The Constitution of the Solar Corona. II. By J. W. Nicholson, M.A., D.Sc.

It has been shown in the first paper that a large part of the coronal spectrum can be interpreted in terms of the vibrations of an elementary substance called protofluorine, to which a definite atomic constitution was assigned; and in an earlier paper that the main nebular lines can be interpreted similarly by the help of nebulium, a substance of an analogous nature. This interpretation led to the prediction of a new nebular line at $\lambda 4353$, which has been found subsequently, by Dr. W. H. Wright, on a plate of the Orion spectrum taken in 1908 at the Lick Observatory. The present paper takes up the further consideration of the spectrum of protofluorine, and connects the sets of wave-lengths corresponding to various degrees of electrical separation within the atom.

The constant of nature in terms of which these spectra can be expressed appears to be that of Planck in his recent quantum theory of energy.[*] It is evident that the model atoms with which we deal have many of the essential characteristics of Planck's "resonators." They possess a principal frequency $\omega/2\pi$, where ω is the angular velocity of the electrons within the atom, and we have seen, in the discussion of spectra, that this is ordinarily the frequency of a strong line in the spectrum of an atom. It is not desirable, in this paper, to give any detailed account of the principles of Planck's theory, but we shall avail ourselves of the main result. This states that the energy possessed by a resonator is of the form $nh\nu$, where ν is the frequency of the resonator, n is an integral number, and h is an universal constant of nature, whose value, derived by Planck from measurements on the radiation of a black body, is

$$h = 6 \cdot 548 \ldotp 10^{-27} \text{ erg seconds.}$$

In other words, interchanges of energy are not continuous, so that it is not possible to represent ultimate dynamics by sets of differential equations. The interchanges of energy are, moreover, regulated in magnitude by the frequencies of the systems concerned in a curious way.[†]

We proceed to examine to what extent this view is in accord with the various protofluorine atoms, neutral and positively charged. The systems with negative charges present some difficulties.

If a connection can be established between the systems on this basis—that is to say, if the ratio of atomic energy to principal frequency follows a regular law,—the investigation will serve the double purpose of confirming the suggested origin of the spectra of

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[*] *Ann. der Phys.*, 1900, Bd. i. p. 69; 1901, Bd. iv. p. 553.
[†] A new mode of presenting the theory has been suggested by Sir Joseph Larmor in his Bakerian Lecture, *Proc. Roy. Soc.* A., vol. lxxxiii. p. 82. A critical discussion has been given recently by Poincaré, *Journal de Physique*, Jan. 1912.

astrophysics, and of giving to Planck's theory an atomic foundation : a foundation of the kind which is now generally believed to be necessary, giving a concrete picture of the possible nature of a resonator.

The energy of an atom.—As more than one writer has pointed out, the absolute energy of an atom cannot be known. We can only deal with changes of energy, or energy relative to that of an arbitrary standard configuration.

Let us examine the energy of a " simple " atomic system, consisting of a ring of n equidistant electrons rotating in a circle of radius a about a positive charge νe, where e is the electronic charge. For a single electron, the equation of energy, as in the theory of orbits, is for an attractive force $\nu e^2/r^2$ at distance r, of the type

$$\tfrac{1}{2}mv^2 = \mathrm{D} - \int \frac{\nu e^2}{r^2}\,dr = \mathrm{D} + \frac{\nu e^2}{a},$$

where m and v are the mass and velocity of an electron, and D is a constant. When n electrons are present together, the distances apart are the various values of the magnitudes of $2a \sin r\pi/n$, so that the potential energy of the atomic system, which would be $-n\nu e^2/a$ in the absence of mutual action between negative charges, is changed to

$$-\frac{ne^2}{a}(\nu - \tfrac{1}{4}\mathrm{S}_n),$$

where

$$\mathrm{S}_n = \sum_{r=1}^{n-1} \operatorname{cosec} r\pi/n.$$

The equation of energy is

$$\tfrac{1}{2}mnv^2 = \frac{ne^2}{a}(\nu - \tfrac{1}{4}\mathrm{S}_n) + \mathrm{D},$$

where D is the energy in a standard configuration. In D is included any intrinsic energy which the electrons and nucleus may possess—such, for example, as may be concerned in their expansion, if Schott's view of the expanding electron is accepted as the cause of the fineness of spectral lines.

By the condition of steady motion,

$$ma^3\omega^2 = e^2(\nu - \tfrac{1}{4}\mathrm{S}_n) = mav^2,$$

and the kinetic energy of motion of the electron being $\tfrac{1}{2}nmv^2$, the potential energy of æthereal strain is of the form

$$\mathrm{D} - mna^2\omega^2,$$

where D is unknown. We shall examine the values of $mna^2\omega^2$, which will for brevity be referred to as " the energy."

Energy of the neutral atom of protofluorine. — Numerical determinations of the energy can have only a limited accuracy on account of the range of uncertainty in the experimental values

of e and e/m. But it is now believed that the most accurate values are

$$e = 4{\cdot}7.10^{-10} \qquad e/mc = 1{\cdot}77.10^{-7}$$

the first being in electrostatic, and the second in electro-magnetic units.*

In the first paper the principal vibration, $q = \omega$, for the proto-fluorine atom has a wave-length $3987{\cdot}1$ tenth-metres, or $3{\cdot}987.10^{-5}$ cms. Thus

$$\omega = 2\pi c/(3{\cdot}987.10^{-5});$$

moreover,

$$\omega^2 a^3 = 3{\cdot}6235 e^2/m,$$

so that after calculation

$$\omega a = 1{\cdot}623.10^8,$$

and the measure of energy/frequency is

$$mna^2\omega^2.2\pi/\omega = 154{\cdot}94.10^{-27}.$$

This is of a similar order of magnitude to Planck's unit, and is nearly 25 of these units. It could be regarded as 25 units without any serious change in the experimental values of e, e/m. If e/m were unaltered, e would need an alteration from $4{\cdot}7$ to $4{\cdot}85$ only. We shall suppose, as is necessary for the complete theory of which this paper gives a slender outline, that the ratio of energy to frequency is exactly 25 of these units. The energy concerned is the potential æthereal energy reckoned in defect of a certain standard.

