to do is to make the ratio $r : R$ exactly equal to $\tan \pi/n$, where n is any integer, and we shall get the harmonic sequence

$$1, n-1, n+1, 2n-1, 2n+1, 3n-1, 3n+1, \text{etc.}$$

For pedal notes the best values of n are 5, 7, or 9; these will give octave harmonics. If we take $n=5$ we shall get a pipe shorter than the stopped diapason of the same pitch in the ratio of 4 : 5; but its harmonic overtones will be the series

$$1, 4, 6, 9, 11, 14, 16, \text{etc.},$$

which should give a far better quality than the ordinary stopped diapason.

But the best of all for quality is the $\pi/3$ *Bicylindron*, the length of which is two-thirds that of the open diapason of the same pitch. This pipe has the harmonic sequence

$$1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, \text{etc.}$$

This pipe, if properly voiced, should make a solo stop with a unique loveliness of tone. Experimentally I have proved the sequence 1, 2, 4, and 5.

I worked out my formulæ in the case of a double conical pipe, made of two cones joined together at the broader equal ends, each tapering to a smaller mouth in opposite directions. The result can be found by a very pretty calculation from the formulæ given by Basset or Lord Rayleigh; it is

$$m \sin m\mathrm{H} = \left(\frac{1}{k} + \frac{1}{k'}\right) \sin mh \sin mh',$$

where $\mathrm{H} = h + h'$; h and k have the meaning given above for the *Pyramidon*; h' and k' the corresponding quantities for the second cone; and m gives as before the vibration-rate of the whole pipe, which is of lower pitch than an open cone of the same length. This double pipe is intended for reed-stops, which need a cone tapering to the narrow reed-tube. By taking $h' = h$ or $h/2$ we can get all the harmonics of the open cone h'; and by giving suitable values to k and k' we can ensure that the fundamental tone shall be an exact double octave below the open cone h', thus gaining the conditions for good quality. Hence we find in either case

$$\frac{1}{k} + \frac{1}{k'} = \frac{\pi}{2h}.$$

In 1868 I made four pipes on this plan, using some harmonium reeds I had by me, of pitch Tenor C, D, E, and F. Mr. Frye tried them in the Cathedral organ. We both agreed that the tone was very beautiful, when it came on. But there was a distinct burr at the beginning and end of the tone, and the pipes were so "slow of speech" that in a quick run they never spoke at all. Such "free reeds" are probably inappropriate: they are never used by English organ-builders. But the experiment shows that in expert hands a reed-pipe of this kind might be made with a tone of unique beauty, free from all defects of "speech." JAMES A. ALDIS.

The Quantum Theory of Dispersion.

IN a recent letter to NATURE (May 10, p. 673) Dr. Kramers advanced a quantum theory of dispersion which is a generalisation of the theory of Ladenburg. The formula proposed by Kramers for the polarisation of an atom when put in a wave is his formula (5). This formula is stated by Kramers to satisfy the condition demanded by the Correspondence Principle, namely, that the dispersion due to an atom in a state of high quantum number is the same on the classical and on the quantum theories.[1] The presence of the second term has been introduced by Kramers for this purpose. From the point of view of the virtual oscillators of Bohr, Kramers, and Slater, the second negative term of Kramers is somewhat dissatisfying, because an oscillator would give rise only to a term of the first positive type.

The present writer has been also considering the question of interaction between radiation and quantised atoms in connexion with the question of the Brownian movement of atoms in black body radiation. A picture similar to the virtual oscillator finds application also in that field. However, the exact form of the interaction has been conjectured by the writer to be somewhat different from that proposed by Kramers.

