EBERHARD KARLS UNIVERSITÄT TÜBINGEN

MATHEMATISCH-NATURWISSENSCHAFTLICHE FAKULTÄT
Fachbereich Physik

Basic Physics Course with MATLAB's Symbolic Toolbox and Live Editor

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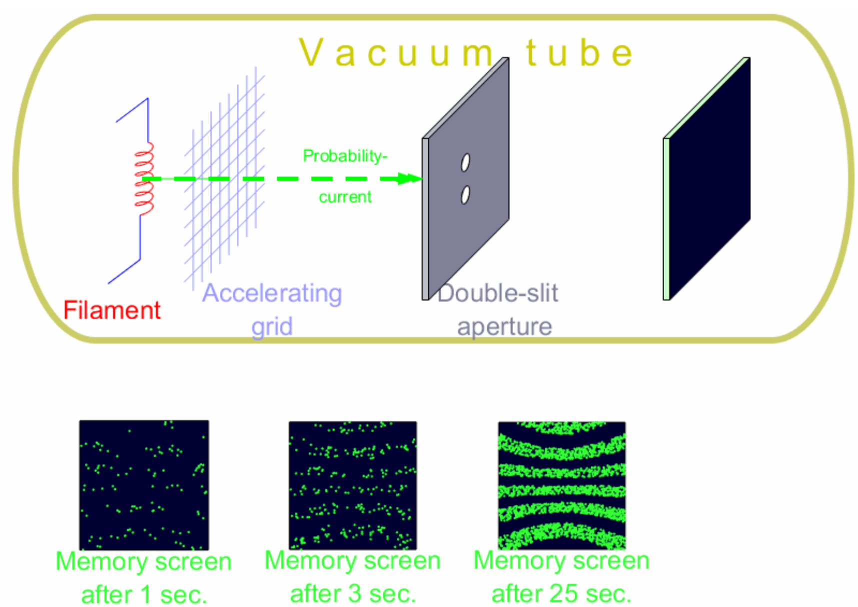
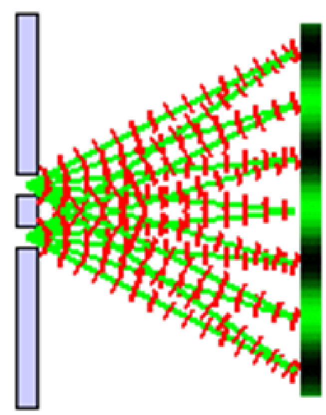
5.1 Quantum mechanics

The quantum-mechanical double-slit experiment

Electrons in a vacuum tube, for example, are only indirectly observable, appearing as blackening on a photographic paper or as a flash of light on a detector screen. If the electron beam is passed through a double-slit aperture, one still observes individual events on the screen, which are assigned to individual electrons. However, many such events form an interference pattern on the screen that clearly indicates a wave phenomenon. This cannot be explained by the effect of single (objective) electrons.





Observation of an electron beam in a vacuum tube. Individual points of light appear on the detector screen, like individual electrons.

Upon prolonged observation, many of these individual events form an interference pattern, like a wave.

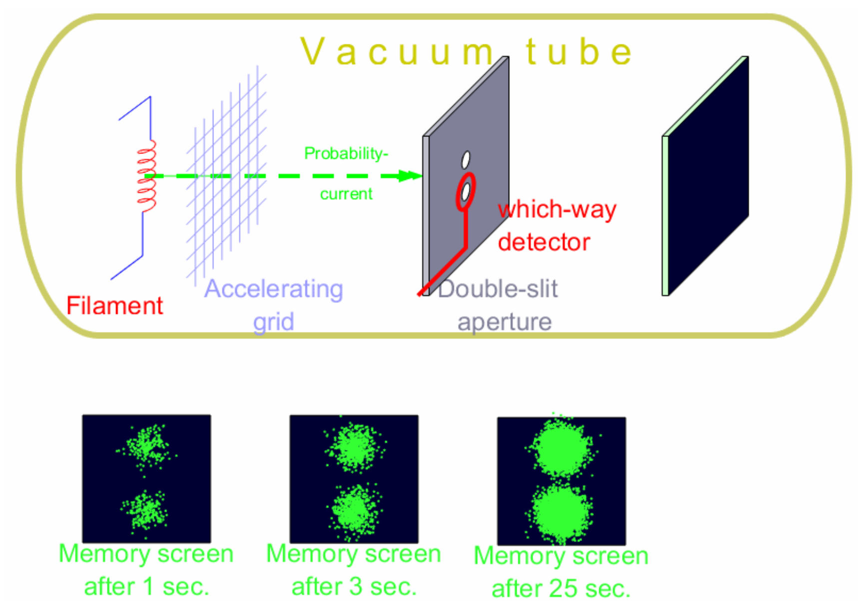
Wave–particle duality

Particle phenomenon	Wave phenomenon
Manifestation of individual events on the screen	Interference pattern