The smallness of the measure is in accord with the view that protofluorine is one of the simplest forms of matter—that is to say, one of the simplest receptacles of energy which can exist.

Angular momentum.—It is possible to hold another view of Planck's theory, which may be briefly pointed out. Since the variable part of the energy of an atomic system of the present form is proportional to $mna^2\omega$, the ratio of energy to frequency is proportional to $mna^2\omega^2$, or $mnav$, which is the total angular momentum of the electrons round the nucleus. If, therefore, the constant h of Planck has, as Sommerfeld has suggested, an atomic significance, it may mean that the angular momentum of an atom can only rise or fall by discrete amounts when electrons leave or return. It is readily seen that this view presents less difficulty to the mind than the more usual interpretation, which is believed to involve an atomic constitution of energy itself.

Energy/frequency measures of non-neutral atoms.—Consider next the atoms with positive charges. For protofluorine, which retains 4 and 3 electrons respectively, we have from the first paper,

No. of electrons $= 4$,   Principal line $= \lambda 4231$,

No. of electrons $= 3$,   Principal line $= \lambda 4586$.

* An excellent summary may be found in Kaye and Laby, *Physical and Chemical Tables*, 1911, pp. 99–100.

The case of two electrons is not included, as the identification given for it was unsatisfactory and only provisional.

The uncertainty of the values of e and e/m makes it convenient to adopt another unit of energy/frequency. From the relations

$$ma^3\omega^2 = e^2(\nu - \tfrac{1}{4}S_n),$$
$$\omega = 2\pi c/\lambda,$$

where λ is the principal wave-length, we deduce

$$mna^2\omega^2 . 2\pi/\omega = \frac{mn}{c}\left\{ \frac{e^2}{m} . (\nu - \tfrac{1}{4}S_n)(2\pi c) \right\}^{\frac{2}{3}} \lambda^{\frac{1}{3}},$$

so that energy/frequency is proportional to

$$n(\nu - \tfrac{1}{4}S_n)^{\frac{2}{3}}\lambda^{\frac{1}{3}},$$

which may be taken as its measure, with λ in tenth-metres instead of centimetres. We shall denote the function by E. Thus, for neutral protofluorine, with the identification suggested in the first paper,

$$E = n(\nu - \tfrac{1}{4}S_n)^{\frac{2}{3}}\lambda^{\frac{1}{3}} = 187\cdot04,$$

which is 25 units if the unit is of magnitude $7\cdot482$.

For protofluorine with four electrons, $n = 4$, $\nu - \tfrac{1}{4}S_n = 4\cdot0429$, $\lambda = 4231$,

$$E = 164\cdot19.$$

For protofluorine with three electrons, $n = 3$, $\nu - \tfrac{1}{4}S_n = 4\cdot4226$, $\lambda = 4586$,

$$E = 134\cdot29.$$

Within one part in 1000, the three systems give respectively

$$25, \quad 22, \quad 18$$

of Planck's units.

These are the first three members of the harmonic sequence

$$25, \quad 22, \quad 18, \quad 13, \quad 7, \quad 0,$$

which would, if it continued valid, give no units to the positive nucleus alone, as would be expected if intrinsic energy were neglected.

The numbers of units *per electron* of (energy)/(frequency) would become, for the five systems in question,

$$5, \quad 5\tfrac{1}{2}, \quad 6, \quad 6\tfrac{1}{2}, \quad 7,$$

exhibiting a regular gradation, so that if the law were valid, the value of $mna^2\omega . 2\pi$ for the system with n electrons and a principal frequency f would be

$$\tfrac{1}{2}(15 - n)nf h,$$

and it becomes possible to express the whole spectrum of proto-fluorine in the form of a series, or a set of series, depending on the natural numbers, though the series is not of the types of Balmer, Kayser and Runge, or Rydberg. These series, however, relate to elements whose structure is more complicated than a single ring system.

If the law continues to be valid in this way, there will be a coronal line for the system with two electrons, leading, within about 1 part in 1000, to a value of E of $6\frac{1}{2}$ units per electron. The line $\lambda5073$ satisfies this necessity, for it would lead to

$$E = 2(4\cdot75)^{\frac{2}{3}}(5073)^{\frac{1}{3}} = 97\cdot107,$$

so that the number of units of magnitude $7\cdot482$ is

$$(97\cdot107)/(7\cdot482) = 12\cdot98,$$

and the number per electron is $6\cdot495$, which is $6\frac{1}{2}$ to less than 1 part in 1000.

An identification for this line was suggested in the first paper, but marked as very doubtful. The line is so weak that a companion line cannot be expected, and the agreement in the present case is so close that the origin of $\lambda5073$ seems established.

This investigation supplies the first instance of the method by which lines can be identified when they have companions either outside the visible spectrum, or too weak to be recorded.

The companion lines.—A correction will now be introduced to certain of the wave-lengths calculated in the first paper, the need for which has been kindly brought to my notice by Dr. T. J. I'A., Bromwich. It had already appeared, as the result of more complete investigation, that the discrepancy between observed and calculated wave-length ratios could not be entirely explained by the neglect of the magnetic effects. But Dr. Bromwich's correction overcomes this difficulty completely. The transverse vibrations of class zero in the paper all have their wave-lengths too high. The mass M of the positive nucleus, although not entering as a rule into the periods of transverse vibrations, does so in the exceptional case $k = 0$, whose proper period equation is

$$mq^2 = ve^2(1 + nm/M)/a^3,$$

where the term in m/M represents the correction.* The mass of the electron is m.

