FEMLAB as a General Tool to Investigate the Basic Laws of Physics

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The common basis of physical laws is the continuity of the action field. It implies Structural mechanics, Hydrodynamics, Quantum mechanics, Electrodynamics and the self-organisation of matter.

A general tool to investigate all the basic equations is FEMLAB. In the academic training it allows to take the concentration away from the manifold mathematical methods of solutions and draw it to the real physical questions.

Both aspects together allow a new holistic insight into physics. I try to represent here the rough idea of this course. More details can be found on www.kbraeuer.de or in a textbook which is in preparation [BRÄ05]. A popular representation of the physical background is in [BRÄ04].

The Action Field as Common Basis of Physical Laws

The central concept of physics is action *S*. *S* connects the observables space *x*, time *t* and force *F*. The product of force and space is energy *E*, the product of force and time is momentum *p*. Action is the product of force, time and space.

Action is so to say the time and space invariant form of forces. This explains its importance. The context independence of the action field is the root the Hamilton-Jacobi field equation. It can be derived by considering small variations of momentum $p \to p+\varepsilon$ and small displacements of the coordinate system $x \to x+\eta$:

$$\underbrace{dS'}_{\substack{\text{Total Differential}\\ \text{of the Action Field}}} = \underbrace{\vec{p}'}_{\vec{p}+\vec{\varepsilon}} \cdot d\vec{x} - \underbrace{E'}_{E+\vec{\nabla}^{(\vec{p})}\cdot\vec{\varepsilon}}\ dt = \vec{p}\cdot\underbrace{d\vec{x}'}_{\vec{x}+\vec{\eta}} - \underbrace{E'}_{E+\vec{\nabla}^{(\vec{p})}\cdot\vec{\varepsilon}}\ dt \overset{!}{=} dS; \qquad (1)$$

$$\Rightarrow$$

$$\underbrace{\frac{\partial S}{\partial t}}_{-E} + \frac{1}{2m}\underbrace{\left(\vec{\nabla}S\right)^2}_{\vec{p}^2} + \underbrace{V}_{\text{Potential}} = 0 \quad (\text{Hamilton-Jacobi Field Equation}).$$

The Hamilton-Jacobi field equation allows very general potentials *V*. They make the differences between the various fields of physics.

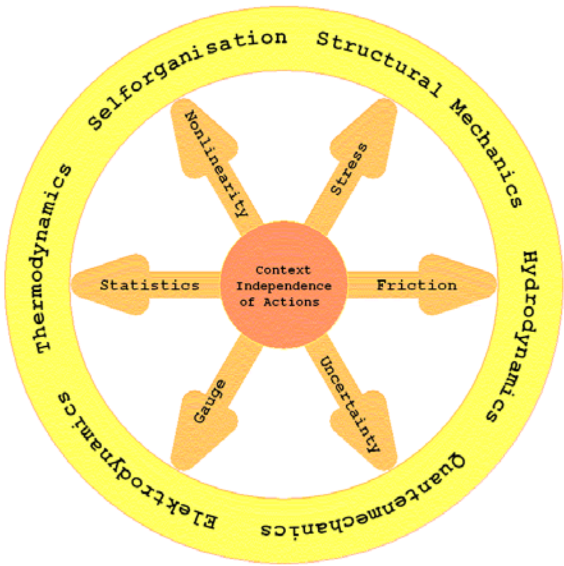

Figure 1
The context independence of the action field implies the Hamilton-Jacobi field equation. Various fields of physics differ mainly by their potentials. Electrodynamics emerges as an aspect of gauge invariance of the action field.

