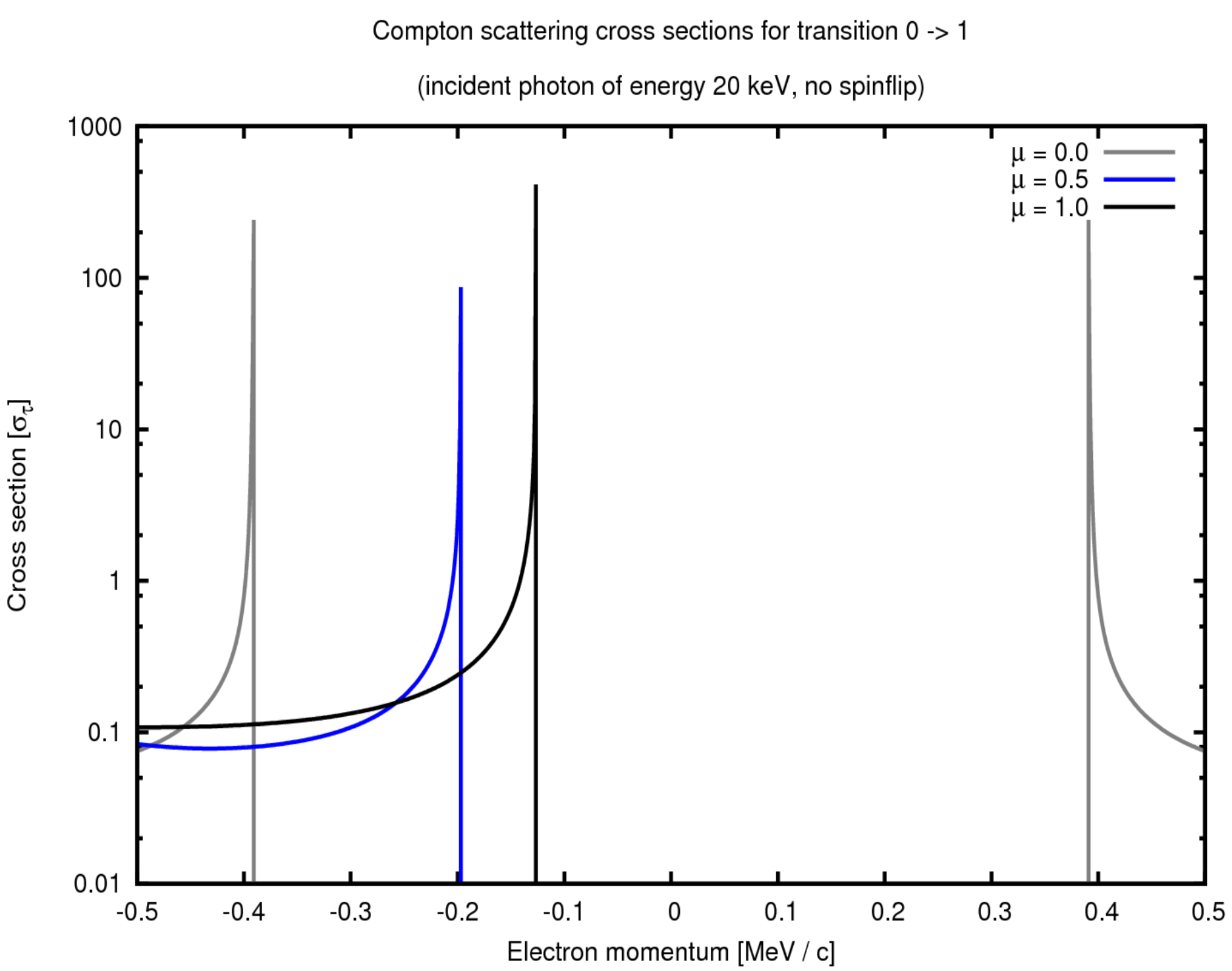
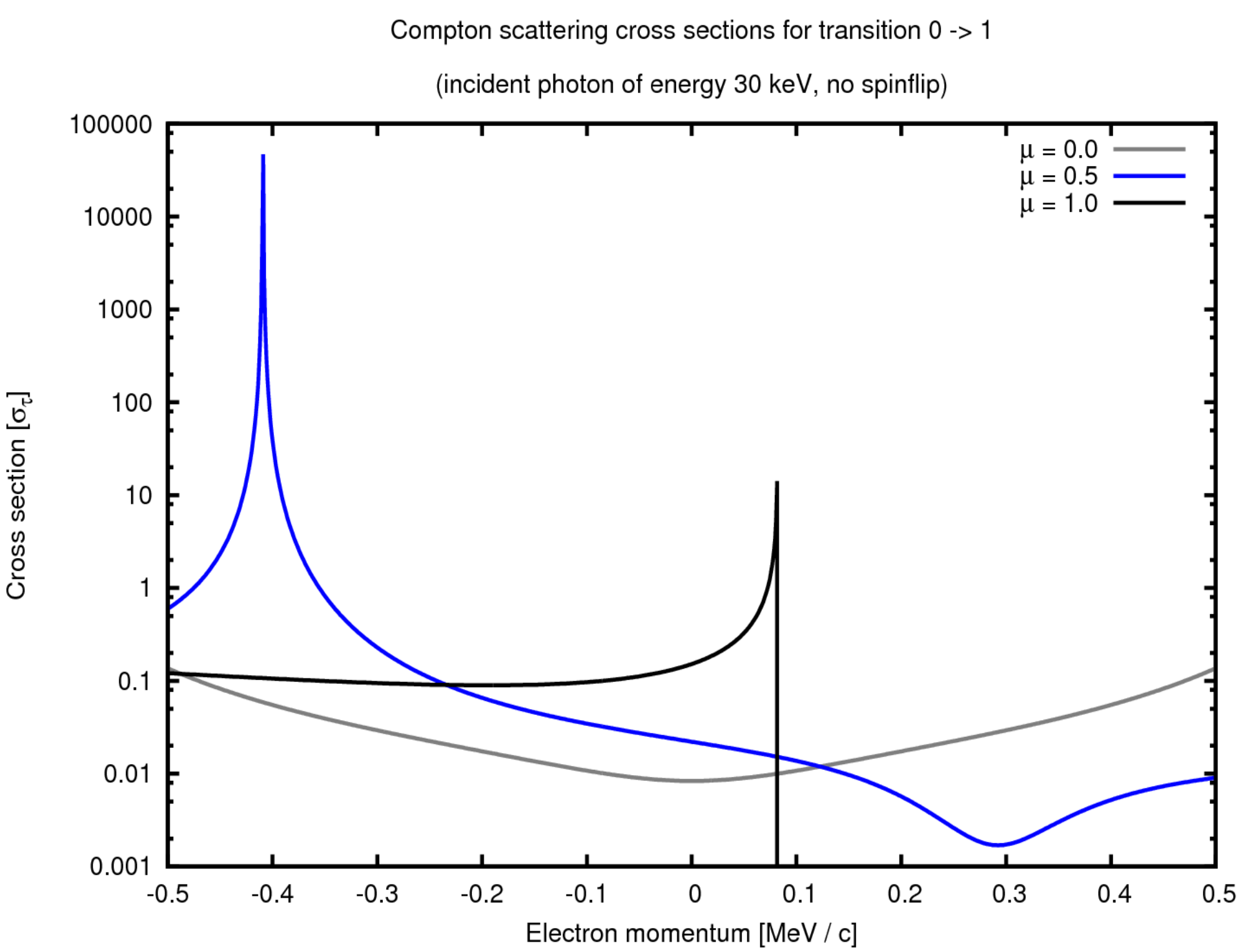
Outline