In each of the above cases, including that of neutral nebulium, an application of this correction makes the coincidence between calculated and observed values even more exact. The value of M is, of course, unknown, but it is an essential point of the view that the atomic weight of this substance is comparable with that of hydrogen, that the value of m/M should be very small. In fact,

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* Vide *Monthly Notices*, Nov. 1911, p. 52, equation (9).

it is generally believed that the electrons do not contribute greatly to the atomic weight.

If H is the value of M for a hydrogen atom, we know from experimental measures that $m/\mathrm{H} = \cdot00054$ approximately. Let us calculate the atomic weight of protofluorine which would make the wave-lengths exact to the tenth of a tenth-metre. Only rough limits can be assigned, as the coronal lines are sometimes uncertain to the extent of a tenth-metre or more.

For three electrons, the calculated value was $\lambda = 4313$ for $4311\cdot3$. Thus

$$(1 + 3m/\mathrm{M})^{\frac{1}{2}} = 1\cdot7/4311,$$

or

$$m/\mathrm{M} = \cdot00026,$$

giving an atomic weight $2\cdot1$ for protofluorine.

For four electrons, the calculated and observed wave-lengths are $3804\cdot5$, $3800\cdot9$, the latter being the mean of Dyson's three results, Campbell * giving $3800\cdot4$.

We derive $m/\mathrm{M} = \cdot00047$, giving an atomic weight about unity. The case of five electrons gives, with Dyson's observation, a somewhat smaller value, Dyson having found the line only in 1901, when it was very strong, at $\lambda3387\cdot9$, but Campbell giving $\lambda3390\cdot7$. Campbell's value again leads to an atomic weight nearly 2. Thus there is undoubted evidence that the atomic weight is comparable with that of hydrogen.

The same method applied to nebulium gives an atomic weight $1\cdot3$. We must note that there is doubt about the line $\lambda3454$ being the *third* line of the neutral atom, for there is not a term in m/M to explain the discrepancy which was found. But the origin of the following lines now seems definitely established :—

Coronal Line.	Origin.		Intensity.
5073	Pf,	$+3e$	1
4586	Pf,	$+2e$	4
4311	Pf,	$+2e$	2
4231	Pf,	$+e$	5
$3987\cdot1$	Pf,	neutral	3
$3800\cdot9$	Pf,	$+e$	3
$3387\cdot9$	Pf,	neutral	12

We proceed now, in the attempt to give an exhaustive treatment of the spectra of neutral and positively charged atoms of protofluorine, to give the theory of the vibrations in the plane of a single ring atom.

Vibrations in the plane of the atom.—The analysis necessary to a determination of these vibrations is similar to that employed by Sir J. J. Thomson for his atomic systems, but the results are very different on account of the change in the law of force.

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* *Lick Obs. Bull.*, No. 131, p. 11.

Moreover, we must suppose the positive nucleus to be capable of motion, for it is sometimes conspicuous in its effect on the periods. It is convenient, at the outset, to use the abbreviations in the following formulæ :—

$$S = \Sigma_1^{n-1} \operatorname{cosec} s\pi/n,$$

$$8a^3 A'/e^2 = 3\Sigma \operatorname{cosec} s\pi/n - \Sigma \operatorname{cosec}^3 s\pi/n,$$

$$8a^3 A_s/e^2 = \operatorname{cosec} s\pi/n + \operatorname{cosec}^3 s\pi/n,$$

$$8a^3 B_s/e^2 = \cos s\pi/n \operatorname{cosec}^2 s\pi/n,$$

$$4a^3 C/e^2 = \Sigma \cos s\pi/n \operatorname{cosec}^2 s\pi/n \ (\cot s\pi/n + \tfrac{1}{2} \sin s\pi/n),$$

$$4a^3 C_s/e^2 = \cos s\pi/n \operatorname{cosec}^2 s\pi/n \ (\cot s\pi/n + \tfrac{1}{2} \sin s\pi/n),$$

$$8a^3 De^2 = \Sigma \operatorname{cosec}^3 s\pi/n,$$

where all the summations range from unity to $n - 1$.

Let a fixed origin be taken at the equilibrium position of the positive nucleus, whose co-ordinates related to any fixed axes in
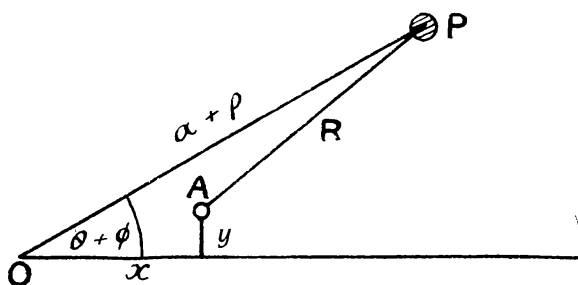
the plane of the atom are (x, y) at time t. The *polar* co-ordinates of the rth electron may be denoted by $(a + \rho_r, \ \theta_r + \phi_r)$ where $(\rho_r, \ \phi_r)$ are small, and angles are measured from the axis of x. The radius of the undisturbed orbit is a. We first find the radial and transversal forces exerted by the moving nucleus on a typical electron in the position $(a + \rho, \ \theta + \phi)$, the nucleus being at (x, y). The arrangement is shown in the figure. The actual force is a radial attraction $\nu e^2/AP^2$ along P A, whose components along x and y are

$$\ddot{x} = -\frac{\nu e^2}{R^3} \left\{ (a + \rho) \cos (\theta + \phi) - x \right\}, \quad \ddot{y} = -\frac{\nu e^2}{R^3} \left\{ (a + \rho) \sin (\theta + \phi) - y \right\}$$