FEMLAB as General Tool to Investigate Physical Laws

FEMLAB allows specifying and solving more or less all kinds of physical equations. Equations and boundaries have to be defined in a graphical user interface or as programming statements. A simple example is structural mechanics, where one can consider the displacement field *u* of a membrane:

Equations of Motion		FEMLAB Commands
$\begin{cases} \underbrace{1}_{a}\dfrac{\partial u}{\partial t} = \underbrace{v}_{f}, \\ \underbrace{1}_{a}\dfrac{\partial v}{\partial t} = \underbrace{\Delta u}_{f}, \end{cases}$	$\rightarrow$	$\begin{array}{l} fem.geom = rect2(0, pi, 0, pi), \\ fem.equ.a = \{'1'\}, \\ fem.equ.f = \{'v' \quad 'u_{xx} + u_{yy}'\}, \\ fem.bnd.r = \{'0' \quad '0'\}, \\ fem.sol = femeig('eigs', \{'8' \quad '0'\}), \\ postplot(fem, 'tridata', 'u'). \end{array}$

(2)

For correct inputs, FEMLAB returns the solution of the physical problem at the touch of a bottom. The same procedure holds for all equations of motion. In this sense, FEMLAB is a common tool to solve all relevant problems in physics.

Propagation and Decay of the Action Fields

This is a very elementary problem, since the Hamilton-Jacobi field equation in (1) has no potential at all. This example demonstrates also the incompleteness of classical mechanics. We consider the Hamilton-Jacobi field equation

$$\underbrace{1}_{a=1}\frac{\partial}{\partial t}S+\underbrace{0}_{\Gamma=0}=\underbrace{-\frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2}_{=F}. \tag{3}$$

a,Γ and F are the quantities to be defined in FEMLAB. The time evolution of the action field S in Figure 2 shows remarkable problems. After a short time, space derivatives of the action field diverge.

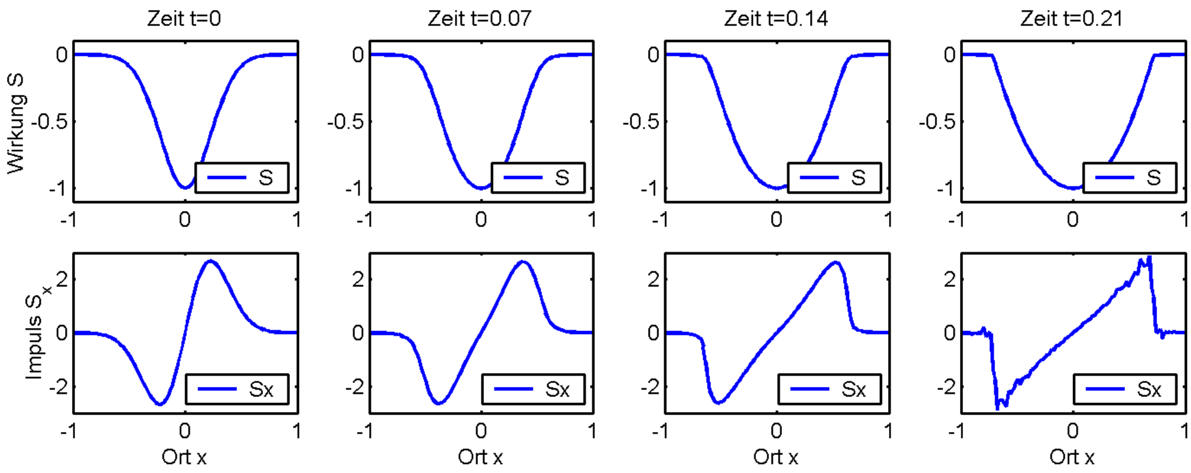

Figure 2
Time evolution of the action field S and its spatial derivatives, the momentum p=∂S/∂x. The action field decays. It describes a motion away from the centre towards the left and right edges. Since the momentum at these edges is zero, the motion from the inside stops more and more abruptly. This is comparable to water waves breaking at the shore.

Quantum mechanics

In Figure 2 the space derivatives of the action field diverge, since all quantities are assumed to have absolute defined values. This is a fundamental problem which was discussed already 2500 years ago by Greek philosophers [BRÄ04]. We can, for example, measure only mean velocities. The well defined velocity at a point in space-time is a pure idea which is obviously unrealistic.

