Objectivity in Space and Time as a Common Basis of Classical Mechanics, Quantum Mechanics and Electrodynamics

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We believe that every physical phenomenon can appear at any place and time. As far as we are right, space and time are homogeneous, space and time coordinates can be defined and the concepts of energy and momentum become meaningful. On this basis a principal equation of motion is developed. If energy is conserved on a spatial average, together with probability conservation this equation is equivalent to Schrödinger's equation of quantum mechanics. For absolute energy conservation, Hamilton's principle or Newton's equation of classical mechanics emerge. If not all momentum and energy is carried by objects we are led to electrodynamics. As a consequence, space, time, action quanta and electromagnetic fields have to be interpreted as mathematical relations between physical objects, they are not objects themselves. The mathematical relations are consequences of the very specific way of human beings to experience themselves subjectively as objects in an objective world. This is a subject of psychology.

Key words: Nonlocality; Relativity; Hamilton's Principle; Quantum Potential; Gauge Invariance; Physics and Psychology.

I. Introduction

Physical theories are based on principles or fundamental equations. In mechanics we have the principles of Newton, in electrodynamics Maxwell's equation, in quantum mechanics among others Schrödinger's equation. All are brilliant abstractions of experimental findings.

The physical theories also involve abstract concepts like energy, momentum, mass, force fields, trajectories, wave functions and so on. Since they have no actual representation in the world of our perception, these abstract concepts are bound up with ideas. And sometimes these ideas puzzle us. This is so, for example, in the case of the speed of light, which is the same in any frame of inertia. Since we imagine light as substantial waves or particles, we have problems to understand this invariance and its consequences in physics. Another drastic example is quantum mechanics with its dubious particle-wave dualism, non-locality and so on. It is only an idea that action quanta propagate on trajectories through the slits of a screen, and this idea is incompatible, for example, with the dramatic change of the phenomenon, when an action is actually observed by a ‘which way’ detector.

Werner Heisenberg emphasized the importance of differentiating between objects of perception or classical objects and the interpretation of mathematical formulas or ideas [1]. This means, glowing wires, screens with slits, photo plates and so on are of one kind, but action quanta, trajectories or force fields are completely different. We can observe nicely ordered iron filings but we cannot observe the electromagnetic field which is assumed to order them. Classical objects are elements of our perception and there is no need to define them. They are also essential elements in the Copenhagen interpretation of quantum mechanics [2].

There exists a fundamental principle of ordering among classical objects. They can be arranged side-by-side, one on the top of the other or one behind the other. This principle of order is the origin of our experience of space [3], [4]. We experience the physical world in the form of objects with spatial relations. The objects themselves do not depend on these relations, and Hermann Weyl points out that only that is why coordinate systems have any meaning and lead to the mathematical description of the physical world [5].

The spatial relations among objects are changing and therefore the world appears different from one time to another time. Time is a further principle of order in our perception. As Hermann Weyl [5], David Bohm [6], [7], [8] and others pointed out, this experience has its origin in our human ability to remember. Animals seem to have no notion of past and future [3]. The condition for a mathematical description of time in the form of coordinates is, according to Hermann Weyl, that a specific change of the spatial relations of classical objects can take place at any time without any modification.

Space and time are fundamental principles of order, which can be expressed in mathematical terms. And the mathematical structure of space and time alone has far-reaching consequences for physics, as we will show in the following.

A physical phenomenon is described by a principal function of space and time. In the simplest case it is a scalar function. The value of this function changes with space and time, and alone this implies the concepts of momentum, energy and action. Combining these concepts leads to the concept of mass. On this basis we shall derive in section II a principal equation of motion.

This equation violates the homogeneity of time, which was the essential assumption for the definition of time coordinates. This is a dilemma, from which there are interesting ways out.

We can restrict the demand for homogeneity of time, or equivalently conservation of energy, to a spatial average. This is discussed in section III. The statistical treatment involves the concept of causality, and this enters the principal equation of motion in form of a causal pressure P. Mean energy conservation determines P uniquely to be the famous quantum potential Q of the causal formulation of quantum mechanics according to David Bohm [8], [9], [10]. The principal equation of motion together with the causality condition is mathematically equivalent to the equations of non-relativistic quantum mechanics.

If homogeneity of time is demanded in full accuracy, Hamilton's principle of classical mechanics is the consequence. As shown in section IV, the causal pressure *P,* which otherwise is nontrivial, then becomes zero and the general equation of motion reduces to Newton's equation of classical mechanics.

In section V, we take into account that an object cannot be completely isolated and therefore cannot carry all momentum and energy alone. Homogenous space-time implies then fields of momentum and energy, which turn out to be the electromagnetic potentials. We are led to electrodynamics.

Section VI is devoted to the discussion. Action quanta, classical trajectories and the electromagnetic field were identified as nothing else but relations between classical objects. These relations are therefore mathematical consequences of our way of perceiving an objective world. And in this objective world, we our selves appear as objects and we can experience us as individuals.

Physical space is 'consciousness space'. The theories of relativity, quantum mechanics, and electrodynamics are clear indications for this.

II. Physical phenomena described by a principal scalar function

Space and time coordinates

The mathematical construction of space and time coordinates is nicely described and discussed by Hermann Weyl [5]. The starting points are relations of order like ‘left-right’ or ‘early-late’. They allow defining an origin *O* of a coordinate system and a basis unit $\overline{OE}$. The coordinate *x* then characterize a point *P* in the coordinate system, by

$$\overline{OP} = x \cdot \overline{OE}\,. \tag{1}$$

Since the origin *O* and the basis unit $\overline{OE}$ are arbitrary, transformations and group theory become important for coordinate systems, and a mathematical description of a physical world can be so constructed.

