EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

MATHEMATISCH-
NATURWISSENSCHAFTLICHE FAKULTÄT
Fachbereich Physik

Basic Physics Course with MATLAB's Symbolic Toolbox and Live Editor

Kurt Bräuer

1.3 Metric of a Mexican hat (Computational example)

```
clear all
syms r s phi real
```

1 Parameterization of the hat and the path over the hat

We parameterize a Mexican hat by

$$\vec{r} = \begin{pmatrix} r\cos\varphi \\ r\sin\varphi \\ r^4 - r^4 \end{pmatrix} \text{ mit } r \in [0, 0.9],\ \ \varphi \in [0, 2\pi]$$

A path over the hat is parametrized by $\varphi \to 0,\ \ r \to s \in [-0.9, 0.9]$

$$\vec{r}_w = \begin{pmatrix} s \\ 0 \\ s^4 - s^4 \end{pmatrix}$$

```
r_v=[r*cos(phi);r*sin(phi);r^4-r^2]
```

r_v =

$$\begin{pmatrix} r\cos(\varphi) \\ r\sin(\varphi) \\ r^4 - r^2 \end{pmatrix}$$

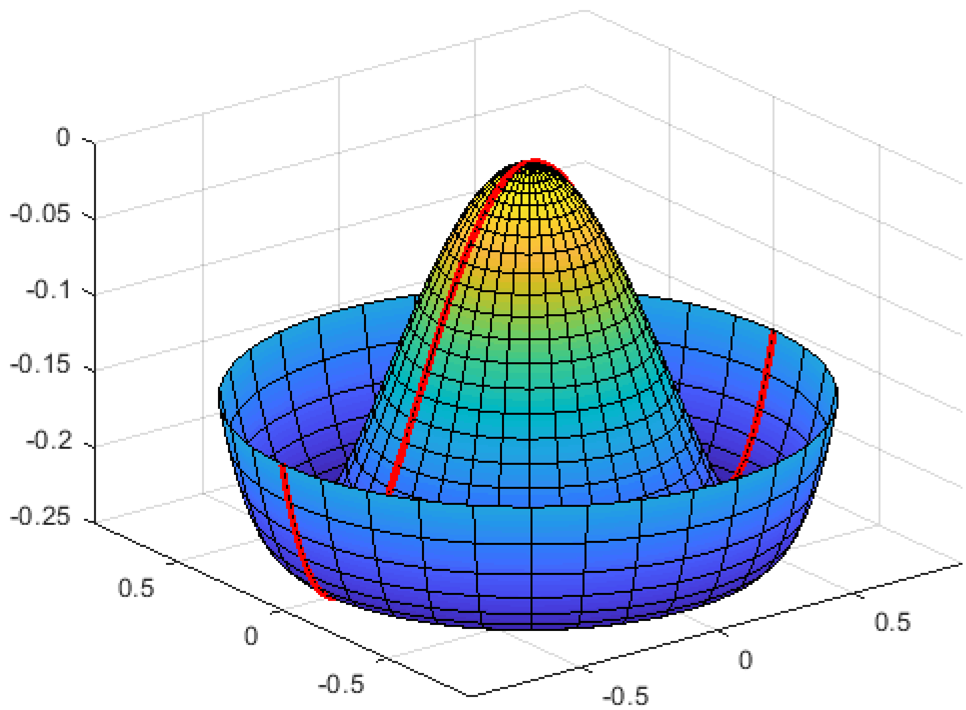
```
r_w=sube(r_v,[phi==0,r==s])
```

r_w =

$$\begin{pmatrix} s \\ 0 \\ s^4 - s^2 \end{pmatrix}$$

2 Graphic representation (includes path over the hat)

```
hold off
fsurf(r_v(1),r_v(2),r_v(3),[0 2*pi 0 0.9])
hold on
fplot3(r_w(1),r_w(2),r_w(3),0.9*[-1 1],'r','LineWidth',3)
```



3 Jacobian *J* und metric *G*

$$J = \left(\frac{\partial x_i}{\partial q_j}\right),\ G = J' J$$

```
J=jacobian(r_v,[r,phi])
```

J =

$$\begin{pmatrix} \cos(\varphi) & -r\sin(\varphi) \\ \sin(\varphi) & r\cos(\varphi) \\ 4r^3 - 2r & 0 \end{pmatrix}$$

```
G=simplify(J'*J)
```

G =

$$\begin{pmatrix} 16\,r^6 - 16\,r^4 + 4\,r^2 + 1 & 0 \\ 0 & r^2 \end{pmatrix}$$

4 Surface of the Mexican hat

Invariant volume element $V_{inv} = \sqrt{(det(G))}$

Surface $O = \int_0^{2\pi} \int_0^{0.9} V_{inv} dr d\varphi$

```
V_inv=sqrt(det(G))
```

V_inv = $\sqrt{r^2\ (16\,r^6 - 16\,r^4 + 4\,r^2 + 1)}$

```
O=vpaintegral(int(V_inv,phi,0,2*pi),r,0,0.9)
```

O = 2.82787

5 Path length across the hat

$J_s = \left(\frac{\partial r_{wi}}{\partial s}\right)$, $S = \int_{-0.9}^{0.9} \sqrt{det(G_s)}\, ds = \int_{-0.9}^{0.9} \sqrt{J_s' J_s}\, ds$

```
J=jacobian(r_w)
```

J =

$$\begin{pmatrix} 1 \\ 0 \\ 4\,s^3 - 2\,s \end{pmatrix}$$

```
S=vpaintegral(sqrt(J'*J),-0.9,0.9)
```

S = 1.96157