ORDER PARAMETER RELAXATION AND HYDRODYNAMICS OF $^3$He NEAR THE A-TRANSITION

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In the vicinity of the phase transition each of the 18 order-parameter components is an independent variable. Generalizing the hydrodynamics to include them, we have found for a homogeneous system five pairs of damped oscillations and three relaxative modes. Also, up to linear order in the wave number only those components connected to the superfluid magnetization couple to the hydrodynamic spin variables, causing excess damping, frequency shifts of the NMR-signals and dispersion of the spin-wave velocity. They also give rise to a new pair of NMR-signals in the high-frequency regime. An estimate of the temperature window, in which it may occur, has been given.

1. Introduction

The variables of the hydrodynamic theory are either conserved quantities, like density and momentum density, or connected to spontaneously broken symmetries, like the superfluid velocity \([I]\). These are but a few out of the enormous number of degrees of freedom of a macroscopic many-body system. The rest are assumed to relax within a microscopically short time to their local equilibrium values determined by the hydrodynamic variables. This assumption is not generally correct: there may be some—actually microscopic—degrees of freedom with an unusually long relaxation time, which confines the range of validity of the hydrodynamic theory to very low frequencies, i.e. to frequencies not only well below the relaxation rate of most of the microscopic variables, \((\text{especially } 1/\tau_c, \text{where } \tau_c \text{ is the average collision time of quasi-particles})\), but also much smaller than the frequencies of the few slow fluctuations. This is reminiscent glass as a very viscous liquid or of its abnormal thermal properties at low temperature.

One readily realizes that this can be a serious invalidation of the hydrodynamic theory for experimentally accessible frequencies. However, if the microscopic variables can be separated into two groups of distinctly different relaxation behaviour, with their frequencies of fluctuations orders of magnitude apart, with the vast majority constituting the much faster one, the hydrodynamic theory is still able to provide an adequate description of the system for frequencies of the “normal” hydrodynamic regime:

\[ \omega \tau_c \ll 1. \]

All that is needed is a slight generalization of the concept of local equilibrium, which now means that all the fast variables have assumed the equilibrium values determined by the hydrodynamic and the slow microscopic ones.

Now, the above described circumstance is exactly what happens in the vicinity of any phase transition. Since the leading coefficient of the Landau expansion vanishes at \(T_c\), the order parameter—with a relaxation time inversely proportional to this coefficient—fluctuates slowly and has to be included in the set of thermodynamic variables. This has been done for superfluid $^4$He by Landau et al. \([2]\), and for the nematic liquid crystal (above \(T_c\)) by de Gennes \([3]\). The theory of the orbital part of $^3$He—A is published elsewhere \([4]\); here, the spin part is presented.

The order parameter \(D_{1\alpha}\) has 18 real components. Five correspond to spontaneously broken gauge and rotational symmetries in spin and orbital space (i.e. neglecting the dipole energy). They are hydrodynamic variables. The remaining 13 are slow, independent, and have to be considered explicitly only in the vicinity of the A-transition.

It turns out that, for a homogeneous system, the order parameter is able to perform five pairs of damped oscillations connected to various internal deforma-
tions; the rest of three modes (there have to be 13 additional modes), connected to the magnitude $\Delta$ of the order parameter and the two transverse components of the superfluid magnetization, are purely relaxative. Including the dipole energy, the latter two couple to the fluctuation of the preferred direction in spin space and influence the NMR experiments, causing excess damping and additional resonance shifts. In addition, a new signal $\omega_\phi$ arises in the high frequency regime, changing the “Pythagorean”\cite{5} to

$$\omega_{\text{transv.}}^2 = \omega_{\text{long.}}^2 + \omega_{\text{Larmor}}^2 - \omega_\phi^2.$$  

Furthermore, expanding to linear order in the wave number $k$, only the magnitude $\Delta$ couples to the orbital variables like the density and superfluid velocity; and only the superfluid magnetization couples to spin variables like the magnetization. They again cause damping and dispersion and give rise to new propagating modes.