To the first order, the radial force along r increasing is

$$- \nu e^2 \{ a + \rho - x \cos \theta - y \sin \theta \}/R^3,$$

and the transverse force increasing θ is

$$\nu e^2 \{ y \cos \theta - x \sin \theta \}/R^3.$$

Writing, to the same order, $R = a + \rho - x \cos\theta - y \sin\theta$, the forces tending to increase r and θ become

$$Radial, \quad -\nu e^2 \{a - 2(\rho - x\cos\theta - y\sin\theta)\}/a^3,$$
$$Transversal, \quad +\nu e^2 \{y\cos\theta - x\sin\theta\}/a^3.$$

The forces R_p, Θ_p acting in the same directions on the pth electron by virtue of the presence of the others are, after Thomson, of the form

$$R_p = \frac{e^2 s}{4a^2} - \rho_p A' - \Sigma\rho_{p+s}A_s - a\Sigma\phi_{p+s}B_s,$$

$$\Theta_p = \Sigma\rho_{p+s}B_s - 2a\,C\,\phi_p + 2a\,\Sigma\phi_{p+s}C_s,$$

in terms of the quantities defined at the beginning of this section. The radial and transversal equations of motion of the pth electron are accordingly

$$m\left\{ \ddot{\rho}_p - (a+\rho_p)(\omega+\dot{\phi}_p)^2 \right\} = R_p - \frac{\nu e^2}{a^2} + \frac{2\nu e^2}{a^3}(\rho_p - x\cos\theta_p - y\sin\theta_p),$$

$$\frac{m}{a+\rho_p}\frac{d}{dt}\left\{ (a+\rho_p)^2(\omega+\dot{\phi}_r) \right\} = \Theta_p + \frac{\nu e^2}{a^3}(y\cos\theta_p - x\sin\theta_p),$$

with the equation of steady motion

$$ma^3\omega^2 = e^2(\nu - \tfrac{1}{4}s_n).$$

If the oscillation has a period $2\pi/q$, so that all quantities relating to the variation from steady motion are proportional to e^{iqt}, then to the first order,

$$-mq^2\rho_p - 2ma\omega i q\phi_p = R_p - \frac{e^2 s}{4a^2} + \rho_p\left(m\omega^2 + \frac{2\nu^2}{a^3}\right) - \frac{2\nu e^2}{a^3}(x\cos\theta_p + y\sin\theta_p)$$

$$= -\rho_p A' - \Sigma\rho_{p+s}A_s - a\Sigma\phi_{p+s}B_s + \rho_p\left(m\omega^2 + \frac{2\nu e^2}{a^3}\right) - \frac{2\nu e^2}{a^3}(x\cos\theta_p + y\sin\theta_p)$$

and

$$2im\omega q\phi_p - maq^2\phi_p = \Sigma\rho_{p+s}B_s - aC\phi_p + a\Sigma\phi_{p+s}C_s$$
$$+ \frac{\nu e^2}{a^3}(y\cos\theta_p - x\sin\phi)_p.$$

If $A = A' - m\omega^2 - 2\nu e^2/a^3$, we get two sets of equations, whose first members are

$$(A - mq^2)\rho_1 + A_1\rho_2 + A_2\rho_3 + \ldots - 2ma\omega i q\phi_1 + aB_1\phi_2 + \ldots$$
$$= -2\nu e^2(x\cos\theta_1 + y\sin\theta_1)/a^3 \quad (A)$$
$$2mi\omega q\rho_1 - B_1\rho_2 - B_2\rho_3 - \ldots + (C - mq^2)a\phi_1 - C_1a\phi_2 - C_2a\phi_3 \ldots$$
$$= \nu e^2(y\cos\theta_1 - x\sin\theta_1)/a^3 \quad (B)$$

Consider now the motion of the positive nucleus. We find on

reduction that the forces (X_1, Y_1) exerted on it by any typical electron, tending to increase (x, y) are, to the first order,

$$X_1 = \nu e^2 \{a \cos \theta + \rho \cos \theta - a\phi \sin \theta - x + 3 \cos \theta (x \cos \theta + y \sin \theta - \rho)\}/a^3$$

$$Y_1 = \nu e^2 \{a \sin \theta + \rho \sin \theta + a\phi \cos \theta - y + 3 \sin \theta (x \cos \theta + y \sin \theta - \rho)\}/a^3$$

and these must be summed for all the electrons.

But

$$\Sigma_0{}^n \cos \theta_n = \Sigma_0{}^n \cos r\pi/n = 0,$$

$$\Sigma_0{}^n \sin \theta_n = \Sigma_0{}^n \sin r\pi/n = 0,$$

so that the first term of X_1 or Y_1 disappears on summation for all the electrons, and the total force is small—as it should be.   Again,

$$\Sigma_0{}^n \cos \theta_n \sin \theta_n = 0,$$

$$\Sigma_0{}^n \cos^2 \theta_n = \Sigma_0{}^n \sin^2 \theta_n = \tfrac{1}{2}\Sigma_0{}^n(1 + \cos 2\theta_n) = \tfrac{1}{2}n,$$

$$\Sigma_0{}^n(3 \cos^2 \theta_n - 1) = \Sigma_0{}^n(3 \sin^2 \theta_n - 1) = \tfrac{1}{2}n,$$

so that the total forces on the positive nucleus, increasing x and y, are

$$\nu e^2 \left\{ -2\Sigma\rho_n \cos \theta_n - a\Sigma\phi_n \sin \theta_n + \frac{nx}{2} \right\} \Big/ a^3,$$

$$\nu e^2 \left\{ -2\Sigma\rho_n \sin \theta_n + a\Sigma\phi_n \cos \theta_n + \frac{ny}{2} \right\} \Big/ a^3,$$