The momentum at a point in space-time is always connected with an uncertainty which has to be treated statistically. This involves a probability field ρ and an energy contribution to the potential V in the Hamilton-Jacobi field equation. The action field S and the probability field ρ can be combined to one complex field ψ, which turns out to be the famous quantum wave function. ψ solves the Schrödinger-equation:

$$\left.\begin{array}{l}\underbrace{\frac{\partial S}{\partial t}+\frac{\left(\vec{\nabla}S\right)^2}{2m}\overbrace{-\frac{\hbar^2}{2m}\frac{\Delta R}{R}}^{\text{Uncertainty Potential}}+V\left(\vec{x}\right)=0}_{\text{Hamilton-Jacobi-Equation with Uncertainty Potential}},\quad \left(R=\sqrt{\rho}\right);\\ \underbrace{\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot\left(\rho\frac{\vec{\nabla}S}{m}\right)=0.}_{\text{Equation of Continuity of the Probability Field }\rho}\end{array}\right\}\underset{\psi=Re^{iS/\hbar}}{\Leftrightarrow}\underbrace{-\frac{\hbar}{i}\frac{\partial\psi}{\partial t}=\left(-\frac{\hbar^2}{2m}\Delta+V\right)\psi.}_{\text{Schrödingers Equation}} \tag{4}$$

We repeat the simulation of Figure 2 with an action field that comprises an uncertainty of Plancks action quantum h:

$$\underbrace{i}_{a}\frac{\partial\psi}{\partial t}+\frac{\partial}{\partial x}\underbrace{\left(\frac{1}{2m}\frac{\partial\psi}{\partial x}\right)}_{\Gamma}=\underbrace{0}_{F}. \tag{5}$$

The action field *S* in Figure 3 decays smoothly and is reflected at the walls.

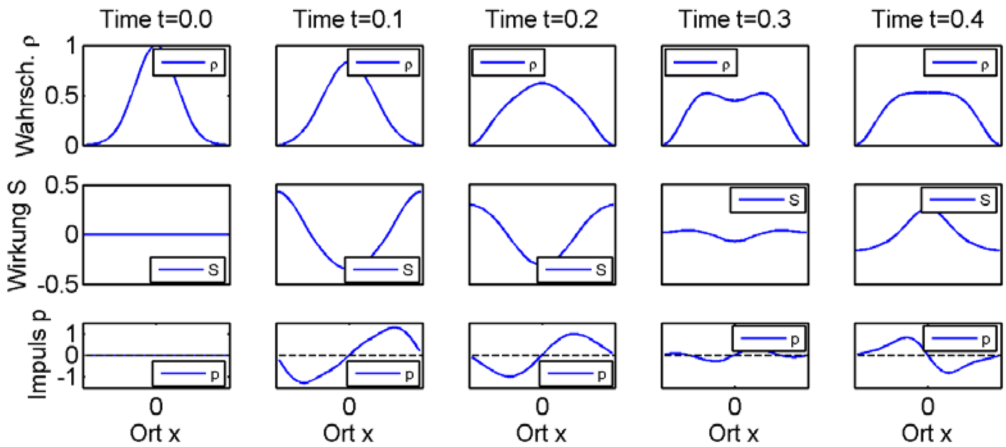

Figure 3
Time evolution of the probability fild ρ, the action field S and the momentum field p. After 0.2 time units, the momentum field changes sign, so the action field is reflected at the edges.

FEMLAB Simulation of the Quantum Mechanical Double-Slit Experiment

Only by a different geometry and different start conditions, (5) describes the quantum mechanical double-slit experiment. The propagation of the probability field ρ is shown in Figure 4.