Double-slit with which-way detector (WWD)

The electron beam in the vacuum tube as an electrical current is the source of a magnetic field. This can be detected by a probe. This way, the 'path' of the electron can be determined through the aperture .

In 50% of the events, the electron is detected at the lower slit, and in 50% at the upper slit. The interference pattern on the detector screen disappears. Many electrons form a shadow of the aperture.



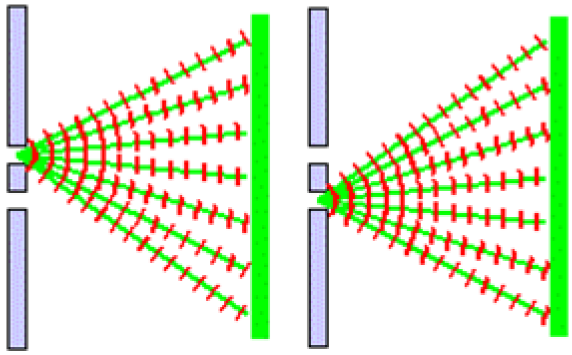
This influence of a detector on the phenomenon is neither compatible with the motion of material particles nor material waves. In the Copenhagen interpretation of quantum mechanics, reference is made solely to the observation at the electron source (filament) and at the detector. The causal relationship is established by the propagation laws of the action and the preservation of the measurement probability.

$$\rho(x,t)=\begin{cases}\cancel{\text{Probability density of a particle}}\\ \text{Probability density for the response of a detector}\\ \rightarrow\quad\text{Copenhagen interpretation}\end{cases}$$

(5-1)

The probability density for the response of a detector includes many possible positions. In the double-slit experiment, for example, these are the interference fringes. In the single measurement, one of these possibilities manifests itself.

In the double-slit experiment with *WWD*, the electron is observed at one slit. The probability contribution of the other slit disappears, and thus the interference pattern on the screen is reduced to a shadow image of the aperture.



'Which way' detector does not respond | 'Which way' detector responds

The role of the observer

The *WWD* has no physical effect on the system, it only extracts information. Nevertheless, it essentially influences the system's behavior.

WWD takes only information, it has no further influence

nevertheless:

without WWD	with WWD
wave–particle duality	particle character
(not compatible with material basis)	(as with classic trajectory)

(5-2)

Nonlocality

The response or non-response of the *WWD* instantaneously changes the amplitude of the probabilities of measurement throughout the system, also far away from the *WWD*.

It is worth noting that: for events at the top of the aperture's shadow, the *WWD* did not respond. Without *WWD*, some of these events are suppressed by destructive interference.

There are dots on the screen

- where without WWD events are not possible
- with WWD events are possible, even if the WWD does not respond

- WWD influences the system even if he does not respond
- Difference: no WWD $\leftrightarrow$ WWD does not respond

  (According to Einstein, this is spooky)

(5-3)

Corrections to classical physics

Classical physics provides us with the basis and concepts for our physical understanding of the world, particularly the strict distinction between content and motion. This distinction is resolved in three steps. With these three steps, we are entering an area of reality that we can no longer consciously grasp. In physics, these steps were first necessary to describe the thermal radiation of black bodies and atomic spectra. They became the foundation of our modern information society. They can help us to see the reality in a new light.

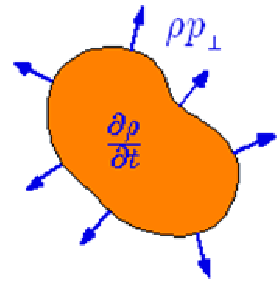
Trajectory → Density of the measurement probability

The measurement density replaces the notion of bodies or objects. There is a probability that one detector at a specific location at a certain time responds.

