EBERHARD KARLS UNIVERSITÄT TÜBINGEN
MATHEMATISCH-NATURWISSENSCHAFTLICHE FAKULTÄT
Fachbereich Physik

Basic Physics Course with MATLAB's Symbolic Toolbox and Live Editor

Kurt Bräuer

7.2 Fractals (Computational example)

```
clear all
```

1 Turtle graphics

Instead of coordinates of lines points, movements of the pen are specified. There are commands for forward movement, rotation, color and storage of states:

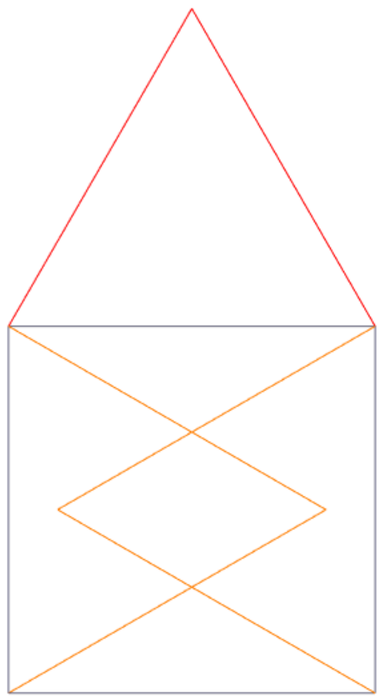
- Line: F, G
- Intermediate space: f, g
- Line color indices: $1, 2, 3, 4, \ldots$
- Rotation angle between lines: α
- Turn by angle α: $+, \ -$
- 90°-rotation: $>, \ <$
- Save and recall an intermediate state: $[\ ]$

Function Turtle (α, commands, colormap, generation index)

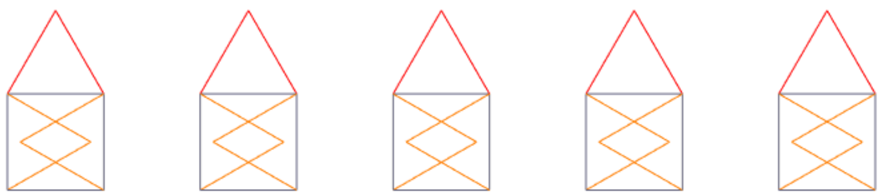
- α: Rotation angle (see above)
- Commands: string with sequence of lines, colors, ...
- Colormap: Assignment of line color indices to rgb colors
- Generation index: for fractals (see below) (-1: no indication of the generation in the plot)

Example: Turtle hut

```
Col=[0.4 .4 .5;1 0 0;1 .5 0;1 1 0];
S='<1F<+2F+F+1F>+3F+F>+1F<F-3F-F>-f';
Turtle(2*pi/3,S,Col,-1)
```

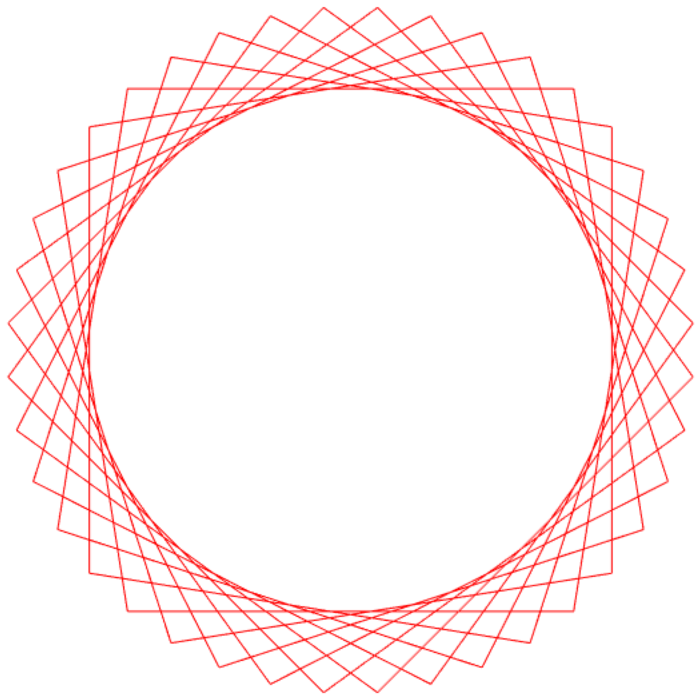


```
Turtle(2*pi/3,[S S S S S],Col,-1)
```



Example: Turtle rosette

```
figure
Col=[1 0 0];
S=[];
for n=1:40
    S=[S 'F+'];
end
Turtle(81*pi/180,S,Col,-1)
```



2 Lindenmayer systems

A fractal is a structure that recursively consists of itself. The structure can be symbolized by letters.

The graphic representation can be done through Turtel graphics.