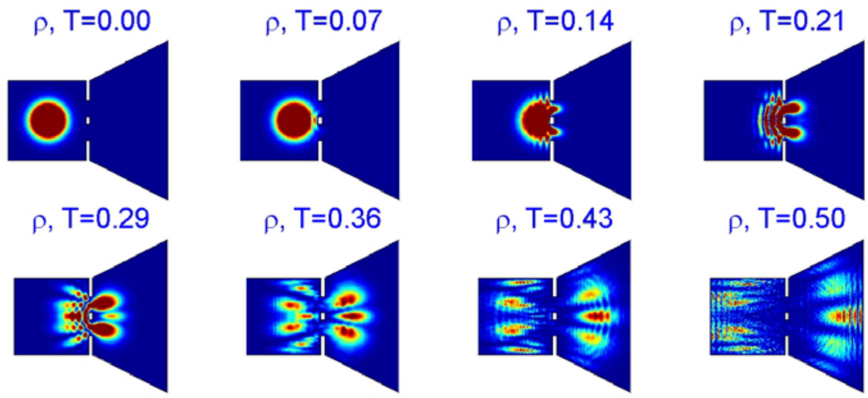

Figure 4
Simulation of the quantum mechanical double-slit experiment. The different colours refer to the probability field ρ, that means to the probability of a detector-respond. On the right side of the geometry appear the well know interference pattern. The region right around the slits are not represented in other methods of solving (5), however they are of special interest. The interference maximum of order 0 in the centre has its origin not in the slits. The idea of something like a particle passing the slits is neither compatible with this nor with the interference pattern.

The simulation can be extended by a so called 'which way detector' behind on of the slits. This allows a very clear discussion of the Copenhagen interpretation of quantum mechanics.

The Nature of Photons

Photons are quanta of the electromagnetic field. What does this means?

In space an time, physical actions appear as forces, which are described as time and space derivatives of the action field *S*. Therefore the action field is not determined uniquely by the observable forces alone. There are gauge degrees of freedom and this is the origin of the electromagnetic potentials A_μ:

$$S \to S^{(Matter)} + S^{(Gauge)}; \qquad (6)$$
$$dS \to \underbrace{p_\mu}_{=\partial_\mu S^{(Matter)}} dx^\mu + \underbrace{eA_\mu}_{=\partial_\mu S^{(Gauge)}} dx^\mu .$$

The dynamics of the electromagnetic potentials is determined by the continuity equation of the electromagnetic current:

$$\underbrace{\overbrace{\partial_\nu c\,\partial^\nu A_\mu}^{\text{Elektro-magn. Current}} = \overbrace{j_\mu}^{\text{Source}} = \overbrace{\rho}^{\text{Quantum Mechanical Probability Density}} \quad \overbrace{p_\mu}^{\text{Mechanical Momentum}}}_{\text{Continuity Equation for the Electromagnetic Current}} . \qquad (7)$$

(7) implies the famous Maxwell equations of electro dynamics [BRÄ05].

For the simulation of the emission of a photon by an atom, we restrict us to the electric component $\varphi = A_0$ of the electromagnetic potential. The coupled system of equations for the atom and the electric potential reads

$$\text{Schrödingers Equation:} \quad \underbrace{-\frac{\hbar}{i}}_{a}\frac{\partial\psi}{\partial t} + \vec{\nabla}\cdot\underbrace{\frac{\hbar^2}{2m}\vec{\nabla}\psi}_{\Gamma} = \underbrace{e\phi\psi}_{F}; \qquad (8)$$

$$\text{Electric Potential Equation:} \quad \underbrace{1}_{a}\frac{\partial^2\phi}{\partial t^2} + \vec{\nabla}\cdot\underbrace{\left(-c\vec{\nabla}\phi\right)}_{\Gamma} = j_e = \underbrace{-ie\left(\psi^*\frac{\partial\psi}{\partial t} - \frac{\partial\psi^*}{\partial t}\psi\right)}_{F}.$$

The FEMLAB simulation is displayed in Figure 5.