and the equations of motion of the nucleus, of mass M, are

$$-Mq^2 x = \nu e^2 \left\{ \frac{nx}{2} - 2\Sigma\rho_n \cos \theta_n - a\Sigma\phi_n \sin \theta_n \right\} \Big/ a^3$$

$$-Mq^2 y = \nu e^2 \left\{ \frac{ny}{2} - 2\Sigma\rho_n \sin \theta_n + a\Sigma\phi_n \cos \theta_n \right\} \Big/ a^3$$

or, if
then

$$a = \tfrac{1}{2}n + Mq^2 a^3/\nu e^2,$$

$$ax = 2\Sigma_1{}^n \rho_n \cos \theta_n + a\Sigma\phi_n \sin \theta_n,$$

$$ay = 2\Sigma\rho_n \sin \theta_n - a\Sigma\phi_n \cos \theta_n,$$

and (x, y) are of order m/M in comparison with any ρ or ϕ.   They will be zero of necessity in all symmetrical vibrations, such, for example, as those of class zero.   This is seen at once, because the ρ's are all equal, and also the ϕ's, in this class of vibration, and come outside the summations.   We seek to solve these equations, with (A) and (B), in the old manner.   Let ω be any nth root of unity, and write

$$\rho_2 = \rho_1\omega, \quad \rho_3 = \rho_2\omega, \ldots$$

$$\phi_2 = \phi_1\omega, \quad \phi_3 = \phi_2\omega, \ldots$$

Then

$$ax = 2\rho_1\sigma_1 + a\phi_1\sigma_2,$$

$$ay = 2\rho_1\sigma_2 - a\phi_1\sigma_1,$$

where

$$\sigma_1 = \Sigma_1^n \omega^{r-1} \cos \theta_r, \quad \sigma_2 = \Sigma_1^n \omega^{r-1} \sin \theta_r, \quad \theta_r - \theta_{r-1} = 2\pi/n.$$

We must now determine the values of σ_1 and σ_2. We find

$$\sigma_1 \pm i\sigma_2 = \Sigma_1^n \omega^{r-1} e^{\pm i\theta_r},$$

and if k be integral or zero, defining the "class" of the vibration as in the case of transverse vibrations, $\omega = e^{2ik\pi/n}$. Thus

$$\sigma_1 \pm i\sigma_2 = e^{\pm i\theta_1} \Sigma_1^n e^{2ir\pi(k\pm1)/n}.$$

If $k = \mp 1$, this summation has the value n, but is otherwise zero when k is not greater than n.

Thus σ_1 and σ_2 separately vanish unless $k \pm 1$ is zero, and therefore x and y vanish, so that the mass of the positive nucleus does not affect the periods. Vibrations of classes ± 1 are therefore exceptional. If $k = 1$, then

$$\sigma_1 + i\sigma_2 = 0, \quad \sigma_1 - i\sigma_2 = ne^{-i\theta_1},$$

or

$$\sigma_1 = \tfrac{1}{2}ne^{-i\theta_1}, \quad \sigma_2 = \tfrac{1}{2}ine^{-i\theta_1}.$$

If $k = -1$, then

$$\sigma_1 - i\sigma_2 = 0, \quad \sigma_1 + i\sigma_2 = ne^{i\theta_1},$$

or

$$\sigma_1 = \tfrac{1}{2}ne^{i\theta_1}, \quad \sigma_2 = -\tfrac{1}{2}ine^{i\theta_1}.$$

There is no loss of generality involved in taking $\theta_1 = 0$. Also

$$ax = 2\rho_1\sigma_1 + a\phi_1\sigma_2$$
$$ay = 2\rho_1\sigma_2 - a\phi_1\sigma_1.$$

Vibrations of classes other than ± 1.—In these vibrations, x and y are zero, and the solution is comparatively simple. The typical equations (A) and (B) become, with ρ_p for ρ_1 and so on,

$$(A - mq^2)\rho_p + A_1\rho_{p+1} + \ldots - 2ma\omega iq\phi_p + aB_1\phi_{p+1} + \ldots = 0$$
$$2im\omega q\rho_p - B_1\rho_{p+1} - \ldots + (C - mq^2)a\phi_p - C_1a\phi_{p+1} - \ldots = 0$$

These equations are of the same form as those obtained by Sir J. J. Thomson for his atomic model, but the coefficients are different. The ratios assumed for the ρ's and ϕ's are the same, and therefore after his analysis we may write down the period equation for the vibrations of class k as

$$(A - mq^2 + L_k)(C - mq^2 - N_k) = (M_k - 2m\omega q)^2,$$

where ω is the angular velocity of rotation, and

$$8a^3L_k = e^2 \Sigma_1^{n-1} \cos 2rk\pi/n (\operatorname{cosec} r\pi/n + \operatorname{cosec}^3 r\pi/n),$$
$$8a^3M_k = e^2 \Sigma \sin 2rk\pi/n \cos r\pi/n \operatorname{cosec}^2 r\pi/n,$$
$$8a^3N_k = 2e^2 \Sigma \cos 2rk\pi/n \cos r\pi/n \operatorname{cosec}^2 r\pi/n (\cot r\pi/n + \tfrac{1}{2} \tan r\pi/n).$$

Moreover, $C = N_0$. The function A is different from that of Thomson, and has the value

$$A = A' - m\omega^2 2\nu e^2/a^3,$$

where A', defined at the beginning of the investigation, is readily reduced to the form

$$A' = -L_0 + e^2 S_n/2a^3,$$

and, finally,

$$A = -(L_0 + 3m\omega^2)$$

by the use of the equation of steady motion.

