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Basic Physics Course with MATLAB's Symbolic Toolbox and Live Editor

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1.2 Spherical coordinates (Computational example)

We consider spherical coordinates $\vec{r} = \begin{pmatrix} r\sin\theta\cos\varphi \\ r\sin\theta\sin\varphi \\ r\cos\theta \end{pmatrix}$

```
clear all
syms r phi theta r_v real
```

1 Space vector $\vec{r}$

```
r_v=[r * sin(theta) * cos(phi); r * sin(theta) * sin(phi); r * cos(theta)]
```

r_v =

$$\begin{pmatrix} r\cos(\varphi)\sin(\theta) \\ r\sin(\varphi)\sin(\theta) \\ r\cos(\theta) \end{pmatrix}$$

2 Jacobian *J* und metric $G = J^t J$

$J = \left(\frac{\partial x_i}{\partial q_j}\right)$, $G = J'J$, $G_{inv} = G^{-1}$

```
J=jacobian(r_v,[r theta phi])
```

J =

$$\begin{pmatrix} \cos(\varphi)\sin(\theta) & r\cos(\varphi)\cos(\theta) & -r\sin(\varphi)\sin(\theta) \\ \sin(\varphi)\sin(\theta) & r\cos(\theta)\sin(\varphi) & r\cos(\varphi)\sin(\theta) \\ \cos(\theta) & -r\sin(\theta) & 0 \end{pmatrix}$$

```
G=simplify(J'*J)
```

G =

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\sin(\theta)^2 \end{pmatrix}$$

```
G_inv=G^-1
```

G_inv =

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & \frac{1}{r^2\sin(\theta)^2} \end{pmatrix}$$

3 Volume of a sphere

Invariant volume element: $V_{inv} = \sqrt{(det(G))}$

Volume: $V = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} V_{inv} dr d\theta d\varphi$

```
syms R V
assume(r>0 & theta>0 & theta<pi)
V_inv=simplify(sqrt(det(G)))
```

V_inv = $r^2\sin(\theta)$

```
V=int(int(int(simplify(V_inv),r,0,R),theta,0,pi),phi,0,2*pi)
```

V =

$$\frac{4\,\pi\,R^3}{3}$$

4 Canonical momentum in spherical coordinates

Generalized momenta: $\begin{pmatrix} p_r \\ p_\vartheta \\ p_\varphi \end{pmatrix} = mG\begin{pmatrix} \dot{r} \\ \dot{\vartheta} \\ \dot{\varphi} \end{pmatrix}$

```
syms m p_v p_r p_theta p_phi T real
syms r(t) theta(t) phi(t)
Gvq=G*[r(t);theta(t);phi(t)];
disp(p_r==m*diff(Gvq(1),t)),disp(p_theta==m*diff(Gvq(2),t)),disp(p_phi==m*diff(Gvq(3),t))
```

$$p_r = m \frac{\partial}{\partial t} r(t)$$

$$p_\theta = m\, r^2 \frac{\partial}{\partial t} \theta(t)$$

$$p_\varphi = m\, r^2 \sin(\theta)^2 \frac{\partial}{\partial t} \varphi(t)$$

5 Kinetic energy

$$T = \frac{1}{2m}(p_r, p_\theta, p_\varphi) G^{-1} \begin{pmatrix} p_r \\ p_\theta \\ p_\varphi \end{pmatrix}$$

```
[p_r;p_theta;p_phi]
```

ans =

$$\begin{pmatrix} p_r \\ p_\theta \\ p_\varphi \end{pmatrix}$$

```
T=1/(2*m)*ans'*G_inv*ans
```

T =

$$\frac{{p_r}^2}{2\,m} + \frac{{p_\theta}^2}{2\,m\,r^2} + \frac{{p_\varphi}^2}{2\,m\,r^2 \sin(\theta)^2}$$