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Basic Physics Course with MATLAB's Symbolic Toolbox and Live Editor

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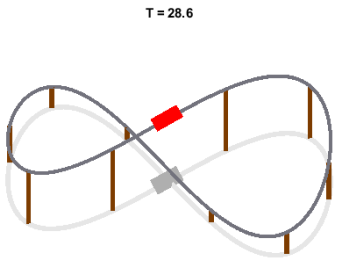
4.4 Roller coaster (Computational example)




```
clear all
```

1 Parameterization of the track

Trajectory: $\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sin(2\pi q) \\ \frac{1}{2}\sin(4\pi q) \\ \frac{1}{4}\cos(2\pi q) \end{pmatrix}, \quad q \in [0,1]$

Parameters: energy $E = 0.2501$, mass and gravitational acceleration $m = g = 1$

```
syms q p E t real
Par=[E==0.25001]
```

```
Par =
```

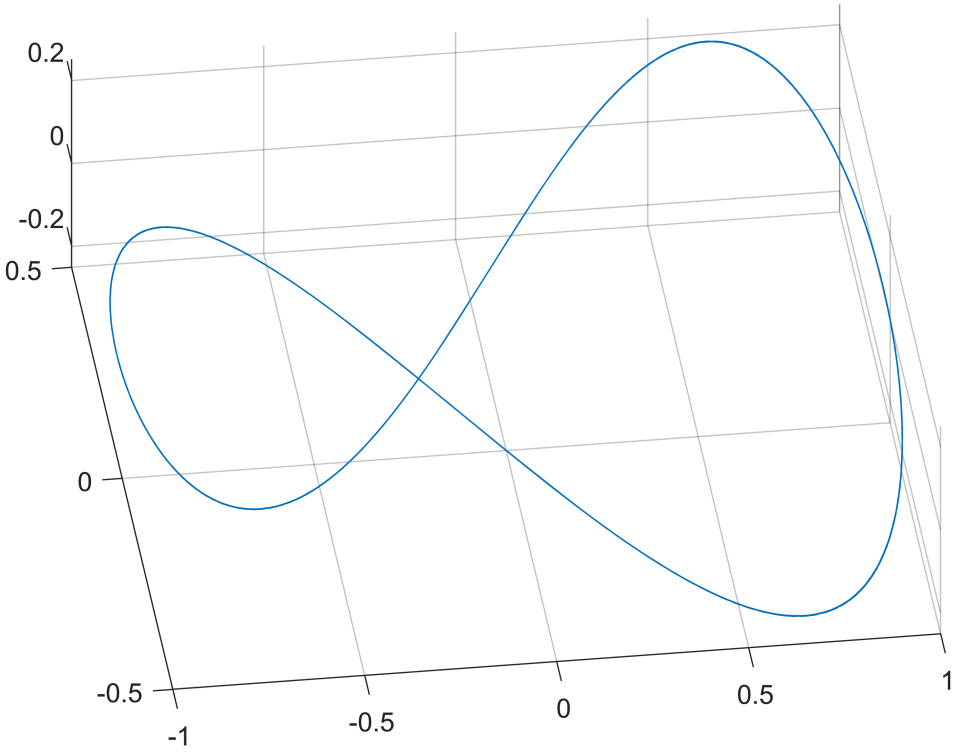
$$\mathrm{E} = \frac{25001}{100000}$$

```
r_v=[sin(2*pi*q);sin(4*pi*q)/2;cos(2*pi*q)/4]
```

```
r_v =
```

$$\begin{pmatrix} \sin(2\,\pi\,q) \\ \frac{\sin(4\,\pi\,q)}{2} \\ \frac{\cos(2\,\pi\,q)}{4} \end{pmatrix}$$

```
figure
fplot3(r_v(1),r_v(2),r_v(3))
view([-7.5 64.])
```



2 Jacobian and metric

Jacobian *J*, metric *G* and contravariant metric G_i: $J = \left(\frac{\partial x_i}{\partial q_j}\right),\ G = J'J,\ G_i = G^{-1}$

```
J=jacobian(r_v,q)
```

```
J =
```

$$\begin{pmatrix} 2\,\pi\cos(2\,\pi\,q) \\ 2\,\pi\cos(4\,\pi\,q) \\ -\frac{\pi\sin(2\,\pi\,q)}{2} \end{pmatrix}$$

```
G=J'*J
```

```
G =
```

$$4\,\pi^2\cos(2\,\pi\,q)^2+4\,\pi^2\cos(4\,\pi\,q)^2+\frac{\pi^2\sin(2\,\pi\,q)^2}{4}$$

```
G_i=G^-1;
```

3 Hamiltonian

$$H=\frac{1}{2m}pG_ip+mgz(q)$$

```
H=p*G_i*p+r_v(3)
```

H =

$$\frac{\cos(2\,\pi\,q)}{4}+\frac{p^2}{4\,\pi^2\cos(2\,\pi\,q)^2+4\,\pi^2\cos(4\,\pi\,q)^2+\frac{\pi^2\sin(2\,\pi\,q)^2}{4}}$$

4 Momentum field

$$H(q,p)=E\ \rightarrow\ p(q,E)$$

```
e1=isolate(H==E,p)
```

e1 =

$$p=\frac{\pi\,\sqrt{4\,\mathrm{E}-\cos(2\,\pi\,q)}\,\sqrt{16\cos(2\,\pi\,q)^2+16\cos(4\,\pi\,q)^2+\sin(2\,\pi\,q)^2}}{4}$$