The period equation therefore becomes

$$(L_k - L_0 - 3m\omega^2 - mq^2)(N_0 - N_k - mq^2) = (M_k - 2m\omega q)^2$$
$$k = 0, 2, 3, \ldots$$

and the functions occurring in it have been given for numbers of electrons up to 6 by Thomson. The equation differs from his by containing the term $3m\omega^2$. Nagaoka has given an investigation of this system with a stationary nucleus, but errors have crept in, and the resulting period equation is of the wrong type.

For vibrations of class $k = 0$, the equation reduces to

$$q^2(q^2 + 3\omega^2) = 4\omega^2 q^2,$$

giving $q = 0$, $q = \pm \omega$.

Thus the period equation of the simplest and most important vibration in the plane is merely $q = \omega$, as we have hitherto stated.

Even for vibrations of classes ± 1, this equation must give a fair approximation to the periods, if the mass of the nucleus is really the source of nearly the whole mass of the atom, as the present theory of material constitution supposes. But we shall find that it is not a sufficient equation. Especially in the case of nebulium, some of these vibrations of class 1 are important spectral lines.

Vibrations of Classes ± 1.—The equations in this more complex case are again of two types :—

$$(A - mq^2)\rho_p + A_1\rho_{p+1} + \ldots - 2ma\omega iq\phi_p + aB_1\phi_{p+1} + \ldots$$
$$= -2\nu e^2(x \cos\theta_p + y \sin\theta_p)/a^3,$$

$$2im\omega q\rho_p - B_1\rho_{p+1} - \ldots + (C - mq^2)a\phi_p - C_1 a\phi_{p+1} - \ldots$$
$$= \nu e^2(y \cos\theta_p - x \sin\theta_p)/a^3,$$

where

$$ax = 2\rho_1\sigma_1 + a\phi_1\sigma_2, \quad ay = 2\rho_1\sigma_2 - a\phi_1\sigma_1,$$
$$(\sigma_1, \sigma_2) = \tfrac{1}{2}n(1, -i)e^{-\iota\theta_1} \qquad (k = -1)$$
$$= \tfrac{1}{2}n(1, i)e^{-\iota\theta_1} \qquad (k = 1)$$
$$\rho_2 = \omega\rho_1, \ \rho_3 = \omega\rho_2 \ \ldots, \quad \phi_2 = \omega\phi_1, \ \phi_3 = \omega\phi_2,$$

the ω in the last formulæ being the nth root of unity.   Consider the case $k = 1$.   Then on reduction

$$x \cos \theta_p + y \sin \theta_p = \frac{n}{2a}(2\rho_1 + \iota a \phi_1)e^{\iota(\theta_p - \theta_1)}.$$

But
$$\theta_p - \theta_1 = 2\pi(p - 1)/n, \qquad \omega = e^{2\pi i/n},$$
$$e^{i(\theta_p - \theta_1)} = \omega^{p-1},$$

so that

$$x \cos \theta_p + y \sin \theta_p = \frac{n}{2a}(2\rho_p + ia\phi_p)$$

on reduction.   In a similar manner,

$$y \cos \theta_p - x \sin \theta_p = \frac{n}{2a}(2i\rho_p - a\phi_p).$$

If therefore

$$\beta = \frac{n}{2a} \cdot \frac{ve^2}{a^3} = \frac{nve^2}{2a^3}\bigg/\left(\frac{Mq^2a^3}{ve^2} + \tfrac{1}{2}n\right),$$

and if we include also the case $k = -1$, whose detailed working follows a similar analysis, we find the period equations to be derived from

$$(A - mq^2)\rho_p + A_1\rho_{p+1} + \ldots - 2ma\omega iq\phi_p + aB, \phi_{p+1} + \ldots$$
$$= -2\beta(2\rho_p \pm ia\phi_p).$$
$$2im\omega q\rho_p - B_1\rho_{p+1} - \ldots + (C - mq^2)a\phi_p - C_1a\phi_{p+1} - \ldots$$
$$= \beta(\pm 2\iota\rho_p - a\phi_p).$$

The upper sign refers to the case $k = 1$, and the lower to $k = -1$. Thus

$$(A - mq^2 + 4\beta)\rho_p + A_1\rho_{p+1} + \ldots + (-2im a\omega q \pm 2ia\beta)\phi_p$$
$$+ aB_1\phi_{p+1} + \ldots = 0$$
$$(2im\omega q \mp 2\iota\beta)\rho_p - B_1\rho_{p+1} - \ldots + (C - mq^2 + \beta)a\phi_p - C_1a\phi_{p+1} - \ldots$$
$$= 0$$

These are again of the old form.   The resulting period equation, derived as before, is

$$(A - mq^2 + 4\beta + L_k)(C - mq^2 + \beta - N_k) = (M_k - 2m\omega q \pm 2\beta)^2,$$

being different for the classes $+1$ and -1.   This may be written as

$$(L_k - L_0 - 3m\omega^2 - mq^2 + 4\beta)(N_0 - N_k - mq^2 + \beta)$$
$$= (M_k - 2m\omega q \pm 2\beta)^2,$$

where the term in β^2 disappears on expansion.