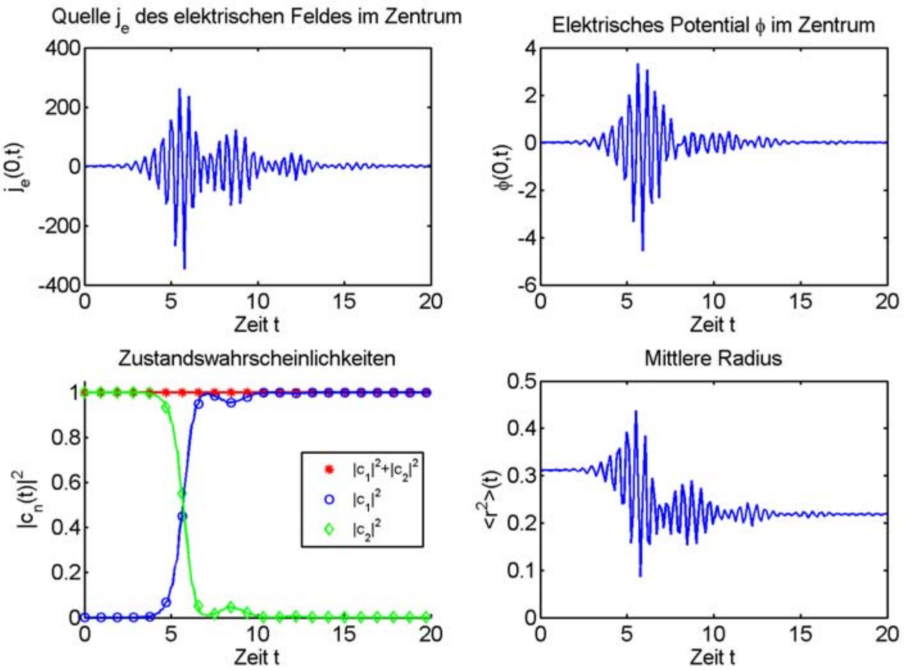

Figure 5
Emission of a photon by an atom.
In the lower left diagram the probability for the exited atom $|c_2|^2$ *comes down to zero after some time. At 9 time units, there is some probability for the reverse excitation of the atom by the electric degrees of freedom.*
In correspondence, the atomic source in the upper left diagram oscillates with an angular velocity proportional to the energy difference of the atomic states $\omega = \Delta E / \hbar$*. This source is not a charge distribution but a probability distribution!*
The upper right diagram shows the electric potential at the centre of the atom. The electric potential absorbs the atomic energy in an oscillation of angular velocity ω*. The energy flows through the walls away from the simulated region.*
In the lower right corner we see the mean radius of the atom. The value for the exited atom is 0.31and for the ground state 0.22.

For the interpretation of this process it's important that the source of the electromagnetic field is not a charge distribution, but the probability distribution of an action! Therefore the excitation of the electric potential is also of probabilistic character. If the edges of the cavity are sensitive to electric forces, they will react only at one isolated point. This reaction is interpreted as photon. Its energy as well as its frequency will be connected to the energy difference of the atomic levels.

The Nature of Eddies and of Biological Forms

Eddies are a consequence of the nonlinear term in the Navier-Stokes equation and this term causes also many complex spatial structures in nature. How this comes about can be studied in a FAMLAB simulation.

The Navier-Stokes equation is derived from the Hamilton-Jacobi field equation in (1) by Hamiltons principle in connection with a 'friction potential':

$$'V^{(Friction)}' = \frac{1}{2}\mu\sum_{ij}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)^2, \quad \text{with} \quad \overset{\text{Velocity}}{\vec{v}} = \frac{\partial \overset{\text{Displacement}}{\vec{u}}}{\partial t}. \tag{9}$$

This potential of course doesn't conserve energy and therefore this consideration is not really complete.