The scalar principal function

In the simplest case, physical phenomena can be described by a scalar principal function. Nothing is said about this function but that it depends on a space coordinate $\vec{q}$ and a time coordinate t. We call this function

$$S(\vec{q},t)\,. \tag{2}$$

S changes with space and time, therefore the total differential of *S* is

$$dS = \vec{\nabla}^{(q)} S \cdot d\vec{q} + \frac{\partial S}{\partial t} dt\,. \tag{3}$$

The time derivative of *S* characterizes the change of the phenomenon with time, and this is our notion of energy. It will be called $-H$. The space derivative of *S* characterizes the change of the phenomenon with space, and this is our notion of momentum. It will be called $\vec{p}$. Here momentum is a purely spatial concept, in contrast to Newton's theory where it is bound up with mass and velocity from the very beginning.

The principal function *S* has the mathematical structure of the action function in a generalized Hamilton-Jacobi-theory. It is called generalized since in this context there is no Hamilton principle. That means generally

$$dS \neq 0. \tag{4}$$

There are no massive particles moving on trajectories. This will be introduced only in section IV. Anyway we call the principal scalar function of (2) 'action function' and find the famous equations

$$\begin{array}{ll} \text{Basic law of motion:} & \vec{\nabla}^{(q)} S \equiv \vec{p}, \\ \text{Generalized Hamilton-Jacobi equation:} & \dfrac{\partial}{\partial t} S \equiv -H, \\ \text{Principal equation of motion:} & \dfrac{\partial}{\partial t} \vec{\nabla}^{(q)} S = \vec{\nabla}^{(q)} \dfrac{\partial}{\partial t} S \quad \Rightarrow \\ & \dfrac{\partial}{\partial t} \vec{p} = -\vec{\nabla}^{(q)} H. \end{array} \tag{5}$$

It is important to be clear at this point that without Hamilton's principle, these equations have a more general meaning than in the classical Hamilton-Jacobi theory. In particular they have nothing to do with particles.

Conservation of momentum and energy

The energy function *H* defines the conditions for energy and momentum conservation. Temporal homogeneity is equivalent to energy conservation, and spatial homogeneity to momentum conservation. Formally,

$$\begin{aligned} H(\vec{q},t) = H(\vec{q},t+\tau) \quad \forall \tau \in \mathbb{R} \quad &\Rightarrow \quad \frac{\partial}{\partial t} H = 0; \\ H(\vec{q},t) = H(\vec{q}+\vec{a},t) \quad \forall \vec{a} \in \mathbb{R}^3 \Rightarrow \quad &-\vec{\nabla}^{(q)} H = 0 = \frac{\partial}{\partial t} \vec{p}. \end{aligned} \tag{6}$$

Propagation of the action function and the concept of mass

For any given time *t,* the action function *S* has a defined spatial form. This form changes with time *t* and we are interested in the speed of the change. We determine a corresponding velocity field $\vec{v}_S$ for the action *S* by

$$\vec{v}_S(\vec{q},t) \equiv \left. \frac{\partial}{\partial t} \vec{q} \right|_{dS=0}. \tag{7}$$

An illustration is given in Fig. 1 for two spatial dimensions. For two different times *t* and *t*+τ*,* equipotential lines of the action function *S* are shown. Each point on the curve for time t moves in time τ to the other curve. The velocity v_s at each point is marked by an arrow and is defined by distance over time.

Without further assumption we derive from (3)

$$dS = \vec{p}\cdot d\vec{q} - Hdt = 0 \;\underset{\vec{\nabla}^{(p)}}{\longrightarrow}\; d\vec{q} - \vec{\nabla}^{(p)}Hdt = 0 \quad\Rightarrow\quad \vec{v}_S \equiv \frac{\partial}{\partial t}\vec{q}\Big|_{dS=0} = \vec{\nabla}^{(p)}H. \tag{8}$$

Obviously the velocity field $\vec{v}_S$ is closely connected with the momentum field $\vec{p}$. The gradient of the action field S is orthogonal to the equipotential lines of the action function S, just as the velocity field $\vec{v}_S$ is. Therefore we introduce the mass m as

$$m\vec{\nabla}^{(p)}H = m\vec{v}_S \equiv \vec{p} = \vec{\nabla}^{(q)}S\,. \tag{9}$$

It turns out that mass is a relation between the spatial and temporal spreading of the action function S. This is quite remarkable. Even more remarkable is that in many cases m is not a field but a scalar.

Energy function H and principal equation of motion

We can immediately integrate (9) and find for the energy function

$$H = \tfrac{1}{2m}\vec{p}^{\,2} + V + P. \tag{10}$$

We have divided the constant of integration into two parts, V and P. The potential V can include all kinds of interaction in this plain theory. The pressure P can include conservation of substance or probability in the following applications.

With (10), we derive from (5) the principal equation of motion

$$\frac{\partial}{\partial t}\vec{p} = -\frac{1}{m}\left(\vec{\nabla}^{(q)}\otimes\vec{p}\right)\cdot\vec{p} - \vec{\nabla}^{(q)}\vec{V} - \vec{\nabla}^{(q)}P. \tag{11}$$

This reads almost like Newton's equation of motion but its much more general. No trajectories or massive particles are involved.

(11) includes a nonlinear term in the momentum $\vec{p}$ that obviously violates conservation of energy. This conservation, however, is closely related to homogeneity of time, which is our major condition for a mathematical description of a physical world. Therefore it is interesting to consider this term in more detail.