Deriving the equation of motion on thermodynamic and symmetry arguments alone, it may be of interest to note that those for the superfluid magnetization keep their structure as derived by Leggett and Takagi\cite{6} up to the temperature range presently under consideration $[1 - (T/T_c) < 10^{-2}]$. However, the parameters are quite different, especially the relaxation time: theirs has been identified with the collision time of the normal phase\cite{7}, while it is expected to diverge at the phase transition in the theory presented here.

2. The order parameter

The order parameter $D_{i\alpha}$ of the ABM-state can be written as\cite{5}

$$D_{i\alpha} = n^\alpha_0 \Delta_\alpha,$$

where $\Delta_\alpha$ is an intrinsically complex vector in orbital space and $n^\alpha_0$ a unit vector in spin space, which can be chosen as real in the absence of an external magnetic field. Under rotation in spin and orbital space the order parameter transforms according to the two vectors, $\Delta_\alpha$ and $n^\alpha_0$, respectively. Under a gauge transformation of $\theta$ each component of $D_{i\alpha}$ is multiplied by $\exp(i2\theta)$, and the operation of time inversion (denoted by $\sim$) is equivalent to complex conjugation. The time inversion behaviour of $\Delta_\alpha$ and $n^\alpha_0$ follows from that of $D_{i\alpha}$, and therefore is not unique. I choose the convention:

\[ n^\alpha_0 = n^\alpha_0, \quad \Delta_\alpha = \Delta_\alpha^* \quad (1) \]

Putting all the transformation properties together, there is enough information for writing down the hydrodynamic equations. However, as a direct product of two vectors of different spaces, without a definite time inversion behaviour (it is neither odd nor even) and not being gauge invariant, $D_{i\alpha}$ is a rather clumsy quantity to handle. Therefore we proceed to look for variables with simpler transformation properties. With the help of the two equilibrium quantities:

\[ N^2 \equiv D_{i\alpha}^* D_{i\alpha} \quad \text{and} \quad n^\alpha_0 \equiv \epsilon_{\alpha\beta\gamma} \Delta_\beta^* \Delta_\gamma / iN^2, \quad (2) \]

we can split up the deviation $d_{i\alpha}$ of the order parameter from its equilibrium value $D_{i\alpha}$ to the following fluctuations:

\[ H = (D_{i\alpha} \cdot d_{i\alpha} + \text{c.c.}) / 2N, \]

\[ G = (D_{i\alpha} \cdot d_{i\alpha} - \text{c.c.}) / 2iN, \]

\[ \Delta = (D_{i\alpha}^* \cdot d_{i\alpha} + \text{c.c.}) / 2N, \]

\[ \varphi = (D_{i\alpha}^* \cdot d_{i\alpha} - \text{c.c.}) / 2iN, \]

\[ H_i = (\Delta_\alpha \cdot d_{i\alpha} + \text{c.c.}) / 2N - H \cdot n^\alpha_0, \quad (3) \]

\[ G_i = (\Delta_\alpha \cdot d_{i\alpha} - \text{c.c.}) / 2iN - G \cdot n^\alpha_0, \]

\[ \Delta_i = (\Delta_\alpha^* \cdot d_{i\alpha} + \text{c.c.}) / 2N - \Delta \cdot n^\alpha_0, \]

\[ \varphi_i = (\Delta_\alpha^* \cdot d_{i\alpha} - \text{c.c.}) / 2iN - \varphi \cdot n^\alpha_0, \]

\[ l_{\alpha} = \epsilon_{\alpha\beta\gamma} \cdot (D_{i\beta}^* \cdot d_{i\gamma} - \text{c.c.}) / 2iN - \Delta^\alpha_0, \]

\[ l_{\alpha i} = \epsilon_{\alpha\beta\gamma} \cdot (\Delta^\beta_\alpha \cdot d_{i\gamma} - \text{c.c.}) / 2iN - [(\Delta_i + \Delta n^\alpha_0) n^\alpha_0, \]

\[ + l_{\alpha i} n^\alpha_0 \] .

(The Einstein summation convention is employed throughout this paper.)