- Compton cross sections for cyclotron scattering
- Profile functions
- Adaptive calculation of profile tables
- Implementation into the CycloMC code
- Cyclotron lines

- The Compton(...) function from CycloMC is used and depends on the following parameter:
  - B: Magnetic field (in units of critical field)
  - p: Electron momentum parallel to magnetic field
  - k: Photon energy
  - θ: Propagation angle with respect to magnetic field
  - n_i, n_f: Initial and final landau level
  - s_i, s_f: Initial and final electron spin
- Four cross sections are returned for different polarization modes
- At the moment averaging over polarization modes
- For profile calculation spin and landau states are summed up

Compton cross sections


Compton scattering cross sections for transition 0 -> 1
(incident photon of energy 20 keV, no spinflip)
μ = 0.0
μ = 0.5
μ = 1.0
Cross section [σ_c]
1000
100
10
1
0.1
0.01
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
Electron momentum [MeV / c]


Compton scattering cross sections for transition 0 -> 1
(incident photon of energy 30 keV, no spinflip)
μ = 0.0
μ = 0.5
μ = 1.0
Cross section [στ]
100000
10000
1000
100
10
1
0.1
0.01
0.001
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
Electron momentum [MeV / c]

Mean free path: $$MFP(\mu,k) = \frac{1}{\sigma_{ave}(\mu,k)}$$

Profile function: $$\sigma_{ave}(\mu,k) = \int_{-\infty}^{\infty} f(p)\cdot\sigma^{lab}_{\mu,k}(p)\,dp$$

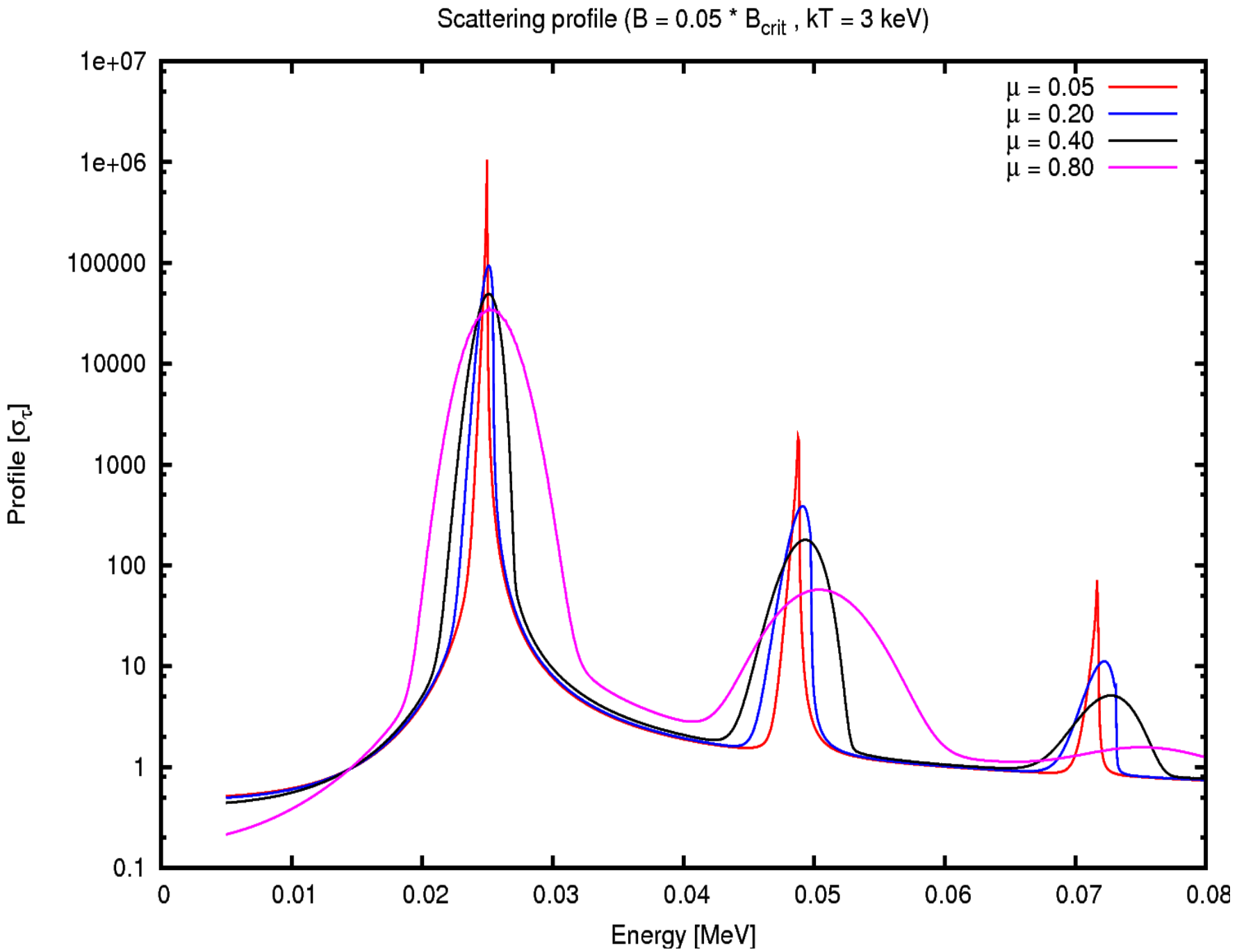
Cross sections in lab frame: $$\sigma^{lab}_{\mu,k}(p) = (1-\mu\beta)\cdot\sigma^{rf}_{\mu,k}(p)$$