III. Causality and quantum mechanics

We first claim energy conservation only on a spatial average. This makes sense if we are not able to observe every detail of the physical phenomenon described by the action function S. The concept of time and its homogeneity makes sense only on the level of observability and energy conservation is only meaningful as far as it is noticeable to an observer.

Now causality becomes an important aspect to the theory. We consider an example. A glowing wire emits some momentum and energy. Due to homogeneity of space and time these momentum and energy have to show up elsewhere, for example in a detector. Emittance and show-up of energy and momentum must have the same probability to ensure that space and time appear homogeneous. This is expressed in form of the continuity equation

$$0=\frac{\partial}{\partial t}\rho+\vec{\nabla}^{(q)}\rho\vec{v}_s=\frac{\partial}{\partial t}\rho+\frac{1}{m}\left(\vec{\nabla}^{(q)}\rho\right)\vec{\nabla}^{(q)}S+\frac{1}{m}\rho\Delta^{(q)}S. \tag{12}$$

ρ is the probability density. Its value is the probability of an action at space-point $(\vec{q},t)$. ‘Action’ means here transfer of momentum and energy to a detector.

The pressure *P* in (10) has to be interpreted as causal pressure. It is generated by the need of causality and induces forces in the principal equation of motion (11).

Surprisingly, *P* can be generally expressed as function of the probability density *ρ:*

$$P=\frac{1}{2m}\left(\hbar\frac{\vec{\nabla}^{(q)}\sqrt{\rho}}{\sqrt{\rho}}\right)^2. \tag{13}$$

We show now that

- (13) is indeed the consequence of energy conservation on the spatial average
- The continuity equation (12) for probability is equivalent to the real part of Schrödinger's equation of quantum mechanics
- The generalized Hamilton-Jacobi equation of (5) is equivalent to the imaginary part of Schrödinger's equation of quantum mechanics
- The constant $\hbar$ in (13) is Planck's action quantum

We deduce alone from the continuity equation (12) the mean energy

$$\langle H\rangle=-\left\langle\frac{\partial}{\partial t}S\right\rangle\equiv-\int\rho\frac{\partial}{\partial t}Sdq=\left\langle\frac{\left(\vec{\nabla}^{(q)}S\right)^2}{2m}+\frac{1}{2m}\left(\hbar\frac{\left(\vec{\nabla}^{(q)}\sqrt{\rho}\right)}{\sqrt{\rho}}\right)^2+\tilde{V}\right\rangle \tag{14}$$

and its change with time

$$\frac{\partial}{\partial t}\langle H\rangle=\left\langle\frac{\partial}{\partial t}\tilde{V}\right\rangle. \tag{15}$$

We identify $\tilde{V}$ as the potential *V* of the energy function in (10) and the causal pressure *P* as in (13).

The generalized Hamilton-Jacobi equation is then

$$-\frac{\partial}{\partial t}S=\frac{\left(\vec{\nabla}^{(q)}S\right)^2}{2m}+\underbrace{\frac{1}{2m}\left(\hbar\frac{\left(\vec{\nabla}^{(q)}\sqrt{\rho}\right)}{\sqrt{\rho}}\right)^2}_{=P}+V \tag{16}$$

P is uniquely determined only in the spatial average that means in the realm of observational possibilities. For a time independent potential *V* (15) expresses mean energy conservation.

It is remarkable that (14) and (15) together imply

$$\frac{\partial}{\partial t}\left\langle\frac{\left(\vec{\nabla}^{(q)}S\right)^2}{2m}+P\right\rangle=0. \tag{17}$$

For a finite constant $\hbar$ only the average of both terms together disappears!

The derivation of (14) and (15) is not quite straightforward and therefore details are given in section VII (Appendix A). It involves the combination

$$\psi=\sqrt{\rho}e^{iS/\hbar} \tag{18}$$

of the action field S and the probability density ρ. According to Madelung [11] any continuity equation like (12) can so be linearized. At this point $\hbar$ is a free constant with dimension of an action.

With (18) the continuity equation (12) can be brought into the form

$$0=\psi^*\left(\frac{\partial}{\partial t}\psi+\frac{\hbar}{2mi}\Delta^{(q)}\psi\right)+\psi\left(\frac{\partial}{\partial t}\psi^*-\frac{\hbar}{2mi}\Delta^{(q)}\psi^*\right) \tag{19}$$

or

$$\frac{\left(\frac{\partial}{\partial t}\psi+\frac{\hbar}{2mi}\Delta^{(q)}\psi\right)}{\left(\frac{\partial}{\partial t}\psi^*-\frac{\hbar}{2mi}\Delta^{(q)}\psi^*\right)}=-\frac{\psi}{\psi^*}. \tag{20}$$

An extension of the fraction ψ/ψ^* in (20) by $\frac{i}{\hbar}\tilde{V}$ implies

$$\begin{aligned}-\frac{\hbar}{i}\frac{\partial}{\partial t}\psi&=-\frac{\hbar^2}{2m}\Delta^{(q)}\psi+\tilde{V}\psi;\\ \frac{\hbar}{i}\frac{\partial}{\partial t}\psi^*&=-\frac{\hbar^2}{2m}\Delta^{(q)}\psi^*+\tilde{V}\psi^*.\end{aligned} \tag{21}$$

These equations are used in the derivation of (14) and (15) in section VII (Appendix A). For any function $\tilde{V}$ they express the conservation of probability as the continuity equation (12) does. If the function $\tilde{V}$ is identified with the potential *V* in (10) then they express also the dynamics of action, since their imaginary parts turn out to be the generalized Hamilton-Jacobi equation of (5) with the energy function *H* of (10) and the causal pressure of (13)!