Effectively, as m/M is small, we may write $\beta = m\omega^2\gamma$, where

$$\gamma = \tfrac{1}{2}nv^2(v - \tfrac{1}{4}S_n)^{-2}\left(\frac{\omega^2}{q^2}\right)\left(\frac{m}{M}\right),$$

and the equation becomes

$$\left(\frac{L_k - L_0}{m\omega^2} - x^2 - 3\right)\left(\frac{N_0 - N_k}{m\omega^2} - x^2\right) - \left(\frac{M_k}{m\omega^2} - 2x\right)^2$$

$$= \gamma\left\{\frac{L_0 - L_k - 4N_0 + 4N_k \pm 4M_k}{m\omega^2} + (3 \mp 5x)(1 \pm x)\right\},$$

where x is q/ω. The solution may be derived by continued approximation. If x_0 is a root of the equation

$$f(x) = 0$$

or

$$\left(\frac{L_k - L_0}{m\omega^2} - x^2 - 3\right)\left(\frac{N_0 - N_k}{m\omega^2} - x^2\right) - \left(\frac{M_k}{m\omega^2} - 2x\right)^2 = 0,$$

the more accurate solutions for classes ± 1 are

$$q/\omega = x_0 + \{\gamma/f^1(x_0)\}\{(m\omega^2)^{-1}(L_0 - L_k - 4N_0 + 4N_k \pm 4M_k)$$

$$+ (3 \mp 5x)(1 \mp x)\}$$

These are the values of q relatively to the moving ring of electrons.

Numerical calculations for the neutral atom.—In proceeding to calculation, we must remember, from the investigations of Schott, that no spectral line of a class higher than 2 will be found ordinarily, although those of class 3 may appear occasionally in strong non-neutral systems.

When $n = 5$, the following results may be obtained, if δ denotes $e^2/8a^3$.

$$M_0 = 0, \qquad M_1 = 4.856\delta, \qquad M_2 = 2.103\delta$$
$$L_0 = 17.6788\delta, \qquad L_1 = 0.511\delta, \qquad L_2 = -9.352\delta$$
$$N_0 = 18.8408\delta, \qquad N_1 = 2.9716\delta, \qquad N_2 = -12.395\delta$$

Consider first the vibrations of class 2. Since by the equation of steady motion,

$$\delta/m\omega^2 = (28.99)^{-1},$$

the period equation is

$$\left(x^2 + 3 + \frac{27.031}{28.99}\right)\left(x^2 - \frac{31.236}{28.99}\right) = \left(2x - \frac{2.103}{28.99}\right)^2$$

or

$$x^4 - 1.1452x^2 + 0.2902x - 4.2428 = 0.$$

This equation, in common with *all* the quartics to which we are subsequently led, has one real and positive, one real and negative, and two complex roots. Thus there are always unstable

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vibrations in the plane of the atom, and these vibrations supply the mechanism by which it is possible for the various positively and negatively charged systems to exist in presence of each other. When an unstable vibration has resulted in the ejection of one or more electrons from a system, the new system can again take them up until the unstable modes arise again. Thus in a vibrating gas of this type, there would ultimately be a kind of equilibrium established in the interchange of electrons, and spectral lines might appear in correspondence with all possible positive or negative degrees of charge.

The real roots of the quartic in the present case are

$$x = 1\cdot61, \ -1\cdot66,$$

or

$$q/\omega = 1\cdot61, \ -1\cdot66.$$

Since they are of class 2, the corresponding vibrations for an observer are derived from

$$q/\omega = 3\cdot61, \ 0\cdot39, \ 3\cdot66, \ 0\cdot34,$$

and since the line $q = \omega$ is at $\lambda 3987$, none of these vibrations are in the visible spectrum.

The real part of the complex roots, moreover, is half the sum of the real roots, or $-\cdot025$, and gives vibrations of about half the wave-length of the principal line, which are not in the spectrum. Thus no vibration of class 2 for this system could be perceptible.

Since the period equations for classes k, $n{-}k$ are identical, we may show that no vibration of class 3 could fall in the visible spectrum.

In general we shall find, in accordance with a remark in the first paper, that the number of vibrations in the plane which give lines falling in the visible spectrum, is very small. Except for the case $q = \omega$ of class zero, the only remaining possibilities for the neutral atom are the vibrations of classes ± 1.

If $q/\omega = x$, the quartic defining these vibrations is found to be

$$x^4 - 0\cdot95519x^2 + 0\cdot6696x - 1\cdot9945 = 0,$$

when the term containing γ is ignored. The two real roots are

$$x_0 = 1\cdot2827, \ -1\cdot5085.$$

As the existence of the term in γ cannot affect the first decimal place, the possible vibrations for an observer are given by

$$q/\omega = \pm 1 + 1\cdot3, \ \pm 1 - 1\cdot5,$$

and none are in the visible spectrum. We are reduced therefore to a consideration of the unstable vibrations. If they are of the type $x = \theta \pm i\phi$ relatively to the rotating electrons,

$$2\theta + 1\cdot2827 - 1\cdot5085 = 0,$$

or $\theta = \cdot 1129$, so that the vibrations for an observer, uncorrected for γ, have wave-lengths determinable from $q/\omega = 1\cdot 1129,\ \cdot 8871$.

Since the wave-length corresponding to $q = \omega$ is $\lambda\ 3987$, these approximate wave-lengths become 3582, 4493. Nothing in the spectrum is close to the second line, and the nearest wave-length to the first is Campbell's line $\lambda 3602$, which is much too strong, although not found by Dyson, to be due to a vibration of this character.

A somewhat laborious application of the correction for m/M indicates that the true wave-length is $3987/(1\cdot 1129 - 4\cdot 6\ m/M)$, and only an increase of about 8 tenth-metres is produced, so that we do not get Campbell's line.

We conclude, therefore, as might be expected from the large argument of the real exponential of these waves, that unstable vibrations of this type are not strong and continuous enough to give observable lines. But the formal verification given here seems necessary for completeness. We must notice that two vibrations for which q/ω is small are introduced when the positive mass occurs in the period equation; but these in the present case, as in most others, are of the unstable type, and manifest themselves with a frequency $q = \omega$, so that they coincide in wave-length with the principal line.

