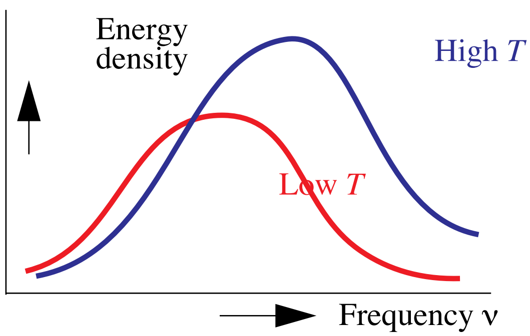
Max Planck and the Enigma of Black-Body Radiation

Heat radiation: at low temperatures infrared light (low frequency, long wavelength), at higher temperatures visible and ultraviolet light (high frequency, short wavelength)

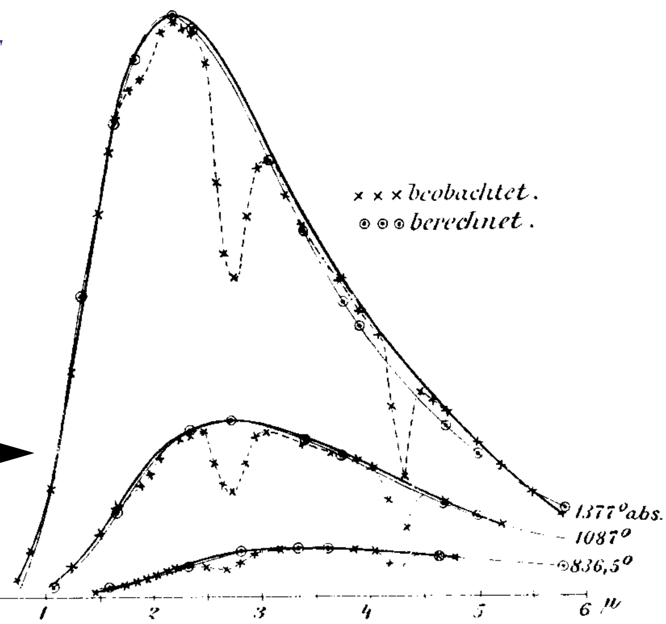
Gustav Kirchhoff (1824–1887). 1859. Thermodynamical arguments:
- Bodies absorbing radiation emit more heat radiation than bodies reflecting radiation. Best emitters: perfect absorbers, i.e., black bodies. Hence, heat radiation called black-body radiation (in German: *schwarze Strahlung* = black radiation]
- Characteristics of black-body radiation depends only on temperature, not on detailed structure of black body.

The challenge of black-body radiation:

Experimentalists: (*a*) for different temperatures, measure the energy density (= the energy per unit volume) of the radiation emitted by a black body at various frequencies/wavelengths; (*b*) plot the results in a series of graphs of energy density against frequency/wavelength for different temperatures; (*c*) compare the experimental results with various theoretical predictions



plots of energy-density against wavelength of black-body radiation at different temperatures (Lummer and Pringsheim)



Theoreticians: derive from first principles (of electrodynamics, thermodynamics etc.) the *black-body radiation formula*, i.e., the formula for how the energy density of black-body radiation depends on temperature T and the frequency ν (wavelength λ) of the radiation.

Constraints on the black-body radiation formula:

(1) **Stefan-Boltzmann law:** the total energy (= energy for all wavelengths combined) is proportional to T^4
Found experimentally: **Josef Stefan (1835–1893)** in 1879 .
Derived theoretically **: Ludwig Boltzmann (1844–1906)** in 1884 (using thermodynamics and electrodynamics).

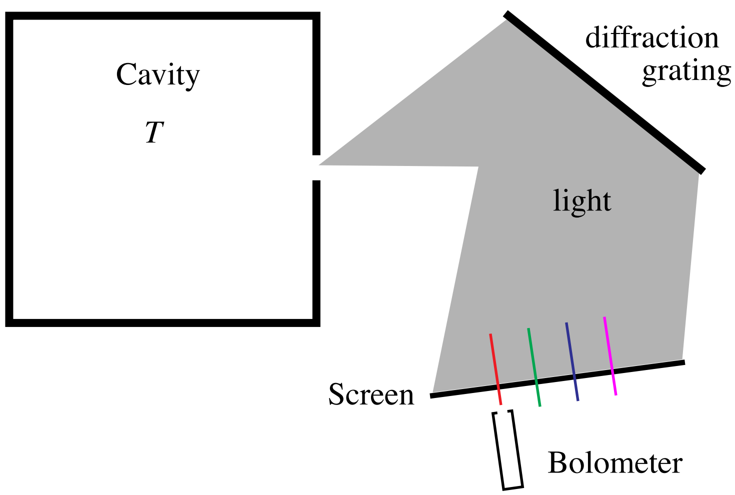
(2) **Wien Displacement Law:** $\lambda_{max} \cdot T = \text{constant}$.
Explication: Temperature T higher → wavelength λ for which the energy of black-body radiation peaks shorter (frequency ν higher).
Example: iron glowing blue [short wavelength] is hotter than iron glowing red [long wavelength]: $\lambda_{blue} \cdot T_{blue} = \lambda_{red} \cdot T_{red}$.
Derivation: **Wilhelm ('Willy') Wien (1864–1928)** from thermodynamics and electrodynamics (1896).