At the same time, (21) are Schrödinger's equations of quantum mechanics and $\hbar$ is Planck's action quantum.

We learn that Schrödinger's equation expresses nothing else than spatially averaged energy conservation in homogeneous space-time and causality. No substance is involved between source of energy and momentum and detector. That (21) is a wave equation is a mathematical feature and only indirectly connected to physics. A physical interpretation must be awarded only to the generalized Hamilton-Jacobi equation in (5) and to the continuity equation for causality (12). The threefold physical information in the wave function ψ is quite interwoven. For any point in space-time $(\vec{q},t)$ is the

$$\begin{aligned} &\text{possibility for a phenomenon} && \rho=\psi^*\psi; \\ &\text{possible energy transfer} && H=-\frac{\partial}{\partial t}S=-\frac{\hbar}{2i}\frac{1}{\psi^*\psi}\left(\psi^*\frac{\partial}{\partial t}\psi-\psi\frac{\partial}{\partial t}\psi^*\right); \\ &\text{possible momentum transfer} && \vec{p}=\vec{\nabla}^{(q)}S=\frac{\hbar}{2i}\frac{1}{\psi^*\psi}\left(\psi^*\vec{\nabla}^{(q)}\psi-\psi\vec{\nabla}^{(q)}\psi^*\right). \end{aligned} \tag{22}$$

Quantum phenomena like interference or non-locality have their origin in the pressure of causality (13). David Bohm intensively discusses this in connection with the causal interpretation of quantum mechanics [8], [9], [10]. Even if we come here to a completely different interpretation, the causal pressure *P* in (13) is formally absolutely equivalent to Bohm’s quantum potential, called *Q*. This also implies that our formalism can be extended to many quanta, spin, Dirac theory, field theory and so on (see [8], [9]).

Non-locality appears for example in connection with a ‘which way’ detector [12]. Here, two effects play a role. Due to the detector the probability density ρ is changed at measurement time. Therefore the probability ρ for an action *S* changes instantaneously over all space. Furthermore the causal pressure *P* is modified, and so completely different forces appear in the principal equation of motion (11).

IV. Hamilton's principle and classical mechanics

We now discuss physics for large actions *S* compared to Planck’s action quantum $\hbar$. At first we consider the propagation of a free quantum, that is to say the potential *V=0*. A standard solution to Schrödinger’s equation (21) is then

$$\psi(q,t)=\sqrt{\frac{1}{1+i\hbar t}}e^{-\frac{q^2}{2(1+i\hbar t)}}. \tag{23}$$

Probability density ρ and action function *S* can be obtained according to (18). Since *S* is in this simple case already quite bulky, we refer to the graphical representations in

Fig. 2 and Fig. 3. The difference between the two figures is that in Fig. 2 Planck's action quantum was chosen as $\hbar$=1 whereas $\hbar$=0.01 in Fig. 3

Fig. 2 shows the probability distribution ρ with the well-known dispersion and the action function S with a clear curvature in q space. Energy is conserved for this system only in the spatial average of (14). When Planck's action quantum approaches zero as in Fig. 3, there is no longer a dispersion of the probability density ρ and the action function S is flat. In this case the action function is known to be

$$S(\vec{q},t) = \vec{p}\cdot\vec{q} - Et, \quad \text{with} \quad \vec{p}, E = const. \tag{24}$$

Obviously the second spatial derivative of the action function S vanishes and this has three consequences:

- The continuity equation for the probability density ρ becomes the particle density of classical mechanics;
- The causal pressure P becomes a constant;
- The principal equation of motion (5) degenerates to Newton's equation of classical mechanics;
- Hamilton's principle of classical mechanics evolves.

First, the continuity equation (12) degenerates due to the flatness of S in (24) and reads

$$0 = \dot{\rho} + \vec{\nabla}^{(q)}\rho\vec{v}_s = \dot{\rho} + \vec{v}_s \cdot \vec{\nabla}^{(q)}\rho. \tag{25}$$

It has the general solution

$$\rho = f\left(\vec{a}\cdot\left(\vec{q} - \vec{v}_s t\right)\right), \quad \forall\vec{a}\in\mathbb{R}^n, f\in\mathbb{L}^2. \tag{26}$$

Therefore is

$$\frac{\vec{\nabla}^{(q)}\sqrt{\rho}}{\sqrt{\rho}} = \frac{\vec{a}}{2} = const\,, \tag{27}$$

and the causal pressure P of (13) becomes an irrelevant constant. Moreover the nonlinear momentum term in the principal equation of motion (11) is zero for a flat action function S as in (24). Formally we arrive at the level of classical statistical mechanics. The principal equation of motion (11) in this case turns out to be equivalent to Newton's equation.

At this point, the interpretation of the equation has to be reconsidered. Until now we did not speak about massive particles or classical objects moving on trajectories. We spoke about a mathematical function S, which describes possible actions at each point in homogeneous space and time.

Our ideas about classical mechanics are very different. We have to assign action to a classical object. It occupies a defined space and time and has a defined momentum, energy and mass. According to (9) its velocity is $\vec{v}_S$ and according to (8) its action function S at each actual space-time position fulfills the condition

$$dS = 0. \tag{28}$$

For any trajectory $\vec{q}(t)$ of the classical object therefore is

$$S = \int_{\vec{q}(t)} dS = \int_{\vec{q}(t)} \left(p \frac{d\vec{q}}{dt} - H \right) dt = \text{extremal.} \tag{29}$$

This is Hamilton's principle, which is said to be the most general expression of classical mechanics. It emerges from the general structure of space and time for actions much larger than Planck's action quantum $\hbar$. It expresses the fact that we experience the world in the form of objects. Objectivity implies homogeneous space and time, and Hamilton's principle. In contrast to that the quanta of quantum mechanics are no objects. We experience classical objects and the action of one classical object on the other but no quanta.