1. Probability density for the response of a detector

Detector response probability density: $\rho(x,t)$

Continuity equation (CE): $$\frac{\partial\rho}{\partial t}+\vec{\nabla}\underbrace{\left(\rho\cdot\frac{\vec{\nabla}S}{m}\right)}_{=\rho\cdot\frac{\vec{\nabla}S}{m}=\rho\cdot\vec{v}}=0$$



The change in density $\dot{\rho}$ in an area is equal to a current $\rho p_\perp$ through the boundary

(5-4)

2. Unification of content and motion

Wave function: $\psi=\sqrt{\rho}e^{iS/\hbar}$

Classical:
$$\left.\begin{array}{l}\underbrace{\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot\left(\rho\frac{\vec{\nabla}S}{m}\right)=0}_{\text{Content: Continuity equation }(CE)}\\ \underbrace{\frac{\partial S}{\partial t}+\frac{1}{2m}\left(\vec{\nabla}S\right)^2+V=0}_{\text{Motion: } HJE}\end{array}\right\}\underset{\psi=\sqrt{\rho}e^{iS/\hbar}}{\Rightarrow}-\frac{\hbar}{i}\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\Delta\psi+\underbrace{V\psi+\frac{\hbar^2}{2m}\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}\psi}_{\text{Quantisation: } V+\frac{\hbar^2}{2m}\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}\to V}$$

(5-5)

3. Quantization

Quantization: $$\underbrace{-\frac{\hbar}{i}\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\Delta\psi+V\psi}_{\text{Schrödinger equation }\neq F(S,\rho)}$$

HJE after quantization: $$\frac{\partial S}{\partial t}+\frac{1}{2m}\left(\vec{\nabla}S\right)^2+V-\underbrace{\frac{\hbar^2}{2m}\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}}_{\text{Bohm's quantum potenzial}}=0$$

(5-6)

The energy relationships in the *HJE* now depend on the measurement probability! If one observes the line spectrum of an atom, for example, it has discrete energy eigenvalues. If one observes the same atom in a scattering experiment, the eigenvalues are continuous. This difference affects the *HJE* or the law of conservation of energy. Without specifying the type of measurement, the energy of the atom is not defined and thus no statements about the atom can be made. It does not exist independently of the measurement!

Quantum mechanics

- $\begin{cases} \text{1. Objects on trajectories} \rightarrow \text{Probability for the response of a detector} \\ \text{2. Content and dynamics} \rightarrow \text{Unified to the wave function} \\ \text{3. Quantization} \left(\text{Elimination of inhomogeneous terms}\right) \end{cases}$
- Complementarity, duality
  - Content and dynamics, particles and wave
- uncertainty
  - Position and momentum - Content and dynamics (uncertainty relation)
- Superposition of possibilities (Interference, beats)
- Manifestation of one of many possibilities
- Non-locality (effect of observation)
- Unity of phenomena and observation

(5-7)

Interpretations

Copenhagen interpretation

Preparation:	Classic terms	$\rightarrow$	$\psi(x,0)$
Temporal evolution:	Schrödinger eq. (Causality)	$\rightarrow$	$\psi(x,t)$
Measurement process:	Probability for the detector to respond:	$\rightarrow$	$\rho(x,t) = \lvert\psi(x,t)\rvert^2$

Complementarity: mutually exclusive terms (Position - momentum, content - movement $\Delta x \Delta p \geq h$) allow a comprehensive description of reality

Acausality: $\begin{cases} \text{W.Heisenberg:} \\ \text{Superposition of possibilities} \underbrace{\rightarrow}_{\text{measuremet}} \text{Manifestation of one possibility} \\ \text{W.Pauli: Note on transcendence} \end{cases}$

(5-8)

Many worlds according to Everett

The wave function describes superposition of possibilities
every possibility is real in a world of its own
The number of worlds is growing exponentially
→ ∘ many worlds
∘ parallel universes

Causal interpretation according to DeBroglie - Bohm

Classical Hamilton-Jacobi equation is extended by the

Quantum potential: $Q = -\frac{\hbar^2}{2m}\frac{\vec{\nabla}^2 R}{R}$

This can be used to calculate classical trajectories

Quantum effects are caused by Q.

(5-9)

Does the dice exist under the cup?

Classic:	The dice exists in the cup as an object; the number of pips is fixed after the cast.
Kopenhagen:	The dice manifests itself upon uncovering; the number of pips manifests itself upon uncovering.
Many worlds:	The cast produces six cups and dice; each of them has a different number of pips in every new universe.
Bohm causal:	All six possibilities are classically causally described, but they interact via the quantum potential

(5-10)