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Basic Physics Course with MATLAB's Symbolic Toolbox and Live Editor

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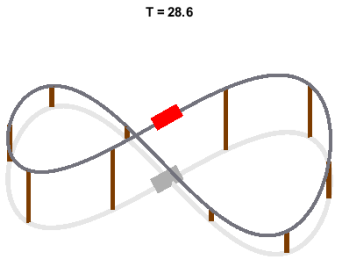
4.4 Roller coaster (Computational example)



1 Parameterization of the track

Trajectory: $\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sin(2\pi q) \\ \frac{1}{2}\sin(4\pi q) \\ \frac{1}{4}\cos(2\pi q) \end{pmatrix}, \quad q \in [0,1]$

Parameters: energy $E = 0.2501$, mass and gravitational acceleration $m = g = 1$

```
clear all
syms q p E t real
Par=[E==0.25001]
```

```
Par =
```

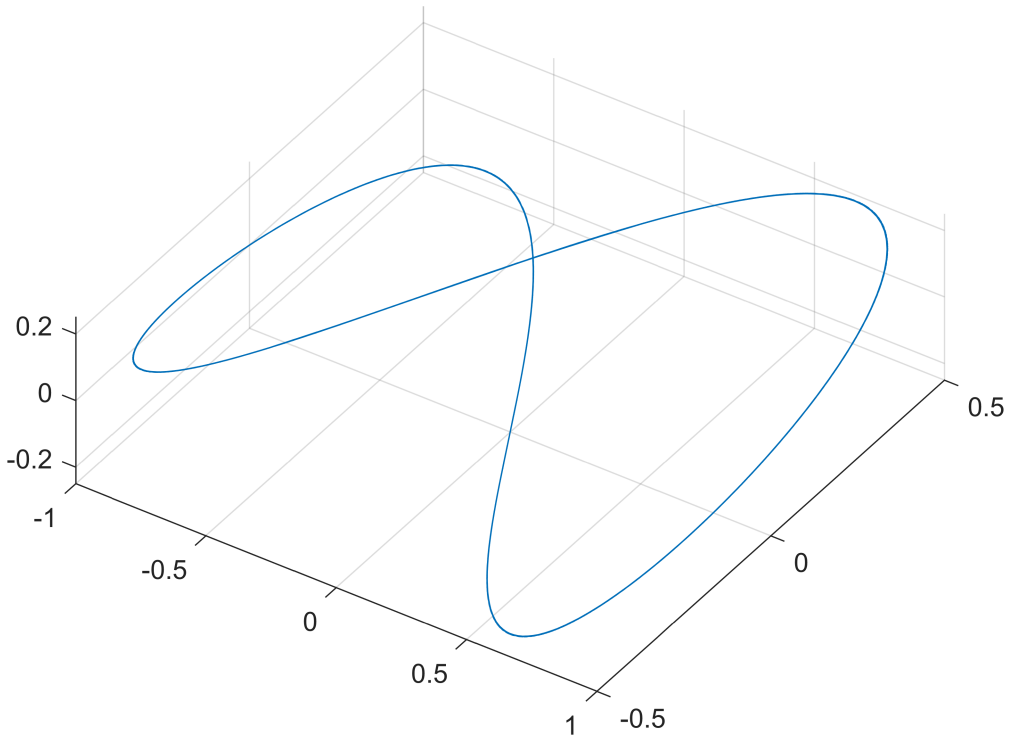
$$\mathrm{E} = \frac{25001}{100000}$$

```
r_v=[sin(2*pi*q);sin(4*pi*q)/2;cos(2*pi*q)/4]
```

```
r_v =
```

$$\begin{pmatrix} \sin(2\,\pi\,q) \\ \frac{\sin(4\,\pi\,q)}{2} \\ \frac{\cos(2\,\pi\,q)}{4} \end{pmatrix}$$

```
fplot3(r_v(1),r_v(2),r_v(3))
view([33.716 66.000])
```



2 Jacobian and metric

Jacobian *J*, metric *G* and contravariant metric G_i: $J = \left(\frac{\partial x_i}{\partial q_j}\right),\ G = J'J,\ G_i = G^{-1}$

```
J=jacobian(r_v,q)
```