These variables are arranged to satisfy the restricting relations:

\[ H n^\alpha_0 = G n^\alpha_0 = \Delta n^\alpha_0 = \varphi n^\alpha_0 = 0, \]

\[ (l_{\alpha})^2 = 1, \quad l_{\alpha i} = 0, \quad l_{\alpha i} n^\alpha_0 = 0, \quad (4) \]

showing that $H_i, G_i, \Delta_i, \varphi_i,$ and $l_{\alpha i}$ have only two and $l_{\alpha i}$ only four fluctuating components. This means eqs. (3) define 18 independent variables. (The independence can be easily recognized by writing down $H,$
The definitions of eqs. (3) are practical and the restrictions of eqs. (4) valid only if \( n^0 \) is real. They can be considered approximately correct for a small external magnetic field. More precisely, for fields whose resulting imaginary part \( \phi^0 \) of \( n^0 \) remains much smaller than its real part \( C^0 \), i.e.

\[
|C^0| \gg |\phi^0|,
\]

where \( C^0 \) and \( \phi^0 \) are defined as

\[
C_i^0 = (\Delta_i^* D_{i\alpha} + \text{c.c.})/2N^2,
\]

\[
\phi_i^0 = (\Delta_i^* D_{i\alpha} - \text{c.c.})/2iN^2.
\]

As long as the condition of eq. (5) is satisfied, \( \phi^0 \) need not be considered as an extra preferred direction (which would make the system become biaxial). Since the magnetic field is parallel to \( (C^0 \times \phi^0) \), it does not constitute a third preferred direction. Well below \( T_c \), eq. (5) holds for any magnetic field and the system is always uniaxial.

The respective transformation properties of \( H \), \( G \), ..., follow directly from their definitions: \( H \), \( G \), \( \Delta \), and \( \varphi \) are scalars, \( l_{\alpha} \) is an axial vector of orbital space, whereas \( H_i \), \( G_i \), \( \Delta_i \), and \( \varphi_i \) are polar vectors of the spin space, and \( l_{\alpha i} \) transforms like, for example \( \Delta_i \). \( H \), \( H_i \), \( \Delta_i \), and \( \Delta \) are even under time inversion and the rest are odd. Furthermore, most of them (i.e. \( \Delta_i \), \( \Delta_i \), \( \varphi_i \), \( l_{\alpha} \), and \( l_{\alpha i} \)) are gauge invariant. \( \Delta_i \), \( l_{\alpha} \) and \( \varphi \) stand for an infinitesimal rotation of \( D_{i\alpha} \) in spin, orbit and gauge space. To see this, one can use the following identity, which is readily verified by being inserted into eqs. (3):

\[
d_{\alpha i} = \frac{1}{N} \left\{ (\Delta + i\varphi) D_{i\alpha} + (H + iG) D_{i\alpha}^* + (\Delta_i + i\varphi_i) \Delta \right\}.
\]

(1) \( \Delta_i \neq 0 \) with all other variables vanishing, i.e. \( d_{\alpha i} = i\varphi D_{i\alpha}/N \). This is a gauge transformation with the phase of \( \varphi/2N \).

(2) \( \Delta_i \neq 0 \) with all other variables vanishing, i.e. \( d_{\alpha i} = \Delta_i \Delta_{\alpha}/N \). This is a rotation of \( D_{i\alpha} \) in spin space. Taking \( X \) to be the preferred axis, it is a rotation by \( -\Delta_x/N \) about the \( Y \) axis, and by \( \Delta_y/N \) about the \( Z \) axis.

(3) \( l_{\beta} \neq 0 \) with all other variables vanishing, i.e. \( d_{\alpha i} = (2i/N) e_{\alpha\beta\gamma} D_{i\gamma} l_{\beta} \). This is a rotation in orbital space. Taking \( X \) to be the preferred axis, it is a rotation by \( -2l_y/N \) about the \( Z \) axis and by \( 2l_z/N \) about the \( Y \) axis.