1896. Wien proposes concrete function meeting requirements (1) and (2): Wien Radiation Law.[1]

Late 1890s. Black-body radiation experiments at the *Physikalisch-Technische Reichsanstalt (PTR,* Imperial Institute of Physics and Technology) in Berlin, by two teams:

- **Otto Lummer (1860–1925)** & **Ernst Pringsheim (1859–1917)**
- **Heinrich Rubens (1865–1922)** & **Ferdinand Kurlbaum (1857–1927)**

What to use as a black body? Recall: perfect absorber is a perfect black body → Use cavity with small opening letting out some light.



- Keep a cavity at temperature *T*.
- Use diffration grating to separate light into its different frequencies.
- Use bolometer to measure energy for different frequencies.
- Plot energies against frequencies

Results: at first good agreement with Wien's formula, but by October 1900 Rubens and Kurlbaum find deviations in low frequency/long wavelength regime. This is where the main character of this story comes in:

Max Planck (1858–1947) had been working on the black-body radiation problem for some years. He had published a series of five papers between February 1897 and June 1899 culminating in the derivation of Wien radiation law

Why was Planck interested in deriving the black-body radiation formula?

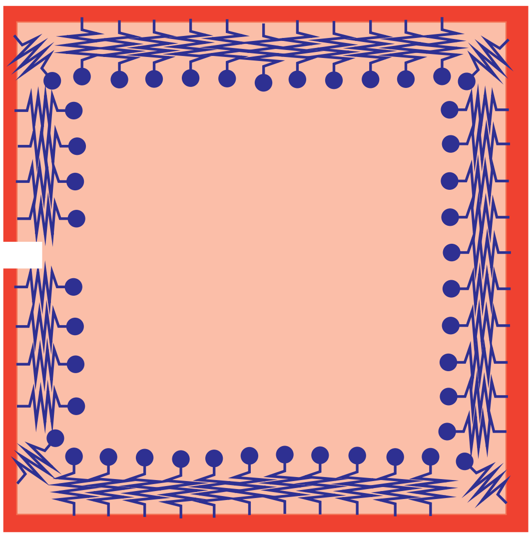
- *Connection with thermodynamics*. Dissertation on second law of thermodynamics (1879): improvement on formulation of **Rudolf Clausius (1822–1888)**.
- *Program for deriving the second law of thermodynamics from electrodynamics*. Planck believed (well into the 1900s!) that the second law of thermodynamics had absolute and not

1. Do not confuse **Wien Displacement Law** with **Wien Radiation Law**.

just statistical validity (as Maxwell and Boltzmann thought). [recall the attack on Boltzmann's interpretation by **Ernst Zermelo (1871–1953)**, one of Planck's students.] Boltzmann had shown that the second law cannot be derived as an exact law from mechanics. Planck now tried to derive it from electrodynamics. Boltzmann showed that that does not work either.

- *Fascination with the absolute and the universal*.

How did Planck derive the black-body radiation formula?



Focus on simple system (Kirchhoff had found black-body radiation is independent of the details of the system producing it): oscillators (= charges on springs) with different resonating frequencies interacting with electromagnetic waves in a cavity.

1899: Planck derives Wien radiation law

October 1900: Rubens and Kurlbaum inform Planck about problems with Wien law for long wavelengths. Planck produces new law, interpolating curve that works for long wavelengths and curve (Wien radiation law) that works for short wavelengths. New law meets requirements (Stefan-Boltzmann law, Wien displacement law) and is in excellent agreement with experimental data.

Session of the German Physical Society, Berlin, October 19, 1900: Kurlbaum presents latest experimental findings. Planck presents his new black-body radiation law ["On an Improvement of Wien's Equation for the Spectrum"]

New law so far just a lucky guess. As Planck said later: "a theoretical explanation had to be found at any cost, no matter how high" (Planck to Robert W. Wood, 1931)

Session of the German Physical Society, Berlin, December 14, 1900 (the birth of quantum mechanics?): Planck presents "quantum" derivation of his new black-body radiation law ["On the Theory of the Energy Distribution Law of the Normal Spectrum"]

The new derivation

- Result of Planck's earlier work for black-body radiation in equilibrium with resonators: *equilibrium energy distribution of radiation* can be obtained from e*quilibrium energy distribution of resonators*.
- *Equilibrium distribution of resonators* is *distribution with maximum entropy*.
- *Distribution with maximum entropy* (= maximal disorder) is *distribution that can be realized in largest number of ways*. [Based on Boltzmann's conception of entropy as a measure of disorder, even though Planck did not accept Boltzmann's statistical interpretation.]