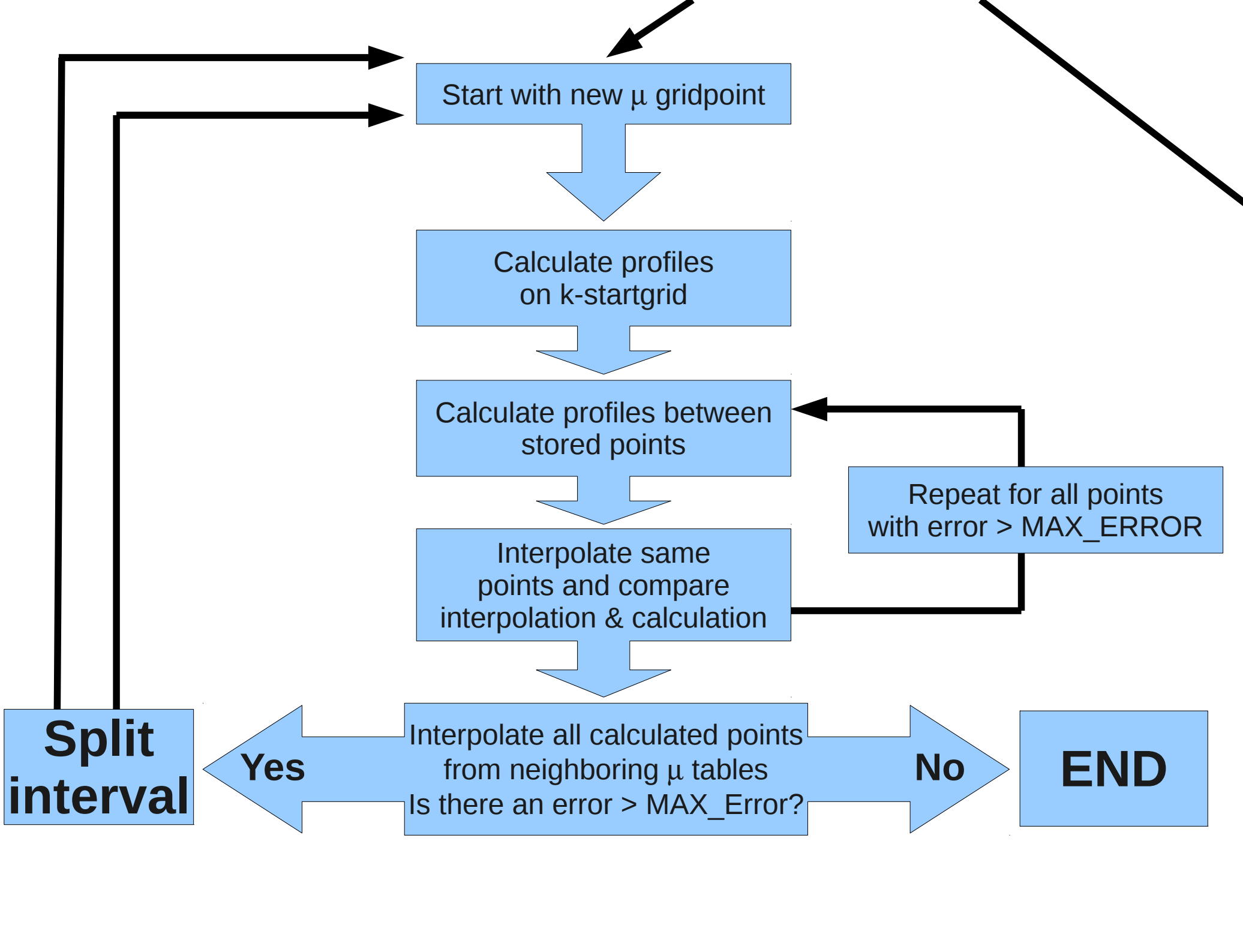
Electron distribution: $$f(p) = N\cdot e^{-\frac{\sqrt{m^2+p^2}-m}{T}}$$

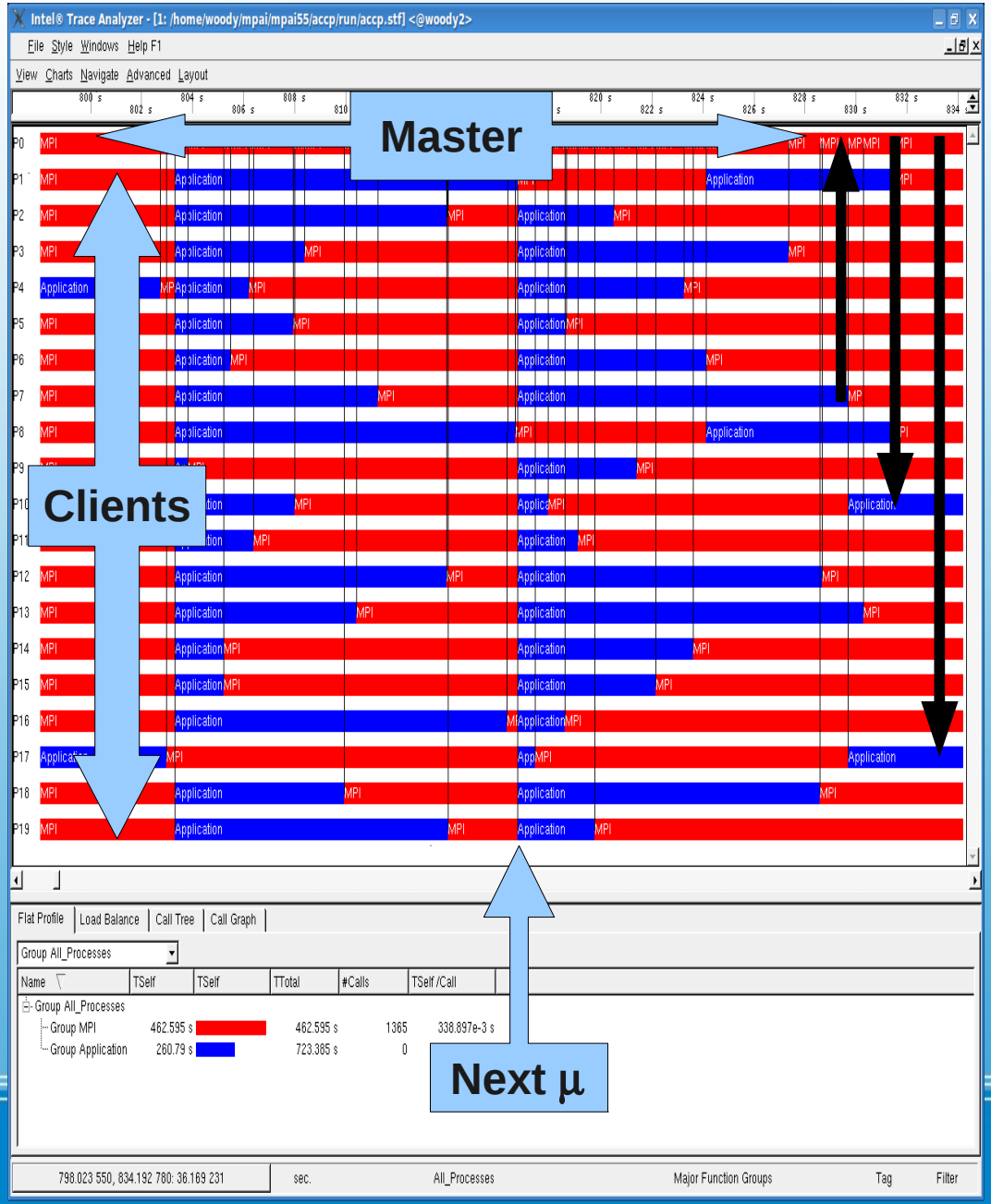
Profile functions

- Adaptive grid refinement:
  - Angle to magnetic field: $\mu = \cos(\theta)$
  - Energy of incoming photon: k
  - Electron momentum: p
- Tables are stored in FITS files
- Multiprocessing support using MPI