V. Interaction and electrodynamics

An object can have any spatial and temporal relation to other objects and to observers and is still exactly the same object. This assumption led in section II to a mathematical description of space and time in form of coordinate systems. In this mathematical space-time physical phenomena were described by a very general scalar field *S*. Only its derivatives were interpreted as momentum field *p* and energy field -*H.* Then this momentum and energy was awarded to a classical object i.e. to an element in our perception.

We considered one single element in complete isolation, which cannot satisfy. Each element in our perception has, beside spatial and temporal, further relations, also to the observer. At least two objects appear in one perception, the observer and the observed. Both are inseparable combined in the perception. And even if we stick in this consideration to one object we have to take connections to other objects into account.

The only way is to award the momentum *p* and the energy –*H* not exclusively to the single object. The spatial change *p* of the action field *S* can concern the considered object only to a fraction p_O, and the rest defines a vector field, say *A*. Analogous the temporal change of the action field concerns only to a fraction $-H_O$ the object and the rest defines a scalar field φ:

$$\vec{\nabla}^{(q)} S = \vec{p} \equiv \vec{p}_O + \vec{A}; \qquad \frac{\partial}{\partial t} S = -H \equiv -H_O + \varphi. \tag{30}$$

Then the energy function of the object according to (10) is

$$H_O = \tfrac{1}{2m} \vec{p}_O^{\,2} + V + P + \varphi + \tfrac{1}{m} \vec{p}_O \cdot \vec{A} + \tfrac{1}{2m} \vec{A}^2 \,. \tag{31}$$

Refrain from the causal pressure *P* this is the famous no-relativistic Hamilton-function of mechanics and electrodynamics. The fields $\vec{A}$ and φ are identified as magnetic and electric potentials of the (non-relativistic) electromagnetic interaction. The only weak point is missing Lorentz invariance. Our natural starting point is space and time and therefore speed of light is relative and space-time is absolute. A deeper set up should start with motion and diffract this one into space and time. Such a proceeding would also introduce spin in a natural way.

It should be mentioned that from (30) we get a clear understand of the connection between gauge transformations of quantum phases and electromagnetic fields. In (30) a gauge transformation of the action field *S* is compensated by the well-known gauge transformation of the electromagnetic fields.

Objectivity appears in a completely new light. The object cannot be completely objective. The object has to be coupled to other objects by electromagnetism, which means contact to other objects. If we see an object, feel it, taste it, smell it or whatever always electromagnetism is involved. The object is an object only in conjunction with other objects, among them also observers. The mathematical consequence of this conjunction is electromagnetism.

Exactly like the action function *S*, the electromagnetic fields $\vec{A}$ and φ are mathematical functions and not physical observables. They all describe possible actions in the objective world of our perception.

VI. Summary and discussion

Besides many others an object has two main features. It appears as something on its own and it appears always and to all observers in the same way. Obviously something like that does not really exist in our experience, but it is the foundation of physics, i.e., of the mathematical description of the world we perceive.

Objects appear spatially and temporally ordered, which can mathematically be expressed with coordinates. Space and time are mathematical structures, which describe aspects of our perception of the world. These structures have far-reaching consequences, as we saw in the preceding sections.

We are led to a principal equation of motion. If we apply this equation to classical objects, we recover Hamilton's principle and Newton's equations of classical mechanics. If we apply it to a classical source and a classical detector of energy and momentum, we recover Schrödinger's equation of quantum mechanics. If we further allow other fields to carry momentum and energy, we are led to electrodynamics.

Common to all are observable objects that are linked by the necessity of energy conservation as a consequence of the homogeneity of time. This yields Hamilton's principle of classical mechanics, electromagnetic interactions and all quantum effects, including the non-local ones. We understand all these aspects on the same footing.

Relativity is a major aspect of these considerations. Space and time are relative. That is known since Einstein. They are a relation between objects. Space and time do not exist without objects. This becomes very clear. But the question arises where do we then find objects, if not in space and time. It is a deep impression of our daily world experience that there is space around us and that things are to be found in this space. But as Herman Weyl argues, we can only speak about our perception of the world and not about the world itself [5]. The things we speak about are in our perception, and there they are ordered spatially and temporally.

This might explain the independence of the speed of light from any motion of an inertia system. If space is individual space of awareness, and if relations between things like electromagnetism are individual relations to observers, then this peculiarity of light is evident.

Action quanta are nothing else but relations between sources and detectors in a homogeneous frame of space and time. The relations are purely mathematical consequences of homogeneity. No waves or particles are propagating, only a purely mathematical wave function. As Bohr expressed it: ‘There is no quantum world’ [2]. That action appears always if form of quanta is due to the discrete energy levels in atoms or solids, the sources of energy, momentum and charge.

Space and time can be considered as psychic structures. C.G. Jung and M.-L.v.Franz describe them as archetypical elements in our collective consciousness [4], [13]-[16]. As the physical action field *S*, archetypes themselves are non-observable, only their actions come out in our perception. They order the elements in our perception and enable us to experience our selves subjectively as an object in an objective world.