5 Period time

$$T=\int_{t(0)}^{t(1)}dt=\int_{t(0)}^{t(1)}\frac{m\dot{q}}{G_ip}dt=\int_0^1\frac{m}{G_ip}dq$$

```
T=double(vpaintegral(G/sube(p,[e1,Par]),0,1))
```

```
T = 47.2180
```

6 Equations of motion and numerical solution

$$m\dot{q}=G_ip(q),\ m=1$$

```
syms Q(t)
N=150;
G_i*sube(p,e1);
odeFunction(sube(ans,[Par,q==Q(t)]),Q(t));
[tn,q_n]=ode45(ans,linspace(0,T,N),0);
```

7 Animation of the movement→ 'Roller_Coaster.gif'

```
figure
```

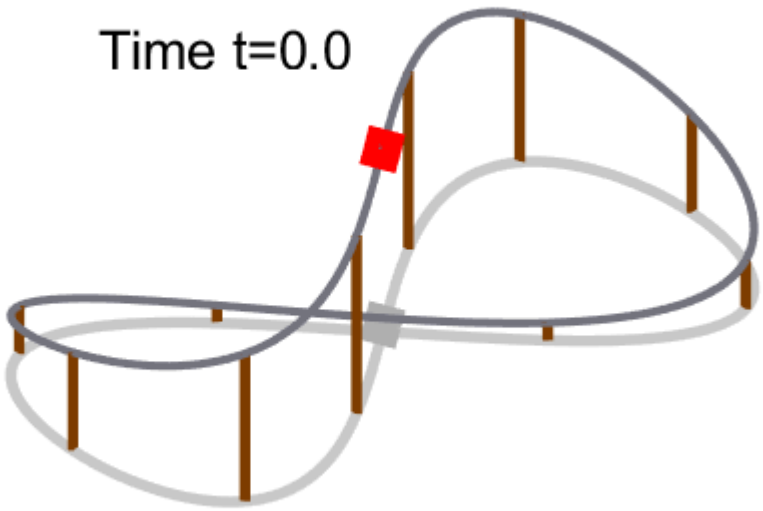
```
n=1;
fplot3(r_v(1),r_v(2),r_v(3),'Color',.5*[.9 .9 1],'LineWidth',3);
hold on
fplot3(r_v(1),r_v(2),0*r_v(3)-0.29,'Color',.8*[1 1 1],'LineWidth',4);
K=10;
for k=1:K
    sube(r_v,q==(k-0.5)/K);
    plot3(ans(1)*[1 1],ans(2)*[1 1],[-0.29 ans(3)],...
        'Color',.5*[1 .5 0],'LineWidth',4)
end
axis equal; axis off
```

Tangent vector for car plot

```
diff(r_v,q);
ta=simplify(ans/sqrt(ans'*ans));

eps=.1;
ta_x=matlabFunction(eps*ta(1));
ta_y=matlabFunction(eps*ta(2));
ta_z=matlabFunction(eps*ta(3));
```

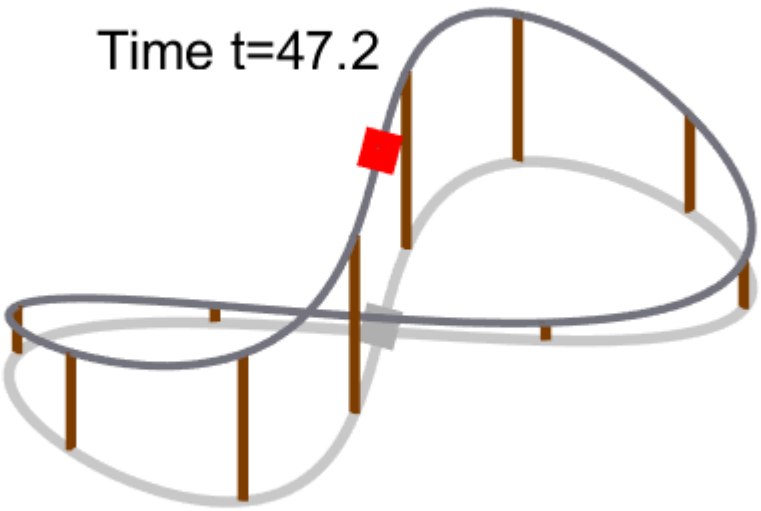
Plot

```
rx=matlabFunction(r_v(1));
ry=matlabFunction(r_v(2));
rz=matlabFunction(r_v(3));

Nt=size(tn,1);
ft=@(t)(tn(round(1+(Nt-1)*t)));
fq=@(t)(q_n(round(1+(Nt-1)*t)));

Car=@(t0)plot3([rx(fq(t0))-ta_x(fq(t0)) rx(fq(t0))+ta_x(fq(t0))],[ry(fq(t0))-ta_y(fq(t0)) ry(fq
Car_S=@(t0)plot3([rx(fq(t0))-ta_x(fq(t0)) rx(fq(t0))+ta_x(fq(t0))],[ry(fq(t0))-ta_y(fq(t0)) ry(

fanimator(Car,'AnimationRange',[0 1],'FrameRate',100);
fanimator(Car_S,'AnimationRange',[0 1],'FrameRate',100);
fanimator( @(t0)text(0,1.2,sprintf('Time t=%.1f',ft(t0)),'FontSize',20), 'AnimationRange',[0 1
```




```
%playAnimation
```

```
writeAnimation('Roller_Coaster.gif','LoopCount',inf)
```


Time t=47.2