Hamiltons principle leads to the Navier-Stokes equation, where a pressure force and a continuity equation is added to take care on the conservation of matter:

$$\begin{cases} \dfrac{\partial^2\vec{u}}{\partial t^2} = \underbrace{\dfrac{\partial\vec{v}}{\partial t} = -m\overbrace{\vec{v}\cdot\vec{\nabla}\vec{v}}^{\text{Nonlinearity}} \overbrace{+\mu\left(\Delta\vec{v}+\vec{\nabla}\vec{\nabla}\cdot\vec{v}\right)}^{\text{Friction Force}} \overbrace{-\vec{\nabla}P}^{\text{Pressure Force}}}_{\text{Navier-Stokes Equation}} ; \\ \underbrace{\dfrac{\partial\rho}{\partial t}+\vec{\nabla}\cdot\left(\rho\vec{v}\right)=0}_{\text{Continuity Equation of the Liquid}} . \end{cases} \tag{10}$$

The nonlinear term in the Navier-Stokes equation is a consequence of momentum conservation. It depends on the change of momentum with space and on the velocity of a droplet moving to the other space region:

$$\underbrace{\frac{d\vec{p}\left(\vec{x},t\right)}{dt}}_{\substack{\text{Totale Change}\\\text{of Momentum}}} = \underbrace{\frac{\partial\vec{p}\left(\vec{x},t\right)}{\partial t}}_{\substack{\text{Change of the}\\\text{Momentum Field}}} \underbrace{+\overbrace{\vec{v}\left(\vec{x},t\right)}^{\frac{\partial\vec{x}}{\partial t}}\cdot\vec{\nabla}\,\overbrace{m\vec{v}}^{\vec{p}}\left(\vec{x},t\right)}_{\substack{\text{Motion to another}\\\text{Position}}}. \tag{11}$$

The term gets important only for curved stream lines and generates a force against the curvature. How can such a term be responsible for eddies? The answer is given in Figure 6.

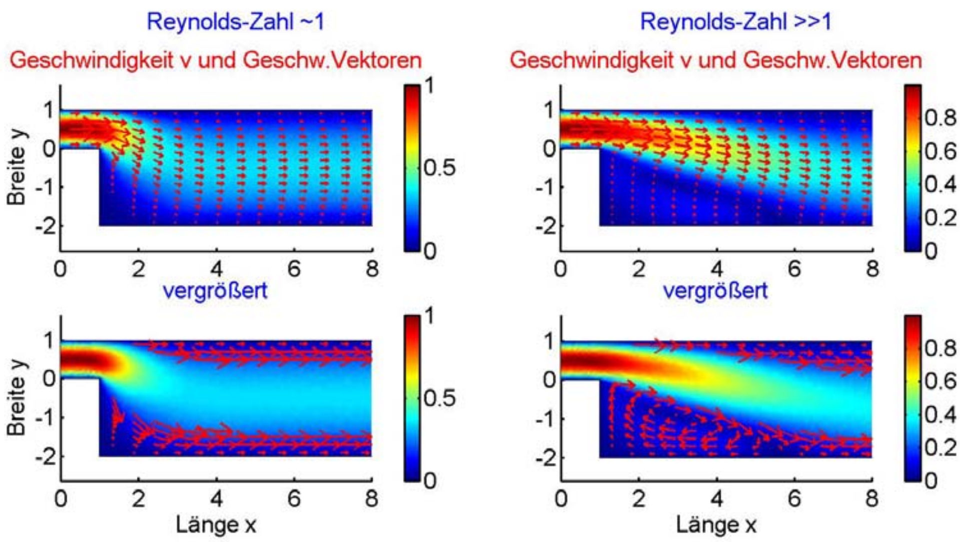

Figure 6
Flow around a step. In the left diagrams the velocity is small and therefore the flow nestles against the step. In the right diagrams the velocity is large and therefore the nonlinear term in (10) becomes effective. As discussed above, it tries to conserve momentum and therefore acts against the curvature of the flow. The flow no longer nestles against the wall. As to be seen in the lower right part of the figure, a calm region appears where an eddy is driven by friction forces. Eddies are a secondary effect of the nonlinearity.