These are the hydrodynamic variables, corresponding to the five spontaneously broken continuous symmetries. The remaining 13 vanish in equilibrium. \( \Delta \) is the deviation of the magnitude from its equilibrium value \( N \), \( H \) describes the difference between the real and imaginary part, and \( G \) allows the right angle between these two parts to fluctuate. \( n^0 \times \phi \) is proportional to the deviation of the transverse (i.e. perpendicular to \( n^0 \)) component of the superfluid magnetization from its equilibrium value determined by the external field. It is interesting to note that, since we have not assumed a homogeneous system anywhere in the discussion, the definitions of eqs. (3) and their geometrical meanings are not altered for small deviations from a non-uniform equilibrium texture [8].

3. The thermodynamical properties

The "generalized" equilibrium is determined by the conserved quantities and the 18 components of the order parameter. The conserved quantities are energy density \( E \), mass density \( \rho \), momentum density \( j \), and magnetization \( m \). With \( S \) as the entropy density we have the Gibbs relation:

\[
dE = TdS + \mu d\rho + \nu d\mu + \lambda d\nu + \varphi_{\alpha\beta} dv_{\beta\alpha} + \Phi d\Phi + \mu G dG + \mu \Delta d\Delta + \mu H dH + \mu^G dG + \mu^\Delta d\Delta
\]

(7)

All conjugate variables are hereby defined. To conform with the usual notations, I have set for the five hydrodynamic variables:

\[
V^x \equiv \nabla(\varphi/2N), \quad n \equiv \Delta/N, \quad l' \equiv l/N.
\]

I hope that dropping the prime of \( l' \) in the following notation will not cause any confusion. The hydrodynamic conjugate variables have been worked out in various publications [9,10]. Here, the non-hydrodynamic conjugate variables have to be introduced. Expanding them in all the thermodynamic variables and eliminating terms not compatible with the symmetry, we are left with five additional elastic coefficients
[11]:
\[ \mu^H = aH, \quad \mu^G = aG, \quad \mu^A = C\Delta, \]
\[ \mu_i^H = bH_i, \quad \mu_i^G = bG_i, \quad \mu_i^F = f\varphi_i, \]
\[ \mu_{ai} = g\alpha_i. \] (8)

The expressions for \( \mu^A \) and \( \mu_i^F \) are valid only if the hydrodynamic variables are held constant.

The inclusion of the dipole energy is accomplished by introducing [9] the term \( \lambda e_i \), i.e. we have to add \( /\alpha_n^2 \) to eq. (7) with
\[ \mu_i^F = \lambda n_i. \] (9)

4. The equations of motion and their solutions

4.1. In a homogeneous system

The time derivatives of the non-hydrodynamic spin variables are expanded in the conjugate variables, for the moment only up to the zeroth order in wave number \( k \). With proper symmetry under space, time and gauge transformations we have [11]
\[ \dot{H}_i + \xi_1 \mu_i^G + \xi_2 \mu_i^H = 0, \] (10)
\[ \dot{G}_i - \xi_1 \mu_i^H + \xi_3 \mu_i^G = 0, \]
\[ \dot{\alpha}_i + [\gamma_1 \rho_i^\alpha + \gamma_2 (\delta_{\alpha\beta} - \rho_i^\alpha \rho_i^\beta)] \mu_i^F = 0, \] (11)
and
\[ \dot{\varphi}_i + \beta_1 \mu_i^F = 0. \] (12)

Due to the fact that \( H_i \) and \( G_i \) are not gauge invariant, one has the equality \( \xi_2 = \xi_3 \). This can be shown by writing down \( (\dot{H}_i + i\dot{G}_i) \) and requiring the gauge invariance of the resulting equation. Because of entropy production, all the irreversible coefficients (i.e. \( \xi_2, \xi_3, \gamma_2, \beta_1 \)) have to be positive. \( \xi_1 \) and \( \gamma_1 \) are reactive coefficients and help to perform two pairs of degenerate damped oscillations:
\[ \omega = (\pm i_1 - i_2)\psi, \quad \omega = (\pm \gamma_1 - i\gamma_2)\varphi. \] (13)