The program

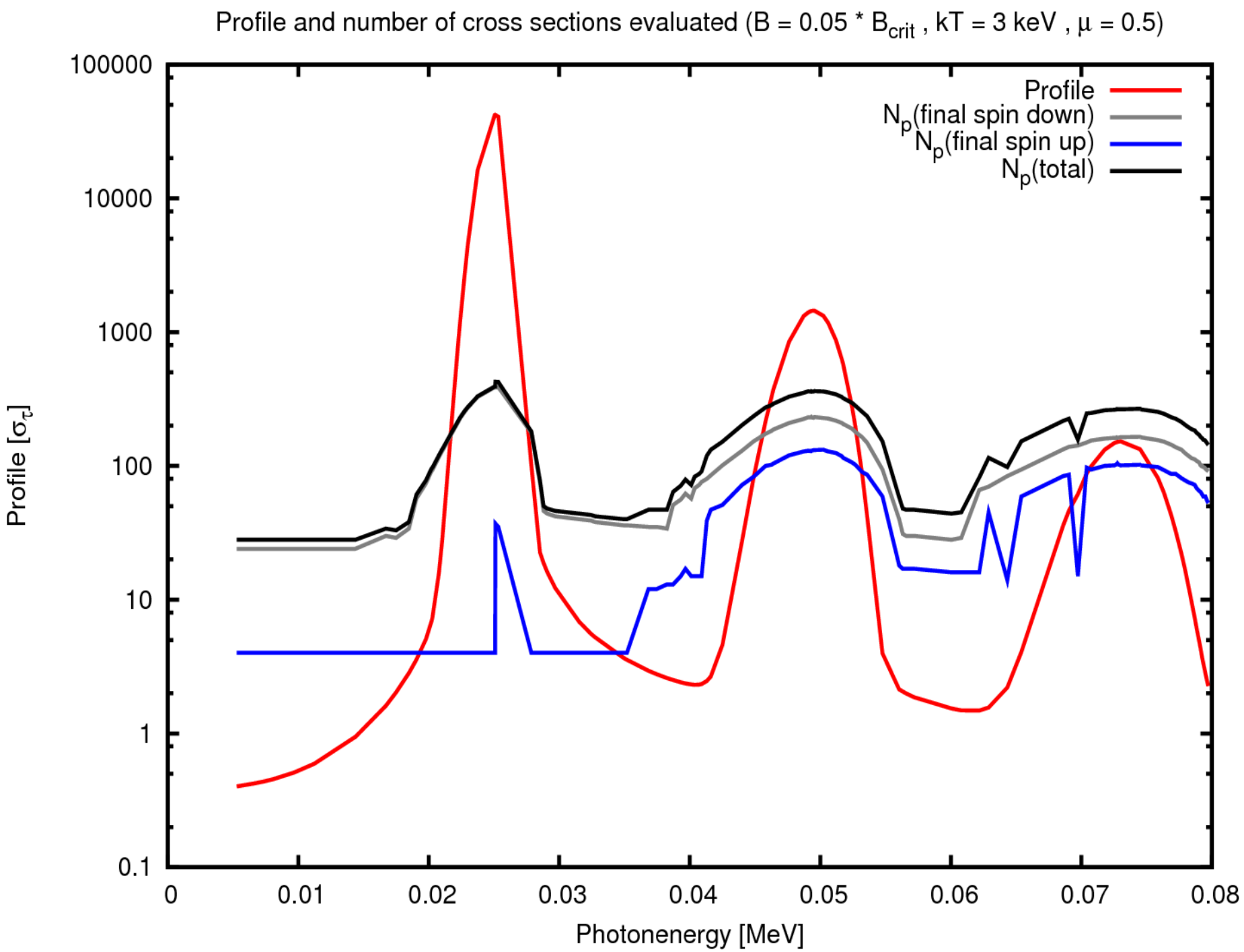

Scattering profile (B = 0.05 * B_crit , kT = 3 keV)
μ = 0.05
μ = 0.20
μ = 0.40
μ = 0.80
Profile [σ_T]
Energy [MeV]
1e+07
1e+06
100000
10000
1000
100
10
1
0.1
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08

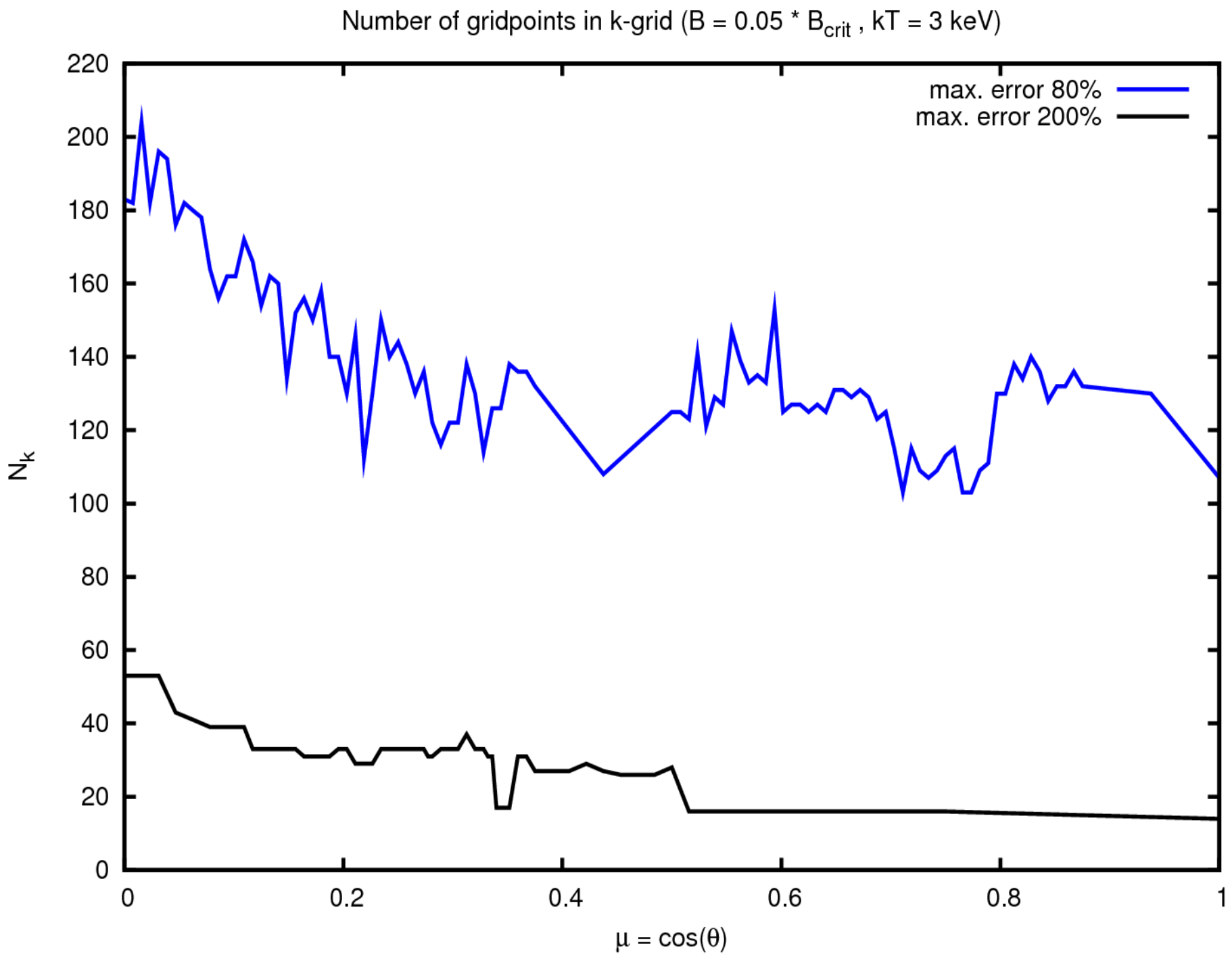

Start with new μ gridpoint
Calculate profiles on k-startgrid
Calculate profiles between stored points
Repeat for all points with error > MAX_ERROR
Interpolate same points and compare interpolation & calculation
Interpolate all calculated points from neighboring μ tables Is there an error > MAX_Error?
Yes
No
Split interval
END