Example: Cantor set (see below)

- a line consists of two lines with an intermediate space f: $F \rightarrow FfF$
- an intermediate space consists of three intermediate spaces $f \rightarrow fff$

Recursion: $F \rightarrow FfF \rightarrow FfF\ fff\ FfF \rightarrow FfFffffFf\ ffffffff$ ...

Function LSys(α,start string,Iteration rules,colormap,generation index)

Iteration rules: cell array with iteration rules, for instance $\{'F : FfF'\ \ 'f : fff'\ \ 'G : GG'\}$

every iteration is then replaced as follows: F by FfF , f by fff and G by GG

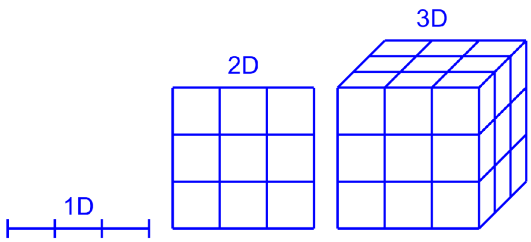
3 Hausdorff dimension

A structure consists of substructures with a specific scaling. It is characterized by the length scaling *s* and the number of substructures *N*.

The Hausdorff dimension *D* is then defined by $s^D = N$, or $D = \frac{\log N}{\log s}$.

For lines, surfaces and spatial structures follow the correct values:

- 3 lines become one line (1D): $s = 3,\ N = 3 \rightarrow D = 1$
- 9 rectangles become one rectangle (2D): $s = 3,\ N = 9 \rightarrow D = 2$
- 27 cubes become one cube (3D): $s = 3,\ N = 27 \rightarrow D = 3$



For fractals, the dimension is fractal.

4 Cantor set

- a line *F* consists of two lines with an intermediate space *f*: $F \rightarrow FfF$
- an intermediate space consists of three intermediate spaces: $f \rightarrow fff$

therefore: $F \rightarrow FfF \rightarrow FfF\ fff\ FfF \rightarrow$ ...

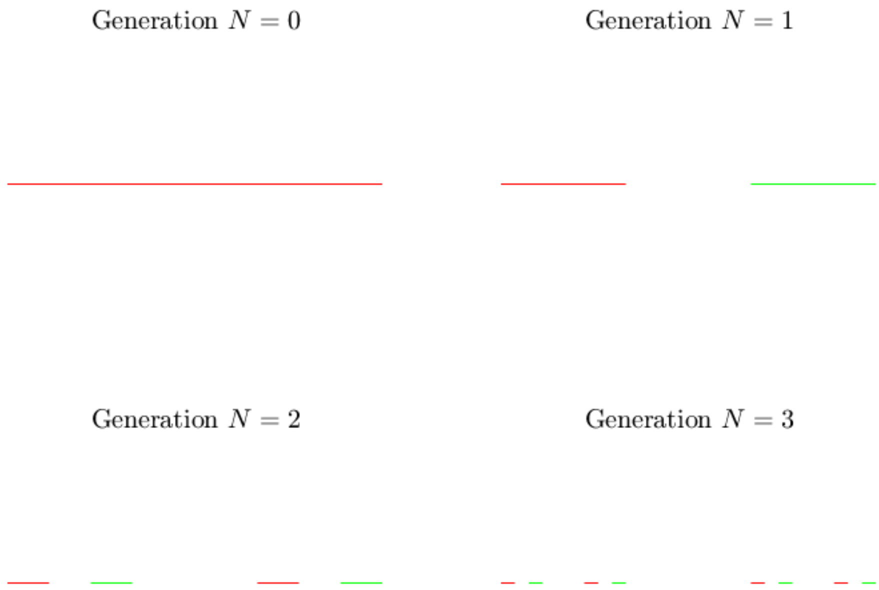
This describes a set with a smaller dimension than a line, but a larger dimension than a point.

Hausdorff dimension:

```
s=3; %A line is three times the size of its parts (scaling)
N=2; %A line consists of two lines
Dimension=log(N)/log(s)
```

```
Dimension = 0.6309
```

```
Col=[1 0 0;0 1 0];
for n=1:4
    subplot(2,2,n)
    LSys(pi/3,'F',{'F:1Ff2F','f:fff'},Col,n-1)
end
```