- *How do you count the number of ways a distribution can be realized?* Planck borrows trick from Boltzmann: divide possible energies into small and equal intervals ε_ν. Look at all possible distributions of the resonators over these energy intervals. Count the number of ways these distributions can be realized.

Counting the number of ways a distribution can be realized:
- Divide energy of a resonator of frequency ν into small and equal intervals ε_ν.
- Look at all possible distributions of all N_ν resonators of frequency ν over these energy intervals. Count the number of ways these distributions can be realized.

Simplified example:[1] total energy $E_\nu = 10\varepsilon_\nu$; $N_\nu = 7$.

Example of distribution

Resonators with energies:	0–1 ε_ν	1–2 ε_ν	2–3 ε_ν	3–4 ε_ν
Number of resonators with that energy:	1	3	2	1

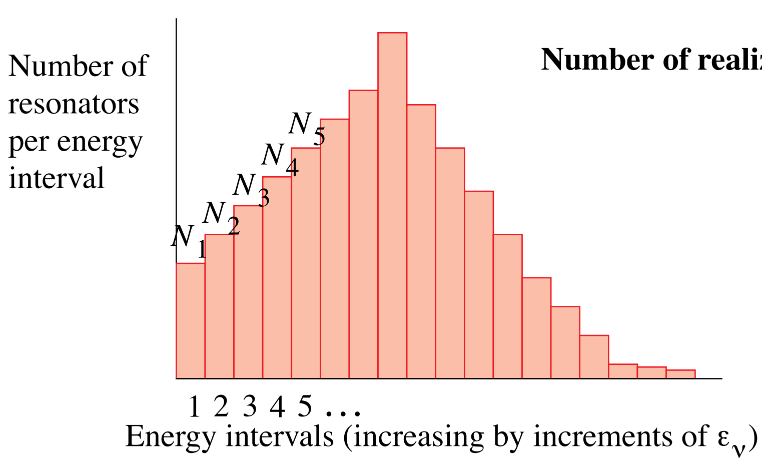
Total energy: $\mathbf{1 \cdot 0\varepsilon_\nu + 3 \cdot 1\varepsilon_\nu + 2 \cdot 2\varepsilon_\nu + 1 \cdot 3\varepsilon_\nu = 10\varepsilon_\nu}$

Example of a concrete realization of this distribution (label 7 resonators *a* through *g*)

Resonators with energies:	0–1 ε_ν	1–2 ε_ν	2–3 ε_ν	3–4 ε_ν
Individual resonators with that energy:	*a*	*b*,*c*,*d*	*e*,*f*	*g*

Number of realizations of this distribution: $\mathbf{\dfrac{7!}{3!2!} = \dfrac{7 \cdot 6 \cdot 5 \cdot 4}{2} = 420}$

More realistic picture:



Number of realizations of this distribution:

$$\frac{N_{total}!}{N_1!N_2!N_3!N_4!N_5!\ldots}$$

1. In reality these numbers are of course much larger.

Crucial assumption to get the Planck radiation law: $\varepsilon_\nu = h\nu$ **with** $h = 6.6 \cdot 10^{-34}\,Js$ **now called Planck's constant.** For this reason, December 14, 1900 is celebrated as the birthday of Quantum Theory.

Controversy among historians: **Did Planck quantize the energy of resonators?**[1] Answer depends on interpretation of Planck's counting procedure:

Standard interpretation: count oscillators with **exactly** energy 0, **exactly** $h\nu$, **exactly** $2h\nu$, etc. → **Yes, he did.**

Alternative interpretation (due to Thomas S. Kuhn): count oscillators with energy **between** 0 and $h\nu$, **between** $h\nu$ and $2h\nu$, **between** $2h\nu$ and $3h\nu$, etc. → **No, he did not.**

It was shown later—by Einstein in 1906—that only the standard interpretation leads to the Planck radiation law. But Planck did not realize that in 1900.

Upshot: Planck didn't quantize much of anything! Quantum theory not launched by Planck in 1900 but by Einstein in 1905.

Historiographical moral: what later generations of physicists think Planck did is different from what he actually did (not unusual in physics: think of Thomson and the discovery of the electron). Once the physics community has credited someone with a discovery, this is unlikely to be revised (Eugene Wigner: "Not everything in physics need be annexed to Einstein"). Credit is sometimes given not just for intellectual contributions but for service to the community as well (Planck was head of the physics department in Berlin, Thomson was head of the Cavendish in Cambridge).

1. Planck certainly did not quantize radiation. Planck did not introduce light quanta in 1900. Einstein did that in 1905. The question is whether Planck at least quantized his resonators.