Julian Jaynes investigates the development of our perception of space and time during the last five thousand years [3]. The appearance of space and time to human beings seems to have changed dramatically during this period. This could imply that our considerations of time evolutions back before this time period are mainly projections of our present-day worldview back to a region where our kind of perception did not yet exist.

VII. Appendix : Mean conservation of energy and Schrödinger’s equation

According to Madelung [11] any continuity equation like (12) can be linearized by an ansatz

$$\psi = \sqrt{\rho}\, e^{iS/\hbar}. \tag{32}$$

At this point $\hbar$ is a free constant with dimension of an action. It follows that

$$\rho = \psi^* \psi \quad \text{and} \quad \mathrm{S} = \frac{\hbar}{2\mathrm{i}}\left(\ln \frac{\psi}{\psi^*}\right). \tag{33}$$

Furthermore

$$\rho \vec{\nabla}^{(q)} S = \frac{\hbar}{2i}\left(\psi^* \vec{\nabla}^{(q)} \psi - \psi \vec{\nabla}^{(q)} \psi^*\right) \quad \text{and} \quad \rho \frac{\partial}{\partial t} S = \frac{\hbar}{2i}\left(\psi^* \frac{\partial}{\partial t} \psi - \psi \frac{\partial}{\partial t} \psi^*\right). \tag{34}$$

With the continuity equation

$$\frac{\partial}{\partial t}\rho+\vec{\nabla}^{(q)}\left(\rho\frac{S_q}{m}\right)=0 \tag{35}$$

we conclude that

$$\begin{aligned}0&=\psi^*\frac{\partial}{\partial t}\psi+\psi\frac{\partial}{\partial t}\psi^*+\vec{\nabla}^{(q)}\left(\psi^*\psi\frac{\vec{\nabla}^{(q)}S}{m}\right)\\&=\psi^*\left(\frac{\partial}{\partial t}\psi+\frac{\hbar}{2mi}\Delta^{(q)}\psi\right)+\psi\left(\frac{\partial}{\partial t}\psi^*-\frac{\hbar}{2mi}\Delta^{(q)}\psi^*\right)\end{aligned} \tag{36}$$

or

$$\frac{\left(\frac{\partial}{\partial t}\psi+\frac{\hbar}{2mi}\Delta^{(q)}\psi\right)}{\left(\frac{\partial}{\partial t}\psi^*-\frac{\hbar}{2mi}\Delta^{(q)}\psi^*\right)}=-\frac{\psi}{\psi^*}=-\frac{\frac{i}{\hbar}\tilde{V}\psi}{\frac{i}{\hbar}\tilde{V}\psi^*}. \tag{37}$$

This holds for any function $\tilde{V}$, but we will see in the next steps that $\tilde{V}$ can be identified with the potential *V* as defined for the energy function *H* in (10).

(37) formally implies Schrödinger's equations

$$\begin{aligned}-\frac{\hbar}{i}\frac{\partial}{\partial t}\psi&=-\frac{\hbar^2}{2m}\Delta^{(q)}\psi+\tilde{V}\psi;\\ \frac{\hbar}{i}\frac{\partial}{\partial t}\psi^*&=-\frac{\hbar^2}{2m}\Delta^{(q)}\psi^*+\tilde{V}\psi^*.\end{aligned} \tag{38}$$

The mean energy can be derived as

$$
\begin{aligned}
\langle H\rangle &= -\left\langle \frac{\partial}{\partial t}S\right\rangle = -\int \rho\frac{\partial}{\partial t}S\,dq \\
&= -\int \frac{\hbar}{2i}\left(\psi^*\frac{\partial}{\partial t}\psi \;-\psi\frac{\partial}{\partial t}\psi^*\right)dq \\
&= -\int\left(\frac{1}{2}\frac{\hbar^2}{2m}\left(\psi^*\Delta^{(q)}\psi+\psi\Delta^{(q)}\psi^*\right)-\psi^*\psi\tilde{V}\right)dq \\
&= \int\left(\frac{\hbar^2}{2m}\left(\vec{\nabla}^{(q)}\psi^*\right)\left(\vec{\nabla}^{(q)}\psi\right)+\psi^*\psi\tilde{V}\right)dq \\
&= \int\left(\frac{\hbar^2}{2m}\left(\frac{\left(\vec{\nabla}^{(q)}\sqrt{\rho}\right)}{\sqrt{\rho}}-\frac{i}{\hbar}\left(\vec{\nabla}^{(q)}S\right)\right)\psi^*\left(\frac{\left(\vec{\nabla}^{(q)}\sqrt{\rho}\right)}{\sqrt{\rho}}+\frac{i}{\hbar}\left(\vec{\nabla}^{(q)}S\right)\right)\psi+\psi^*\psi\tilde{V}\right)dq \\
&= \frac{1}{2m}\left\langle\left(\vec{\nabla}^{(q)}S\right)^2+\left(\hbar\frac{\left(\vec{\nabla}^{(q)}\sqrt{\rho}\right)}{\sqrt{\rho}}\right)^2\right\rangle+\left\langle\tilde{V}\right\rangle \\
&= \frac{1}{2m}\left\langle\left(\vec{\nabla}^{(q)}S\right)^2-\hbar^2\frac{\Delta^{(q)}\sqrt{\rho}}{\sqrt{\rho}}\right\rangle+\left\langle\tilde{V}\right\rangle.
\end{aligned}
\tag{39}
$$