5 Koch curve

A line consists of four lines in a cone shape

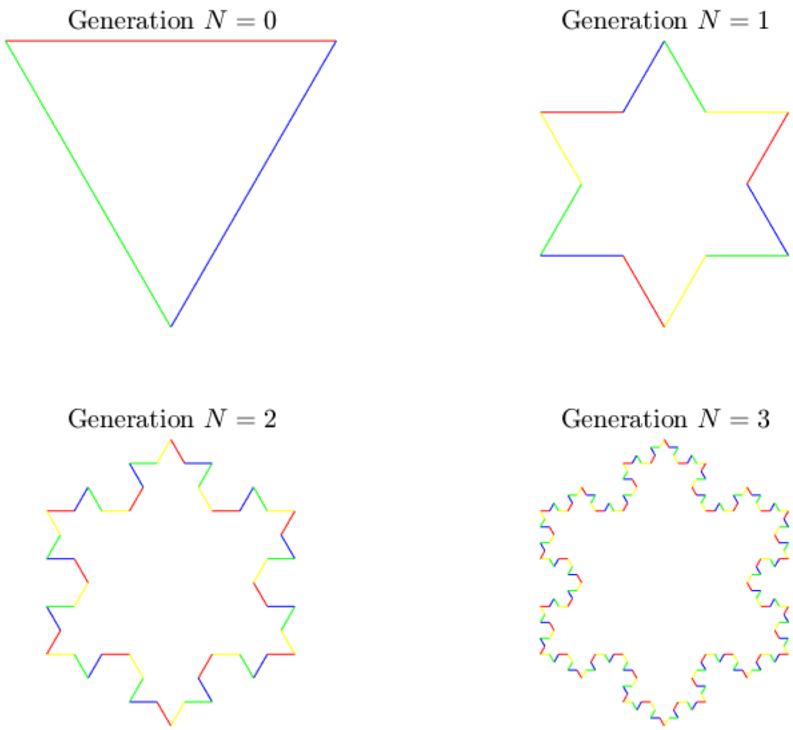
$F \to F - F + +F - F$, or graphical : $\_ \to \_/\backslash\_$

With $D \in [1, 2]$ the linear structure is already somewhat planar

```
s=3; % a line has three times the size of its parts (scaling)
N=4; % a line consists of four lines
Dimension=log(N)/log(s)
```

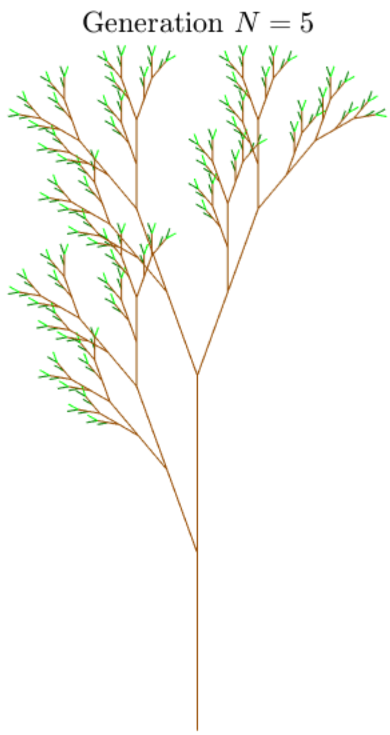
```
Dimension = 1.2619
```

```
Col=[1 0 0;0 0 1;0 1 0;1 1 0];
for n=1:4
    subplot(2,2,n)
    LSys(pi/3,'1F++2F++3F',{'F:1F-2F++3F-4F'},Col,n-1)
end
```



6 Farn

```
subplot(1,1,1)
Col=[.6*[1 .5 0];0 .5 0;0 1 0;0 1 0];
LSys(pi/9,'<F',{'F:1G[-2F]1G[-2F]+3F' 'G:GG' },Col,5)
```



Lindenmayer systems

```
function LSys(phi,S,s,Col,N)
for n=1:N
    T=S;
    S=[];
    K=size(s,2);
    for m=1:size(T,2)
        TM=T(m);
        if isletter(TM)
            for k=1:K
                s0=s{k};
                if strcmp(s0(1),TM)
                    S=[S s0(3:end)];
                end
            end
        else S=[S TM];
        end
    end
end
Turtle(phi,S,Col,N)
end
```

Turtle graphics

```
function Turtle(phi,S,Col,N)
X=[1;0];
Y=[0 0;0 0];
L=[cos(phi) sin(phi);-sin(phi) cos(phi)];
R=[cos(phi) -sin(phi);sin(phi) cos(phi)];
col=Col(1,:);
cnt=0;
hold on
for n=1:size(S,2)
    switch S(n)
        case {'F' 'G'}
            Y=[Y(:,2) Y(:,2)+X];
            plot(Y(1,:),Y(2,:),'Color',col)
        case {'f' 'g'}
            Y=[Y(:,2) Y(:,2)+X];
        case {'1' '2' '3' '4' '5' '6' '7' '8'}
            col=Col(str2double(S(n)),:);
        case '-'
            X=R*X;
        case '+'
            X=L*X;
        case '<'
            X=[0 -1;1 0]*X;
        case '>'
            X=[0 1;-1 0]*X;
        case '['
            cnt=cnt+1;
            Mem{cnt}={Y, X, col};
        case ']'
            [Y,X,col]=Mem{cnt}{:};
            cnt=cnt-1;
    end
end
set(gcf','DefaultTextInterpreter','LaTex');
if N+1
    txt=['${\rm Generation}\ N=' num2str(N) '$'];
    title(txt)
end
axis equal
axis off
hold off
end
```