Protofluorine with two electrons.—If $\delta = e^2/8a^3$, we have for this system,

$$M_0 = M_1 = 0, \quad L_0 = 2\delta, \quad L_1 = -2\delta, \quad N_0 = -N_1 = \delta,$$

and the first approximation to vibrations of class 1 is given by

$$\left(x^2 + \frac{59}{19}\right)\left(x^2 - \frac{1}{19}\right) = 4x^2,$$

a soluble biquadratic. But the real roots are nearly equal to ± 1, and the imaginary roots have no real part. The correction to the latter is, in its real part, of magnitude $7\cdot 4\ m/M$. The real roots lead to no vibrations in the visible range, and the corrected imaginary roots to no visible lines. If it might be supposed that the considerable radiation in this case, when an electron is departing, should be manifested, it must be remembered that the system is very weak, and that even one of the vibrations of class zero is missing, so that unstable vibrations can hardly appear, especially when absent from the neutral atomic spectrum.

Protofluorine with three electrons.—For this special case, since the quartics for classes $k,\ n-k$ are always identical except for a possible term in m/M, the quartic for class 2 is the first approximation to that for class 1, namely

$$x^4 - 0\cdot 9347x^2 + 0\cdot 1304x - 0\cdot 5276 = 0.$$

The roots are ultimately found to be $x = 1\cdot 1148,\ -1\cdot 2096$, so that for an observer, there are no vibrations of class 1 in the visible range of spectrum. For class 2 there are theoretically

two, but it is not likely that they can ever be observed, for the system is weak in the corona, and vibrations of class 2 are only important even to any moderate extent for negatively charged systems. Their wave-lengths are 5180, 5802.

Protofluorine with four electrons.—This is a much stronger system, whose principal line is $\lambda 4231$. The quartic for class 2 is

$$x^4 - x^2 - 1 \cdot 8494 = 0,$$

whose real solution is $x = 1 \cdot 3960$. The vibration nearest to the visible spectrum is given to an observer as $q/\omega = 0 \cdot 6040$, and has a wave-length 7005, far in the infra-red.

The vibrations of class 3 accurately, and of class 1 approximately, satisfy

$$x^4 - 0 \cdot 9382 x^2 + 0 \cdot 3498 x - 1 \cdot 1053 = 0,$$

which has real solutions $x = 1 \cdot 1926, -1 \cdot 3451$. With a principal line $\lambda 4231$, we find that neither class 1 nor class 3 give any lines in the visible range.

Summary.—In the present paper, the suggested theory of the coronal spectrum has been put upon a definite basis which is in accord with the recent theories of emission of energy by bodies. It is indicated that the key to the physical side of these theories lies in the fact that an expulsion or retention of an electron by any atom probably involves a discontinuous change in the angular momentum of the atom, which is dependent on the number of electrons already present.

This view leads to a determination of spectral lines which is free from the dependence on the method of ratios.

It has been shown that coincidences between observed and calculated wave-lengths can be rendered exact by a process which shows at the same time that the atomic weights of nebulium and protofluorine are comparable with that of hydrogen.

The complete spectrum of protofluorine due to neutral or positively charged atoms is given in the table.

Wave-length.	Intensity.	Origin.	
5073	1	Pf,	$+3e$
4586	4	Pf,	$+2e$
4311	2	Pf,	$+2e$
4231	5	Pf,	$+e$
3987·1	3	Pf,	neutral
3800·9	3	Pf,	$+e$
3387·9	12	Pf,	neutral

The origin of these lines is now fairly certain. All other lines of protofluorine in the spectrum must be due to negatively charged atoms.

Trinity College, Cambridge :
1912 *April* 28.

On the New Nebular Line at λ4353.   By J. W. Nicholson, M.A., D.Sc.

At the meeting of the Society of 1912 March the writer announced the discovery of the new nebular line at λ4353 which had been predicted in his paper on "The Spectrum of Nebulium." *   A plate of the spectrum of the Orion nebula, on which the line was found, and which had been taken with a long exposure at the Lick Observatory in 1908 by Dr. W. H. Wright, was also exhibited.   In the meantime the line has been recorded again by Dr. Max Wolf, of Heidelberg, who has, in a letter, given an account of its discovery, and this brief note gives a record of some of the details of the observation.

The plate on which the line is shown was exposed at Heidelberg between 1912 January 20 and February 28, with an exposure of 40^h 48^m.   The most northern star of the Trapezium is in the centre of the photographed region, and the new line is visible fairly strongly, especially in the spectrum of the star and on both sides.

The wave-length in the Orion nebula, obtained by plotting from an iron curve, is 4353·9, which is, of course, too large, as all the lines in this nebula are shifted to greater wave-lengths, on account of the motion of the nebula.†   But the correction is not so large as a tenth-metre.

The wave-length of the line on the Lick plate, as measured at the Cambridge Observatory by Mr. Stratton, is 4353·3, the value calculated in the paper being 4352·9.

Trinity College, Cambridge :
    1912 May 8.

Photographic Magnitudes of Stars brighter than $9^{m}·0$ within 5° of the North Pole.   By F. W. Dyson and C. Davidson.

The determination of the photographic magnitudes of the stars in the Greenwich section of the Astrographic Catalogue is being made by comparison with Prof. Pickering's North polar sequence. As there are no stars brighter than $8^{m}·9$ within 1° of the North Pole, the photographs taken with the Astrographic telescope have been supplemented by a series taken with the Franklin-Adams 6-inch lens, which well covers a field of 3° radius.   Each field is photographed when its centre is at the same altitude as the pole, and the polar area containing Pickering's standards is photographed on the same plate with an equal exposure.

It was found to be desirable to amplify the number of stars of the polar area available for purposes of comparison so as to include

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* *Monthly Notices*, vol. lxxii., No. 1 (November 1911).
† Keeler, *Lick Observatory Publications*, iii., 1894 ; Hartmann, *Ast. Jour.*, xv. 1902, p. 287.