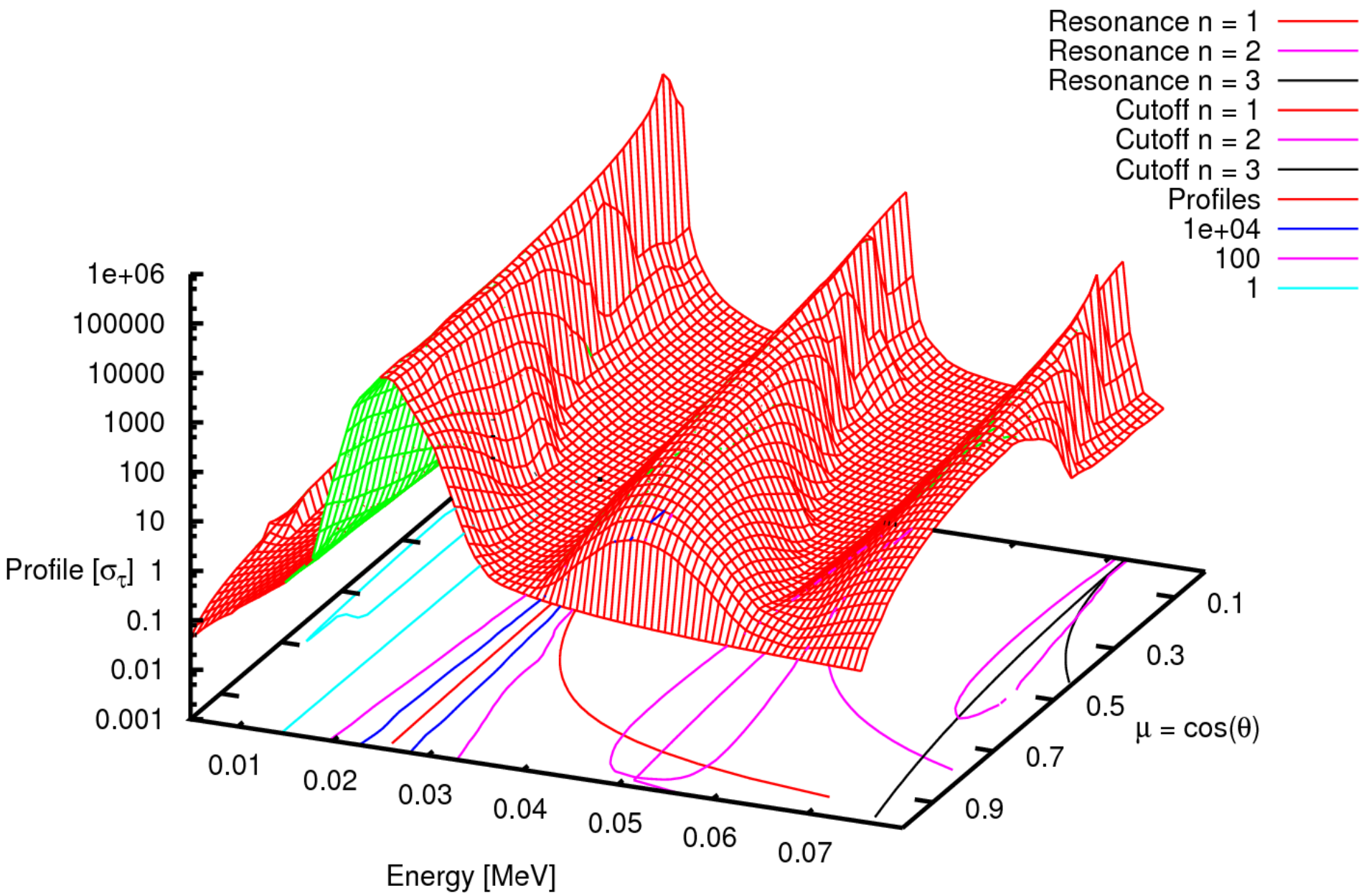


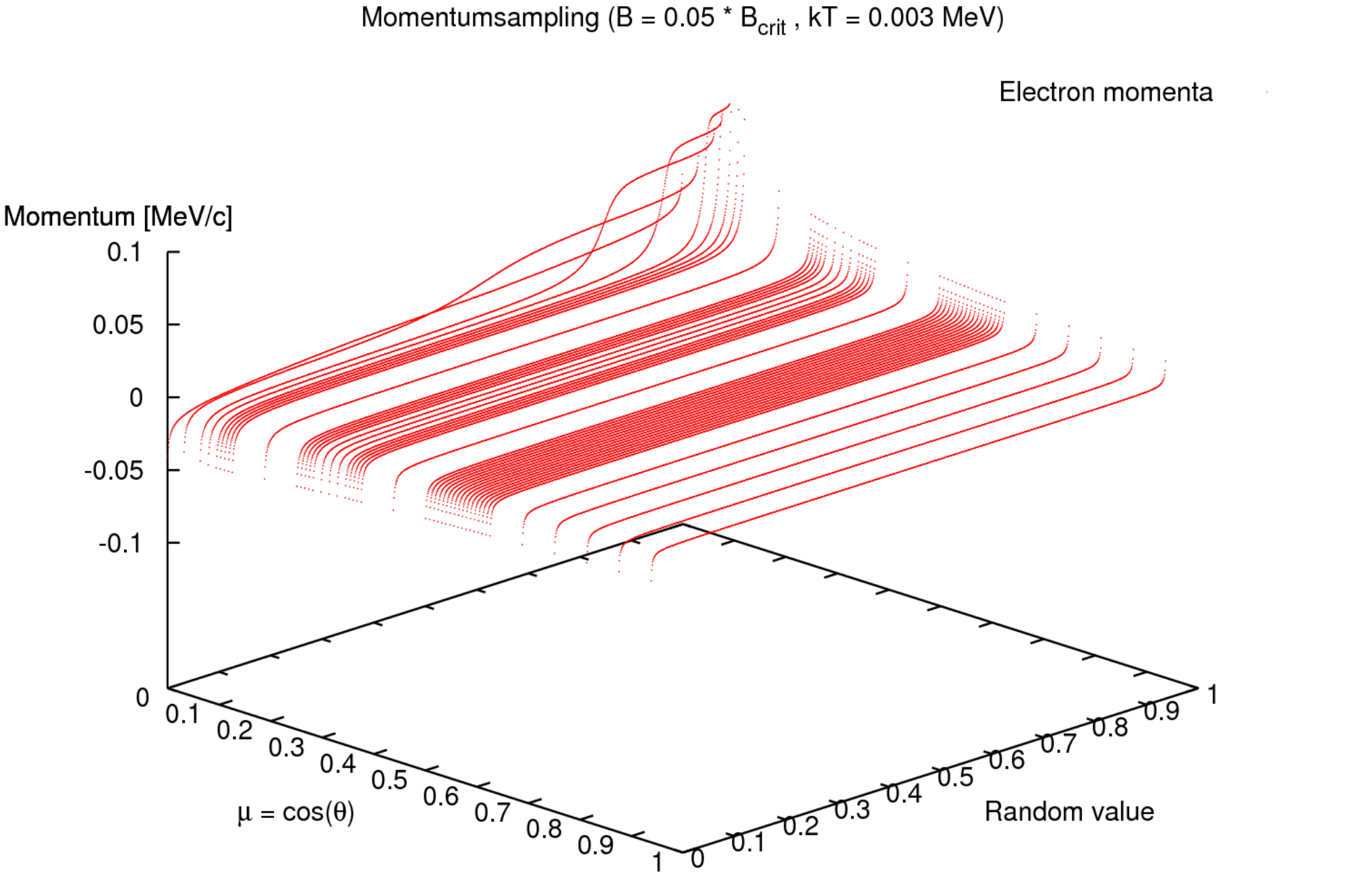
- Master / Worker scheme
- Message passing via MPI
- Master:
  - Interpolation
  - Adaptive refinement of μ, k
  - Data reduction and storage
- Client:
  - Adaptive simpson integration
- Scalability of about 50%