Lacking a reactive coupling, \( \varphi_i \) is only able to relax:
\[ \omega = -i\beta_1 f \equiv -i/\tau. \] (14)

Here, \( \tau \) is introduced as the relaxation time of the superfluid magnetization. It clearly diverges as \( f \) goes to zero at the phase transition, assuming the transport coefficient \( \beta_1 \) to be uncritical. In order for \( \varphi \) to reach equilibrium, it seems necessary to have at the same time two mechanisms, characterized by the collision time \( \tau_c \) and the relaxation time \( \tau \), respectively. Well below \( T_c \), \( \tau \) is microscopically short and \( \tau_c \) dominates. Close to \( T_c \) it is the diverging \( \tau \) which is relevant.

Eqs. (13) and (14), each being once degenerate, represent ten modes. With the three collective modes of the orbital part [4] and the five Goldstone modes [5], there are altogether 18, as could be expected. Comparing these results with those calculated by Wölffe [12] for a collisionless regime, one finds that the grouping of the variables and their coupling to the fluctuations of the conserved quantities (see section 4.2) have undergone some drastic changes.

Including the dipole energy term, eqs. (10) and (11) retain their form, while \( \dot{\varphi}_i \) now couples reactively to \( n_i \):
\[ \dot{\varphi}_i - \alpha \mu_i^F + \beta_1 \mu_i^F = 0, \] (15)
\[ \dot{n}_i + \alpha \mu_i^F + \beta_2 \mu_i^F + \gamma n_k \epsilon_{kli} h_i = 0, \] (16)
and because of the spin–orbit coupling the magnetization is no longer a conserved quantity [9]:
\[ \dot{m}_i - \gamma (m \times H^F)_i + \gamma n_k \epsilon_{kli} \mu_i^F = 0, \] (17)
where, \( H^F \) is the external magnetic field. Eqs. (15), (16) and (17) have the same structure [13] as those derived by Leggett and Takagi [6], indicating the correctness of their assumption, that the fluctuations of the nonhydrodynamic components of \( D_{j\alpha} \) other than \( \varphi_i \) do not influence the damping of the NMR-signals.

These seven equations form two independent subsets, longitudinal and the transverse, because a reflection under a plane perpendicular to the magnetization (which is a symmetry element of the system) changes the signs only of the longitudinal variables.

Before solving these equations, I have redefined \( \varphi_i \):
\[ \varphi_i \equiv [\Delta_0 (D_{j\alpha} + d_{j\alpha}) - \text{c.c.}]/2iN \varphi \cdot n_i^0, \] (18)
to be the actual value rather than the deviation. It has
non-vanishing thermodynamic cross derivatives. Linearized, the conjugate variables become:

\[
\delta h_i = \left[ \chi^{-1}_{ii} + f h^2 \cdot \left( \delta_{ii} - n^0_\parallel n^0_\parallel \right) \right] \delta m_j + f h n^0_\parallel \epsilon_{kj} \delta \phi_j ,
\]

\[
\delta \mu_f = f \left( \delta \phi_i - h n^0_\parallel \epsilon_{kj} \delta m_j \right) ,
\]

(19)

where \( \chi^{-1}_{ii} = \chi^{-1}_0 \delta_{ii} \) is the inverse magnetic susceptibility in true equilibrium (i.e. \( \mu_f^0 = 0 \)), and \( h = (\partial \phi_2 / \partial m_3)_{eq.} = -(\partial \phi_3 / \partial m_2)_{eq.} \) is the additional elastic coefficient, coupling both fluctuations thermodynamically. \( h \) vanishes well below \( T_c \) and is, for \( |c_0^c| \gg |c_0^\phi| \), proportional to \( N^{-1} \) near the A-transition [14].