The mean change of energy with time is then

$$
\begin{aligned}
\frac{\partial}{\partial t}\langle H\rangle &= \int\left(-\frac{\hbar^2}{2m}\left(\frac{\partial}{\partial t}\psi^*\right)\Delta^{(q)}\psi-\frac{\hbar^2}{2m}\left(\frac{\partial}{\partial t}\psi\right)\Delta^{(q)}\psi^*+\psi^*\psi\frac{\partial}{\partial t}\tilde{V}+\psi\tilde{V}\frac{\partial}{\partial t}\psi^*+\psi^*\tilde{V}\frac{\partial}{\partial t}\psi\right)dq \\
&= \int\left(-\left(\frac{\partial}{\partial t}\psi^*\right)\underbrace{\left(\frac{\hbar^2}{2m}\Delta^{(q)}\psi-\tilde{V}\psi\right)}_{\frac{\hbar}{i}\frac{\partial}{\partial t}\psi}-\left(\frac{\partial}{\partial t}\psi\right)\underbrace{\left(\frac{\hbar^2}{2m}\Delta^{(q)}\psi^*-\tilde{V}\psi^*\right)}_{-\frac{\hbar}{i}\frac{\partial}{\partial t}\psi^*}+\psi^*\psi\;\frac{\partial}{\partial t}\tilde{V}\right)dq \\
&= \left\langle\frac{\partial}{\partial t}\tilde{V}\right\rangle.
\end{aligned}
\tag{40}
$$

The conservation of energy therefore implies

$$
\frac{1}{2m}\frac{\partial}{\partial t}\left\langle\left(\vec{\nabla}^{(q)}S\right)^2+\left(\hbar\frac{\vec{\nabla}^{(q)}\sqrt{\rho}}{\sqrt{\rho}}\right)^2\right\rangle=0. \tag{41}
$$

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IX. Figure caption

Fig. 1 Propagation of the action function S in two spatial dimensions x and y. For two different times t and $t+\tau$ equipotential lines of the action function S are shown. The points on the line for time t move in time τ to the other line. The velocity v_s at each point is marked by an arrow and is defined by distance over time.

Fig. 2 Probability distribution ρ and action function S for a free particle $(V=0)$. The values for S are in the range of $\hbar=1$.The density distribution ρ gets broader with time due to dispersion. The action function S has a significant curvature in space q.

Fig. 3 Probability distribution ρ and action function $S>>\hbar=0.01$ for a free particle $(V=0)$. For the density distribution ρ there is no dispersion and the action function S is flat.