Multiprocessing support


Profile and number of cross sections evaluated (B = 0.05 * B_crit , kT = 3 keV , μ = 0.5)
Profile
N_p(final spin down)
N_p(final spin up)
N_p(total)
Profile [σ_τ]
Photonenergy [MeV]
100000
10000
1000
100
10
1
0.1
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08


Number of gridpoints in k-grid (B = 0.05 * B_crit , kT = 3 keV)
max. error 80%
max. error 200%
N_k
μ = cos(θ)
0
20
40
60
80
100
120
140
160
180
200
220
0
0.2
0.4
0.6
0.8
1


Compton scattering profiles (B = 0.05 * B_crit , kT = 3 keV)
Resonance n = 1
Resonance n = 2
Resonance n = 3
Cutoff n = 1
Cutoff n = 2
Cutoff n = 3
Profiles
1e+04
100
1
1e+06
100000
10000
1000
100
10
1
0.1
0.01
0.001
Profile [σ_τ]
0.01
0.02
0.03
0.04
0.05
0.06
0.07
Energy [MeV]
0.1
0.3
0.5
0.7
0.9
μ = cos(θ)


Momentumsampling (B = 0.05 * B_crit , kT = 0.003 MeV)
Electron momenta
Momentum [MeV/c]
0.1
0.05
0
-0.05
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
μ = cos(θ)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Random value

Mean free path

- Inverse profiles
- Photon propagation

Momentum sampling

- Rn $\Leftrightarrow$ $\dfrac{\int_{-\infty}^{p} f(p)\cdot\sigma_{\mu,k}^{lab}(p)\,dp}{\int_{-\infty}^{\infty} f(p)\cdot\sigma_{\mu,k}^{lab}(p)\,dp}$

Profile usage in CycloMC

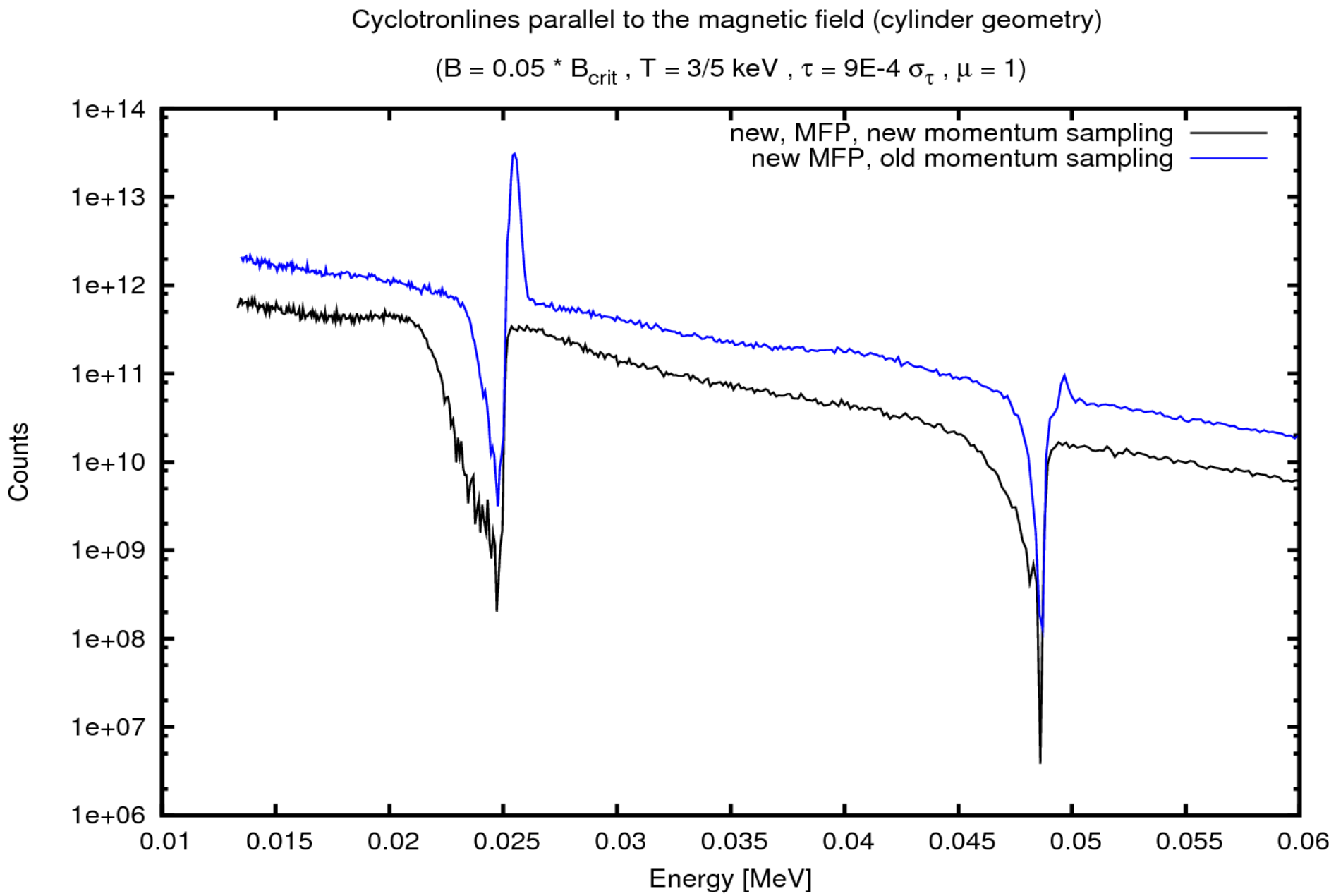
- libaccp.a: Library including functions for:
  - Initialization / Finalization
  - Interpolation of profiles from FITS tables (lin. / log.)
  - Momentum sampling from FITS tables
  - Already includes libcfitsio and libpil