The solution of the longitudinal subset is

\[
\omega^2 = (\omega_L^2 + i \omega \lambda [\beta_2 + (\gamma - \omega \tau)^2 / (1 - i \omega \tau) \cdot \beta_1] ,
\]

with \( (\omega_L^2)^2 = \lambda \gamma^2 \chi^{-1}_0 \) as the Leggett frequency. The solution of the transverse subset is

\[
\omega^2 = \omega_{LA}^2 + (\omega_L^2)^2 - i \omega \lambda \left[ \beta_2 \left( \frac{\omega_L^2}{\omega} \right)^2 \right.
\]

\[
+ \frac{1}{\beta_1} \frac{\omega - \omega_L^2}{1 - i \omega \tau} \left( \frac{\omega^2 - \omega_{LA}^2}{\omega^2} - \omega h \right) \right] ,
\]

(21)

where \( \omega_{LA} \) is the Larmor frequency.

Not too near the A-transition \( h \) and \( \omega \tau \) are negligible quantities. Here, we reproduce the narrowing of the transverse resonance [6,15]:

\[
\Delta \omega_L = \lambda \beta_2 + (\gamma^2 / \beta_1) ,
\]

\[
\Delta \omega_T = \omega_L \cdot \omega_L^2 / [\omega_{LA}^2 + (\omega_L^2)^2] .
\]

(22)

The term in the damping with \( \beta_2 \) results from the relaxation of \( n \) toward its equilibrium position determined by \( P^0 \), \( \beta \). \( \omega_L \) comes from the fluctuation of the superfluid magnetization. If \( h \) is much greater than \( \phi \) and \( \beta_2 \), there will be no narrowing effect left. For \( h \) and \( \beta \) comparable, the damping of the transverse mode is only for a vanishing external magnetic field necessarily positive definite. This is not astonishing, since the magnetic field may be used as an energy source.

Neglecting the dissipation arising from \( \beta_2 \) (which is an order in \( \omega \) smaller), eq. (21) is a biquadratic equation in \( \omega \) for the high frequency regime (\( \omega \tau \gg 1 \)).

Its roots can be expanded to look much simpler under the assumption

\[
\omega_{LA}^2 + \lambda \gamma^2 \chi^{-1}_0 + f(\gamma h - \omega^2) \\
\gg 2 \omega_L \left| \lambda \gamma h - \omega \right|^{1/2} .
\]

(23)

The two roots are

\[
\omega_{L}^2 = \lambda \gamma h - \omega \phi ,
\]

\[
\omega_{T}^2 = \omega_{LA}^2 + (\omega_{LA}^2)^2 - \omega_{LA}^2 
\]

(24)

(25)

where \( \omega_{LA}^2 = \lambda \gamma^2 \chi^{-1}_0 + f(\gamma h - \omega^2) \) is the high-frequency longitudinal resonance, \( \omega_{T}^2 \) the corresponding transverse resonance, now satisfying this extended form of Pythagorean, and \( \omega \phi \) is a new NMR-signal. Eq. (23) implies \( \omega_T \gg \omega_L \).

4.2. In an inhomogeneous system

Expanding the time derivatives an order further in wave number \( k \), eqs. (10) keep their form, while \( \dot{\mu} \) now couples to hydrodynamic variables of both spaces, e.g.

\[
\dot{\mu}_f = \gamma_2 \left( \delta_{ii} - n^0_\parallel n^0_\parallel \right) + \gamma_1 \frac{\partial^2}{\partial r^2} \left( n_\parallel \phi \right) \nabla \cdot \nabla \phi = 0 .
\]

But as long as eq. (5) holds, this coupling is almost non-linear. In the second order of \( k, H_i \) and \( G_i \) do couple to hydrodynamic variables, e.g.

\[
\Delta \omega \dot{\delta}_{i} = (\delta_{i} - n^0_\parallel n^0_\parallel) \Delta \omega \cdot \nabla \phi = 0 ,
\]

(27)

and the corresponding oscillation may therefore, at least in principle, be excited.