Another simulation in hydrodynamics gives a hint to the origin of the form of a jellyfish. We consider this in Figure 7.

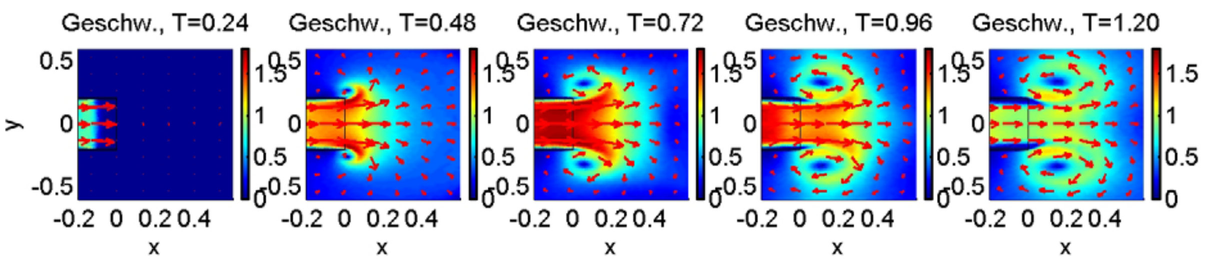

Figure 7
Flow from a tube into a basin. The flow starts whirling and generates a pattern comparable the form of the jelly-fish.

The jellyfish moves in water by pressing it to the back. The water starts whirling as does our tube flow in the basin. By looking on Figure 8 on could say that a jellyfish mirrors its own form in the water.

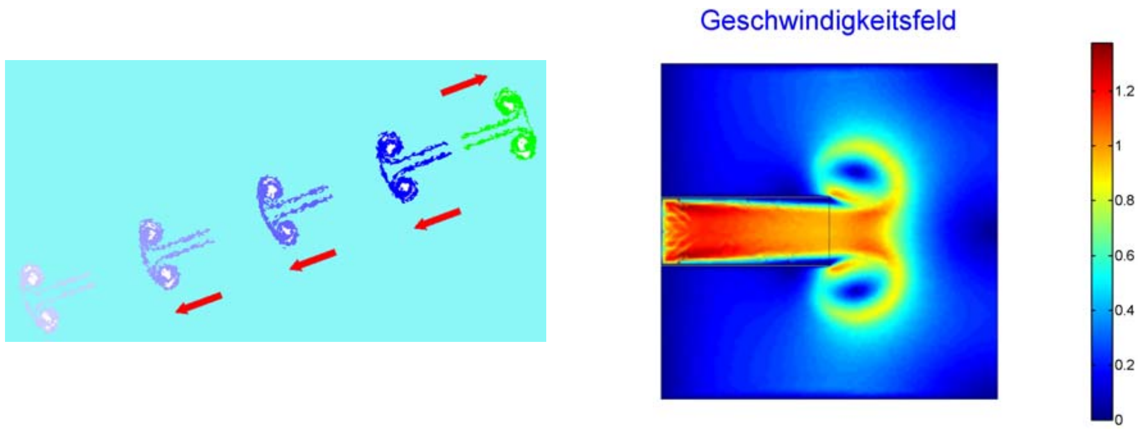

Figure 8
By moving through water, a jellyfish (left part upper right) generates eddies behind itself. In this way it morrows its own form in the water. In the right part of the figure we simulate this phenomenon by the flow from a tube into a basin.

Much more spatial structures of nature can be explained by hydrodynamic forces [BRÄ04].

Closing words

Since the length of this article is restricted, only same examples of FEMLAB simulation could be given and the theoretical argumentations are quite rough. A more detailed publication is in preparation [BRÄ05]. A colour version of this article can be found on www.kbraeuer.de.

I would like to thank the FemLab GmbH and specially Dr. Bernhard Fluche for their support.

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