X. Figures

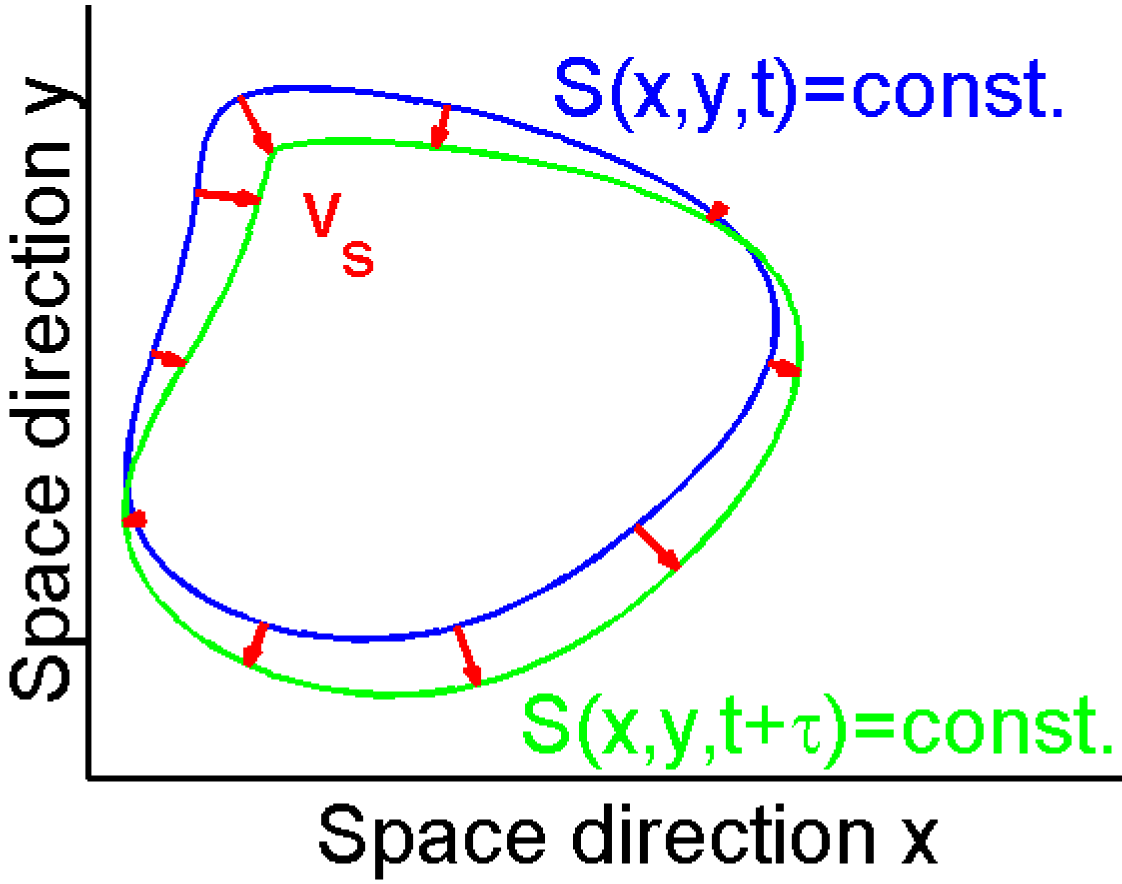


Fig. 1

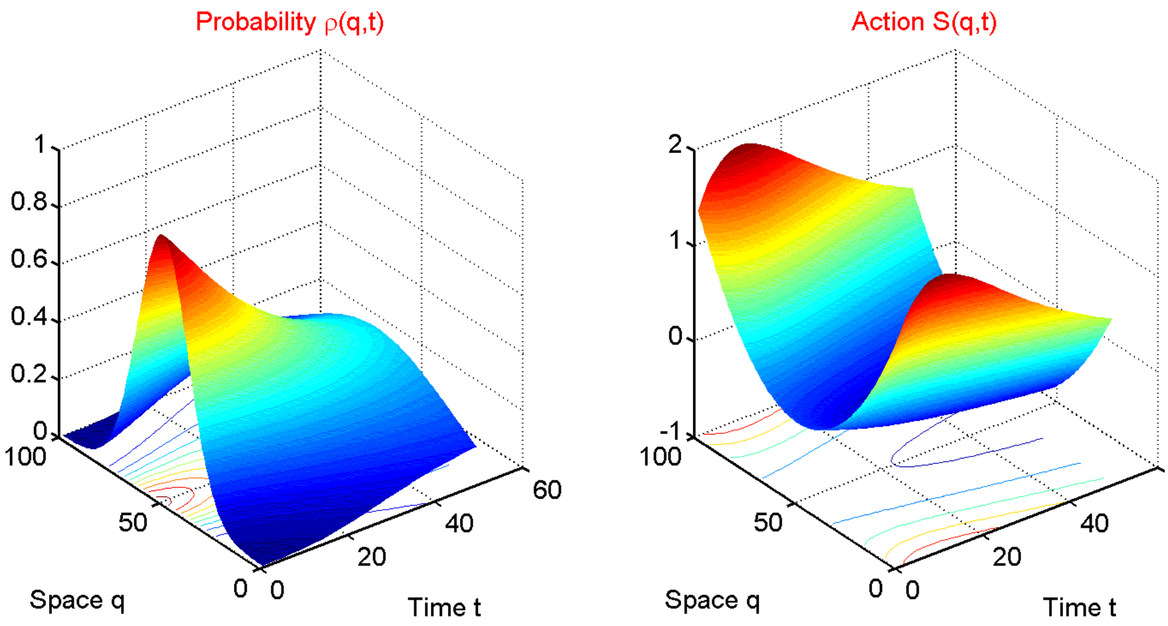
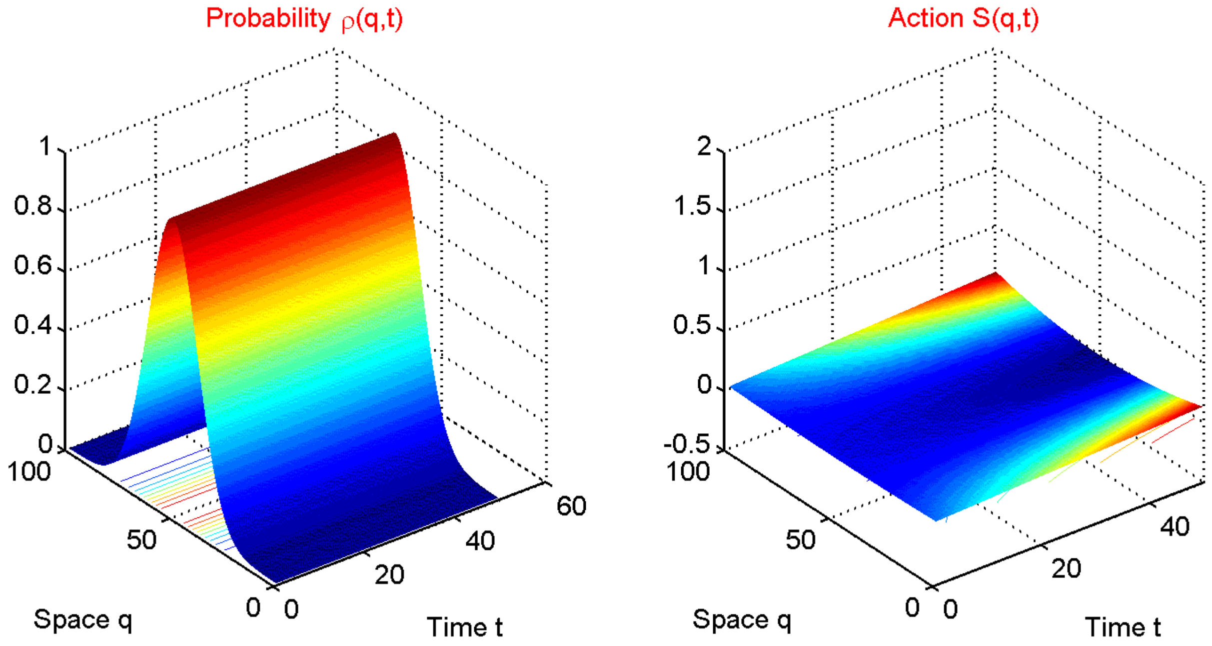

Probability ρ(q,t)
1
0.8
0.6
0.4
0.2
0
100
50
0
Space q
0
20
40
60
Time t
Action S(q,t)
2
1
0
-1
100
50
0
Space q
0
20
40
60
Time t

Fig. 2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1
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0
Space q
0
20
40
60
Time t
Action S(q,t)
2
1.5
1
0.5
0
-0.5
100
50
0
Space q
0
20
40
Time t

Fig. 3