It can be concluded that, to linear order in \( k \) and for a uniaxial spin system, the fluctuation of \( \phi_0 \) again is solely responsible for the damping of the spin waves. Considering the dipole energy as a small symmetry breaking force, one can introduce the bending energy of the preferred direction \( n^0 \) via [9]

\[
\lambda \rightarrow \lambda + (M_1 \cos^2 \Theta + M_2 \sin^2 \Theta) k^2 ,
\]

(28)

with \( \Theta \) as the angle between \( k \) and \( P^0 \), and \( M_i \) as the respective elastic coefficients. A substitution of eq. (20), (21), (23), (24) and (25) generates the corresponding results for the spin waves.

5. Possible experimental consequences

All the many elastic constants and transport coefficients introduced in this and previous papers [4,9] are within the framework of the hydrodynamic theory — unknown phenomenological parameters.
Thus, it seems difficult to tell the range of validity and where to look for those predicted effects. Nevertheless one can find some rough estimates of the relevant frequency, wavelength, and temperature range either within the mean field approximation or in analogy to more familiar systems like superconductors.

For the order parameter to become a slow variable, the relaxation time $\tau$ has to be much greater than the collision time $\tau_c$. Assuming $\tau$ to be of the same form as the relaxation time of the superfluid density in an uncharged superconductor [16], i.e.

$$\tau = \hbar [32k_B(T - T_c)]^{-1} = 5 \times 10^{-10} e^{-1} s,$$

where $\epsilon \equiv 1 - (T/T_c)$ is the reduced temperature, then $\tau \gg \tau_c$ is equivalent to

$$\epsilon \ll 5 \times 10^{-2}. \quad (29)$$

The collision time is here taken to be $10^{-8}$ s. It restricts the frequency range to

$$\omega \ll 10^8 \text{ Hz}, \quad (30)$$

because the local equilibrium has to be built up. Following Leggett [15], eq. (30) only applies to the frequency of the longitudinal resonance and not to that of the transversal one.

The question of where to look for the high frequency $\varphi$-signal is somewhat more complicated. The high frequency regime $\omega \tau \gg 1$ can be reached by either

$$\omega_L \tau \gg 1 \quad \text{or} \quad \omega_{LA} \tau \gg 1.$$  

Since $\omega_L \sim \epsilon^{1/2}$ and $\tau \sim \epsilon^{-1}$, the first condition can always be satisfied by going arbitrarily close to $T_c$.

Taking [14] $\omega_L = 2 \times 10^6 \epsilon^{1/2}$, it is equivalent to $\epsilon \ll 10^{-6}$, which seems quite unrealistic concerning the experimental possibilities. The second condition can be satisfied by increasing the magnetic field. With [14] $\omega_{LA} = 2 \times 10^4 \cdot H$ (in Gauss), it implies

$$\epsilon \ll 10^{-5} \cdot H \text{ (in Gauss)}. \quad (31)$$

But too high a magnetic field changes the structure of the order parameter, invalidating eq. (5), which is equivalent to

$$\epsilon \gg T_{A_1}/T_A - 1 = 2 \times 10^{-6} \cdot H \text{ (in Gauss)} \quad (32)$$

where $T_{A_1}$ and $T_A$ are the respective transition temperatures. Eqs. (31) and (32) give an estimate of the temperature window in which the $\varphi$-signal may occur.

Because of the relation $\omega^2 = \omega_{LA}^2 + \omega^2 + C_{sp}k^2$, one may try to increase the frequency by letting the system become inhomogeneous. This, however, is not promising for the following reason: The correlation length $\xi$ diverges with $\epsilon^{-1/2}$ at the phase transition, and the condition $k\xi \ll 1$, valid for any hydrodynamic theory, requires the permitted wave number range to vanish with $\epsilon^{1/2}$. Taking the spin wave velocity [5,14] $C_{sp} = 20 \times \epsilon^{1/2}$ m/s, the spin wave part of $\omega \tau$, $C_{sp}k \cdot \tau$, is independent of the reduced temperature $\epsilon$ and much smaller than unity.

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References

[1] However, as has been shown by F. Jähnig and H. Schmidt, Ann. Phys. (NY) 71 (1972) 129, for the broken rotational symmetries, the reason for the hydrodynamic behaviour may be traced to a (more or less hidden) conserved quantity.