J =

$$\begin{pmatrix} 2\,\pi\cos(2\,\pi\,q) \\ 2\,\pi\cos(4\,\pi\,q) \\ -\frac{\pi\sin(2\,\pi\,q)}{2} \end{pmatrix}$$

```
G=J'*J
```

G =

$$4\,\pi^2\cos(2\,\pi\,q)^2+4\,\pi^2\cos(4\,\pi\,q)^2+\frac{\pi^2\sin(2\,\pi\,q)^2}{4}$$

```
G_i=G^-1;
```

3 Hamiltonian

$$H=\frac{1}{2m}pG_ip+mgz(q)$$

```
H=p*G_i*p+r_v(3)
```

H =

$$\frac{\cos(2\,\pi\,q)}{4}+\frac{p^2}{4\,\pi^2\cos(2\,\pi\,q)^2+4\,\pi^2\cos(4\,\pi\,q)^2+\frac{\pi^2\sin(2\,\pi\,q)^2}{4}}$$

4 Momentum field

$$H(q,p)=E\ \rightarrow\ p(q,E)$$

```
e1=isolate(H==E,p)
```

e1 =

$$p=\frac{\pi\ \sqrt{4\,\mathrm{E}-\cos(2\,\pi\,q)}\ \ \sqrt{16\cos(2\,\pi\,q)^2+16\cos(4\,\pi\,q)^2+\sin(2\,\pi\,q)^2}}{4}$$

5 Period time

$$T=\int_{t(0)}^{t(1)}dt=\int_{t(0)}^{t(1)}\frac{m\dot{q}}{G_ip}dt=\int_0^1\frac{m}{G_ip}dq$$

```
T=double(vpaintegral(G/sube(p,[e1,Par]),0,1))
```

T = 47.2180

6 Equations of motion and numerical solution

$$m\dot{q}=G_ip(q),\ m=1$$

```
syms Q(t)
N=150;
G_i*sube(p,e1);
odeFunction(sube(ans,[Par,q==Q(t)]),Q(t));
[tn,q_n]=ode45(ans,linspace(0,T,N),0);
```

```
matlabFunction(r_v);
r_vn=ans(q_n');
```

7 Animation of the movement→ 'Roller_Coaster.gif'

Tangent vector for plot of the car

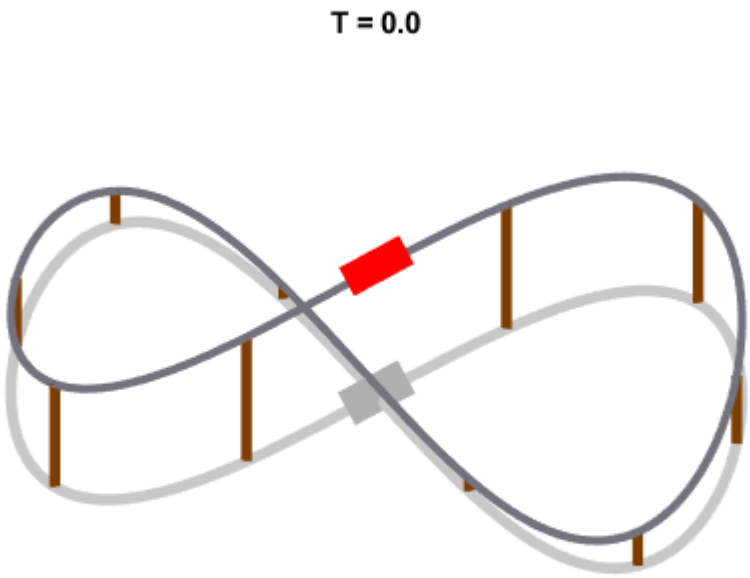
```
tang=(r_vn(:,2:N)-r_vn(:,1:N-1));
tang=[tang(:,1) tang];
nor=1./sqrt((diag(tang'*tang)'));
```

```
figure
n=1
```

```
n = 1
```