The difference can be illustrated in the case of the linear oscillator. In this case the expression of Kramers becomes at long wave-lengths

$$P = [n - (n-1)]\frac{e^2}{m}\frac{1}{4\pi^2(\nu_i^2 - \nu^2)} = \frac{e^2}{m}\frac{1}{4\pi^2(\nu_i^2 - \nu^2)}.$$

The same result may be also derived as

$$P = [\tfrac{1}{2} + \tfrac{1}{2}]\frac{e^2}{m}\frac{1}{4\pi^2(\nu_i^2 - \nu^2)}.$$

In this manner the negative term may be avoided. In order to satisfy the Principle of Correspondence, the dependence of P on ν must be in general slightly more complicated than that for the oscillator. This dependence can be derived from a consideration of a "virtual orbit" rather than a virtual oscillator. (I am indebted to Prof. Van Vleck for this term.) The "virtual orbit" has the same frequency as the "virtual oscillator." However, its reaction to the external field is comparable with that of an electron, the orbit of which is the mean of all the orbits between the two stationary states. (The meaning of "mean" is of necessity somewhat indefinite.) It is clear that in the general case a properly taken sum of the contributions of the various ν_i^a, ν_i^e will give the required result. Thus it is sufficient to attribute to a transition between a quantum state of quantum number $(n_1+\tau_1, \ldots n_u+\tau_u)$ to $(n_1, \ldots n_u)$ one half of the contribution to the polarisation on the classical theory due to the terms in frequencies $\tau_1 w_1 + \ldots + \tau_u w_u$ (in Bohr's notation) in order to satisfy the Principle of Correspondence.

It appears that the form of the theory here outlined is better capable of explaining dispersion at long wave-lengths (say for an atom in the normal state) than the form of Kramers because the characteristics of the motion are of greater influence on the "virtual orbit" than on the "virtual oscillator" point of view. By introducing a properly taken mean, one may hope to obtain the influence of a static field as a limiting case of a field considered in the theory of dispersion. G. BREIT.

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THROUGH the courtesy of the Editor of NATURE, I have been permitted to see Mr. Breit's letter, and I welcome the opportunity thus afforded me to add some further remarks on the theory of dispersion, in order to elucidate some points which were only briefly touched upon in my former letter.

In addition to the empirical applicability of a dispersion formula of the type (4), the arguments which

[1] Prof. J. H. Van Vleck, of this University, has shown, in a publication which is to appear soon, that this formula satisfies the Correspondence Principle for the case of the general non-degenerate multiple periodic orbit.

led to the proposal of formula (5) rested on the classical expression for the amplitude of the secondary wavelets which an incident plane wave sets up in a system of electrified particles. Consider a system, the motion of which is of multiple periodic type, and let the electrical moment M in a given direction of the undisturbed system, which is supposed to possess u independent fundamental frequencies $\omega_1, \ldots \omega_u$, be represented by

$$M = \Sigma C \cos(2\pi\omega t + \gamma), \quad . \quad . \quad . \quad (1^*)$$

where the frequencies $\omega = \tau_1\omega_1 + \ldots \tau_u\omega_u$ and the amplitudes C depend on the quantities $I_1, \ldots I_u$, which in the theory of stationary states are equal to integer multiples of Planck's constant h, as well as on the set of integer τ-values characteristic for the considered harmonical component of the motion. Let next the incident wave be linearly polarised with its electrical vector parallel to the given direction, and let the value of this vector at the point where the system is situated be given by $E \cos 2\pi\nu t$. The electrical moment of the forced vibrations of frequency ν set up in the system will then be equal to

$$P = \frac{E}{2}\Sigma\frac{\partial}{\partial I}\left(\frac{C^2\omega}{\omega^2 - \nu^2}\right)\cos 2\pi\nu t, \quad . \quad . \quad . \quad (2^*)$$

where $\frac{\partial}{\partial I}$ stands as an abbreviation for $\tau_1\frac{\partial}{\partial I_1} + \ldots \tau_u\frac{\partial}{\partial I_u}$.