- 2-3h for spectra with 10^6 photons
- Old decay rates are still in use

- Less wings in spectra
- Broader lines, third resonance disappears

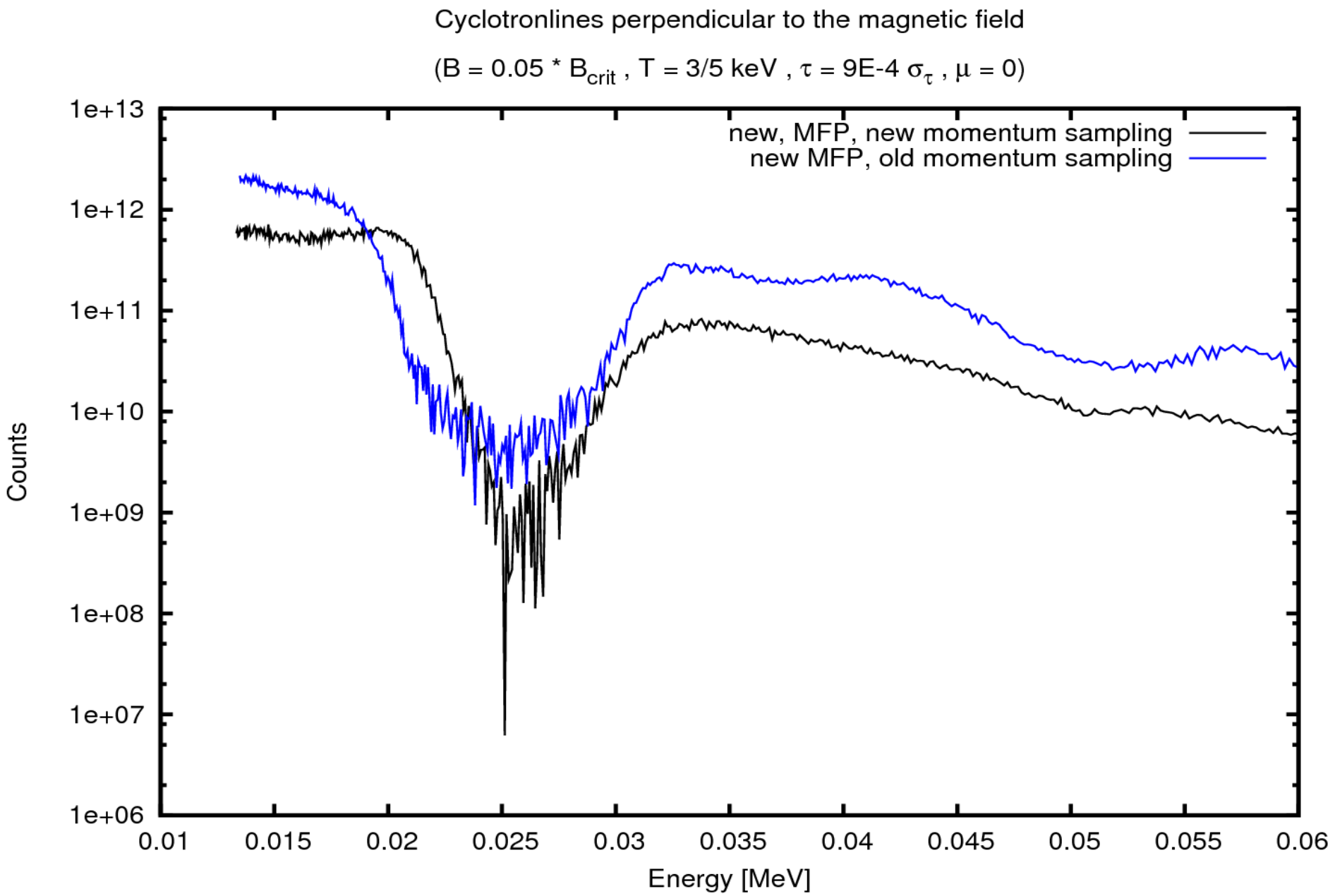
CycloMC - Implementation

Viewing angle of 0° to B-Field:



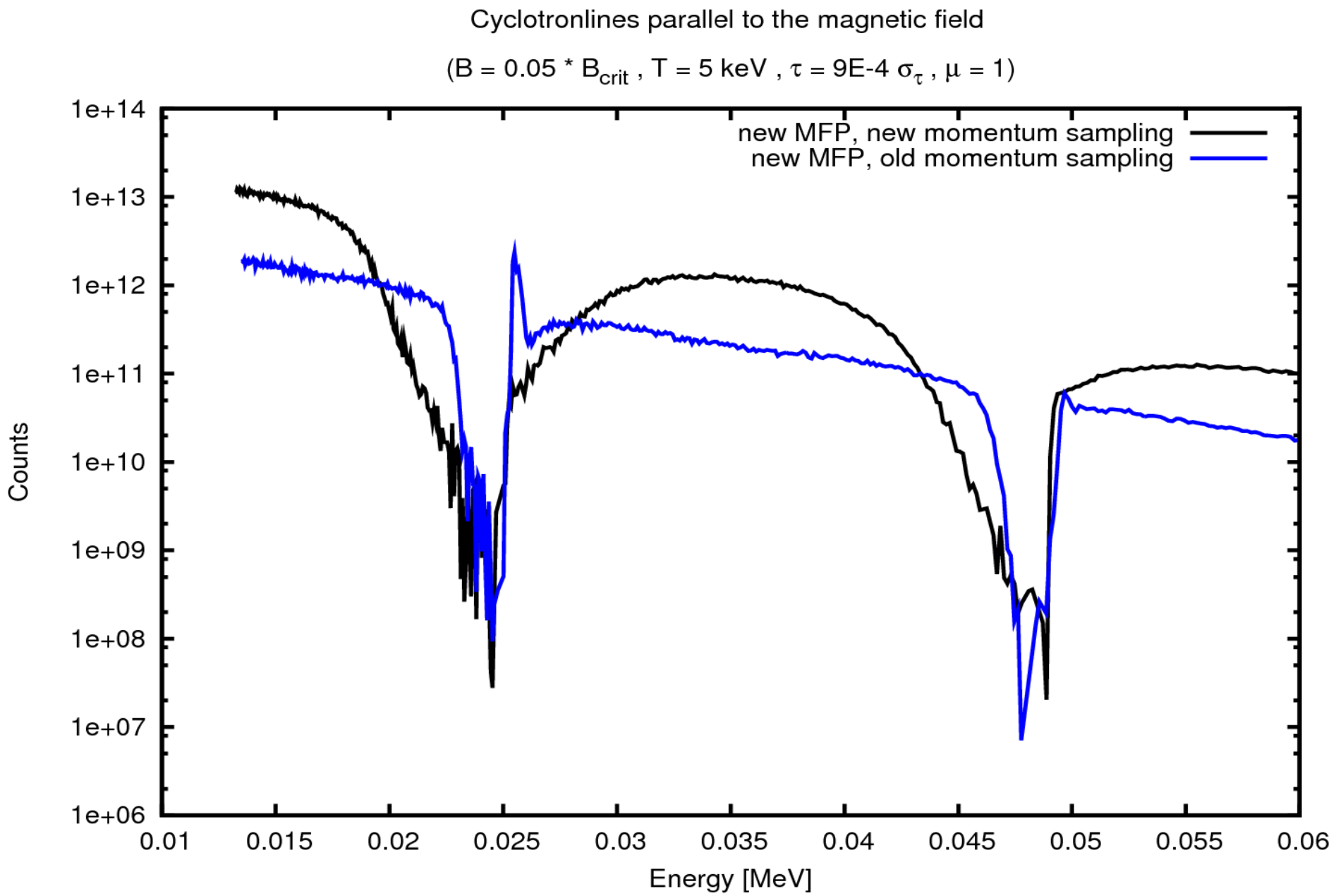
Cylinder geometry

Viewing angle of 90° to B-Field:



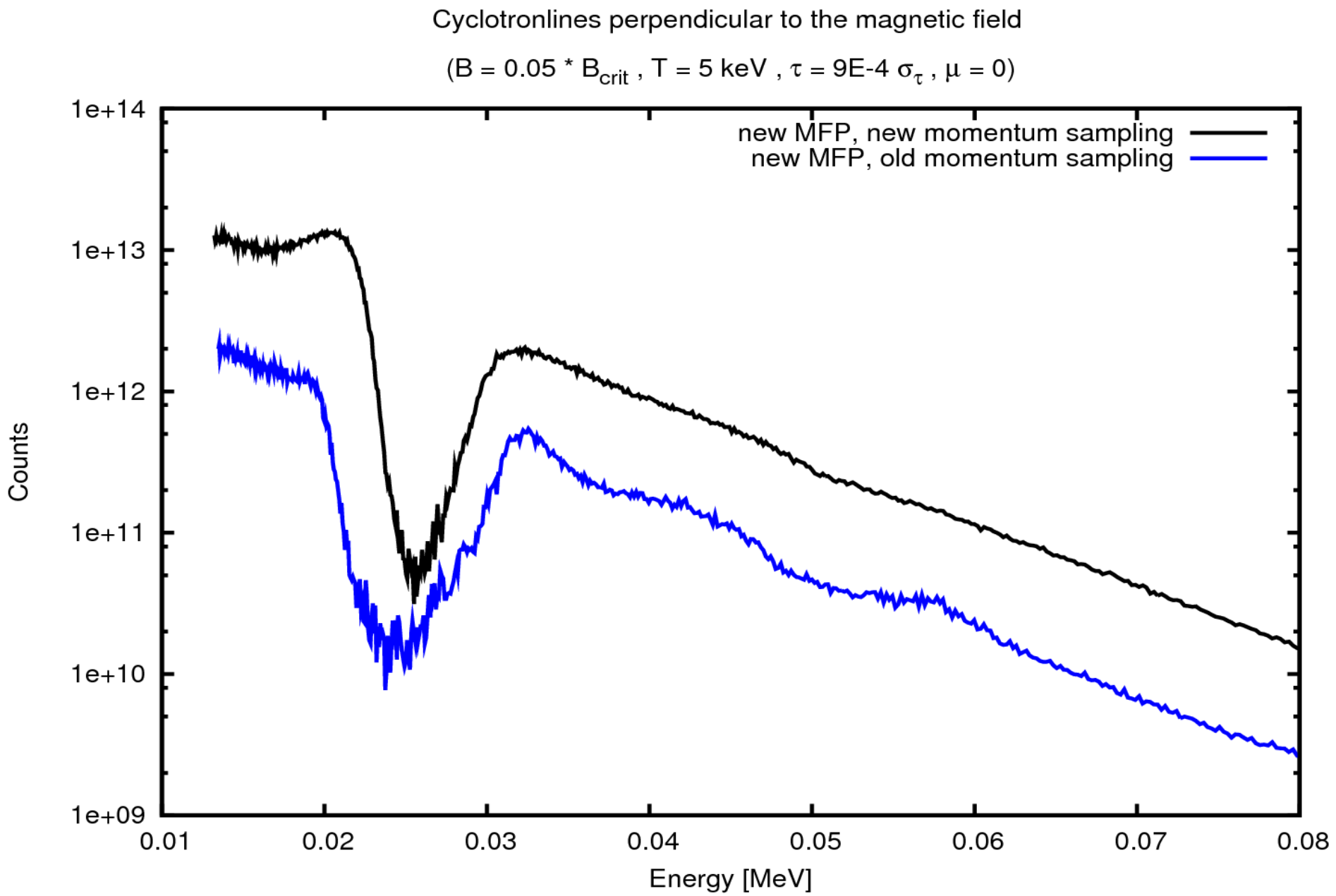
Cylinder geometry

Viewing angle of 0° to B-Field:

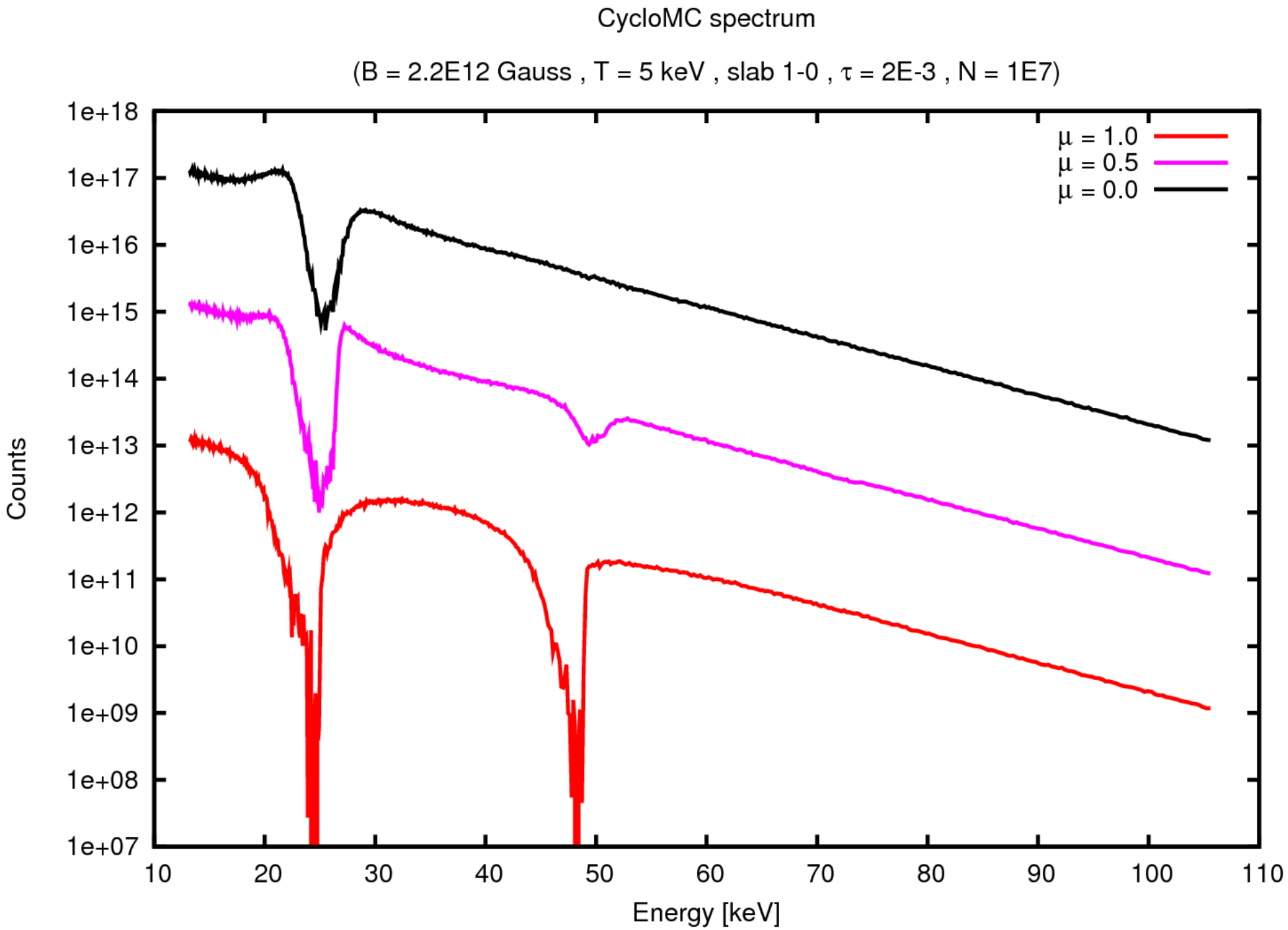


Slab 1-0 geometry

Viewing angle of 90° to B-Field:



Slab 1-0 geometry


CycloMC spectrum
(B = 2.2E12 Gauss , T = 5 keV , slab 1-0 , τ = 2E-3 , N = 1E7)
μ = 1.0
μ = 0.5
μ = 0.0
Counts
Energy [keV]
1e+18
1e+17
1e+16
1e+15
1e+14
1e+13
1e+12
1e+11
1e+10
1e+09
1e+08
1e+07
10
20
30
40
50
60
70
80
90
100
110

- Program adaptively creates profile tables
- Library is used to interpolate from those tables
- Implementation in CycloMC:
  - Momentum sampling strongly influences spectra, especially the line wings
  - Broader lines without strong wings
  - Higher harmonic CRSF less significant for low optical depths and magnetic fields
- TODO: Tables for larger parameter grids (also B, T), Greensfunctions, XSPEC model

Summary