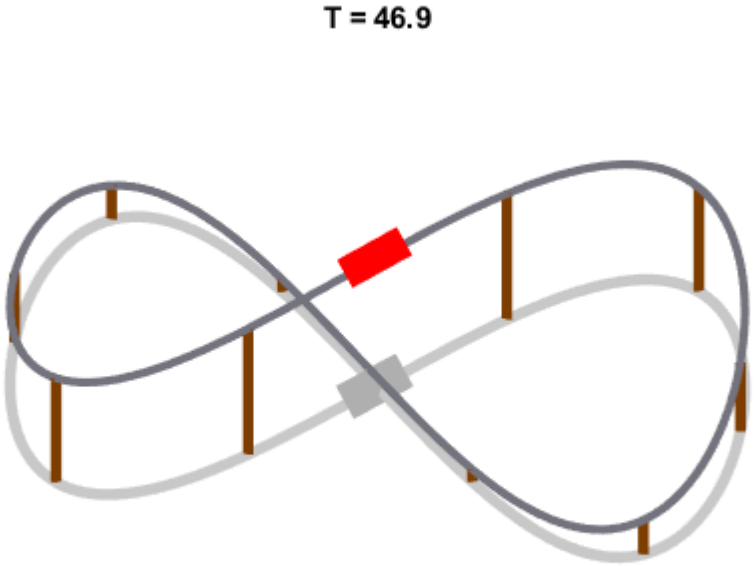
```
FiNa='Roller_Coaster.gif';
fplot3(r_v(1),r_v(2),r_v(3),'Color',.5*[.9 .9 1],'LineWidth',3);
hold on
fplot3(r_v(1),r_v(2),0*r_v(3)-0.29,'Color',.8*[1 1 1],'LineWidth',4);
K=10;
for k=1:K
    sube(r_v,q==(k-0.5)/K);
    plot3(ans(1)*[1 1],ans(2)*[1 1],[-0.29 ans(3)],...
        'Color',.5*[1 .5 0],'LineWidth',4)
end
eps=.1;
d=0.01;
dr=eps*tang(:,n)*nor(n);
HB=plot3(...
    [r_vn(1,n)-dr(1) r_vn(1,n)+dr(1)],...
    [r_vn(2,n)-dr(2) r_vn(2,n)+dr(2)],...
    [r_vn(3,n)-dr(3)+d r_vn(3,n)+dr(3)+d],...
    'Color','r','LineWidth',12);
HB2=plot3(...
    [r_vn(1,n)-dr(1) r_vn(1,n)+dr(1)],...
    [r_vn(2,n)-dr(2) r_vn(2,n)+dr(2)],...
    -0.28*[1 1],...
    'Color',.7*[1 1 1],'LineWidth',14);

HT=title(sprintf('T = %2.1f',tn(n)));
axis equal
axis([-1.1 1.1 -0.6 0.6])
axis off
view_kb(n,T,tn)
hold off
im=frame2im(getframe(gcf));
```




```
[A,map] = rgb2ind(im,32);
imwrite(A,map,FiNa,'gif','LoopCount',Inf,'DelayTime',0);
```

```
for n=2:N-1
    dr=eps*tang(:,n)*nor(n);
    HB.XData=[r_vn(1,n)-dr(1) r_vn(1,n)+dr(1)];
    HB.YData=[r_vn(2,n)-dr(2) r_vn(2,n)+dr(2)];
    HB.ZData=[r_vn(3,n)-dr(3)+d r_vn(3,n)+dr(3)+d];
    HB2.XData=[r_vn(1,n)-dr(1) r_vn(1,n)+dr(1)];
    HB2.YData=[r_vn(2,n)-dr(2) r_vn(2,n)+dr(2)];
    HT.String=sprintf('T = %2.1f',tn(n));
    view_kb(n,T,tn)
    im=frame2im(getframe(gcf));
    [A,map]=rgb2ind(im,32);
    imwrite(A,map,FiNa,'gif','WriteMode','append','DelayTime',0);
end
```




```
function view_kb(n,T,tn)
    view([20*sin(tn(n)/T*2*pi)+10 50+10*sin(tn(n)/T*2*pi)])
end
```