Now in the limit of high quantum numbers the frequencies of the spectral lines connected with the different possible transitions will coincide asymptotically with the frequencies of the harmonic components of the motion, and, according to the Correspondence Principle, the energy of the spontaneous radiation per unit time combined with each of these frequencies will be asymptotically represented by the expression $\frac{(2\pi\omega)^4 C^2}{3c^3}$. We will now make the assumption that, in this limit, formula (2*) gives an asymptotical expression for the dispersion. In order to obtain a general expression holding for all quantum numbers we note that, while the frequencies ω of the harmonic components of the motion are given by the general formula

$$\omega = \frac{\partial H}{\partial I},$$

the exact expression for the frequencies of the spectral lines is given by the general quantum relation

$$\nu_q = \frac{\Delta H}{h},$$

where ΔH signifies the difference of the energy H in two stationary states for which the values of $I_1, \ldots I_u$ differ by $\tau_1 h, \ldots \tau_u h$ respectively. The assumption presents itself that, in a generalisation of formula (2*), the symbol $\frac{\partial}{\partial I}$ has to be replaced by a similar difference symbol divided by h. This is just what has been done in establishing formula (5). In fact, this formula is obtained from (2*) by replacing the differential coefficient multiplied by h, by the difference between the quantities $\frac{3c^3 A^a h}{(2\pi)^4 \nu^{a2}(\nu^{a2} - \nu^2)}$ and $\frac{3c^3 A^e h}{(2\pi)^4 \nu^{e2}(\nu^{e2} - \nu^2)}$ referring to the two transitions coupled respectively with the absorption and emission of the spectral line which corresponds with the harmonic component under consideration.

Apart from the problem of the validity of the underlying theoretical assumptions and of any eventual restriction in the physical applicability of formula (5), the dispersion formula thus obtained possesses the advantage over a formula such as is proposed by Mr. Breit in that it contains only such quantities as allow of a direct physical interpretation on the basis of the fundamental postulates of the quantum theory of spectra and atomic constitution, and exhibits no further reminiscence of the mathematical theory of multiple periodic systems.

In this connexion it may be emphasised that the notation "virtual oscillators" used in my former letter does not mean the introduction of any additional hypothetical mechanism, but is meant only as a terminology suitable to characterise certain main features of the connexion between the description of optical phenomena and the theoretical interpretation of spectra. This point is especially illustrated by the appearance of negative as well as positive oscillators, which helps to bring out the new feature, characteristic of the quantum theory of spectra, that the emission and absorption of a spectral line is coupled with two separated types of physical processes. The fundamental importance of this general feature for the interpretation of optical phenomena is, as mentioned in my former letter, indicated by the necessity, pointed out by Einstein, of introducing the idea of negative absorption in order to account for the law of temperature radiation.

H. A. KRAMERS.

Institute for Theoretical Physics,
Copenhagen, July 22.

Lake Victoria and the Flow of the Yala River.

DURING a recent investigation into the epidemiology of sleeping sickness along the east coast of Lake Victoria, north of the opening of the Kavirondo gulf, I observed the following phenomena which I trust some readers of NATURE may be able to explain, for I cannot.

The river Yala, which enters the lake at this part of the coast, passes through swamps which are kept back from the lake by sand-banks. Having landed here I noted many channels cut through the sand, running inland from the lake. The local natives said, "These were made by our forefathers, six generations ago, to drain the swamps." I pointed out that at that moment (about 9.30 A.M.) the water was not running out *from* the swamp but *into* it from the lake at quite a considerable rate, which I thought might be two miles per hour.

The natives replied that the current ran thus from daybreak until two in the afternoon, when it was reversed and flowed back into the lake. This is the daily routine so long as the lake is at the higher levels of its seasonal variations; when it is low, in the dry season, the flow through these channels is always from the swamp.

I am much puzzled as to the explanation of this diurnal variation, which cannot be accounted for by changes in direction of wind. I was working along that coast for several weeks and soon noted that the breeze from lake to shore (westerly, here) does not commence before 9.30 A.M., often not till ten; until that time, from the small hours, there is a strong cool breeze from off shore.

I omitted to ask the natives in which direction the current flowed during the night: they had said "from daybreak," but that probably means merely the earliest time when they noticed it.

Since it occurred to me that the great loss which must take place from the open lake by evaporation during the heat of the day might conceivably lower