On-chip atom interferometry

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Interference of two condensates

Separation between the condensates: \( d \)

Relative momentum of the condensates after an expansion time \( \tau \):

\[
\Delta p \approx m \frac{d}{\tau}
\]

The corresponding wavelength is the period of the fringe pattern:

\[
\Delta x \approx \frac{h \tau}{md}
\]

Interference of two condensates

Interference of condensates

Condensate wavefunctions:

\[ \Psi_1(x,t) = f_1(x-x_1,t) \cdot e^{i\varphi_1(x-x_1,t)} \]
\[ \Psi_2(x,t) = f_2(x-x_2,t) \cdot e^{i\varphi_2(x-x_2,t)} \]

Interference:

\[ |\Psi_1 + \Psi_2|^2 = f_1^2 + f_2^2 + 2f_1f_2\cos(\varphi_1 - \varphi_2) \]

Time evolutions of the amplitude \( f_i(x-x_i,t) \) and phase \( \varphi_i(x-x_i,t) \) have to be taken into account!

Thomas-Fermi approximation

We approximate the GP equation for repulsive interaction \((a>0)\) in the limit

interaction energy >> kinetic energy

\[
\mu \psi(\vec{r}) = \left( -\frac{\hbar^2 \nabla^2}{2m} + V(\vec{r}) + g |\psi(r, t)|^2 \right) \psi(\vec{r}) \\
\approx \left( V(\vec{r}) + g |\psi(r, t)|^2 \right) \psi(\vec{r})
\]

The density the shape of the potential (TF limit):

\[
n(r) = |\psi(r, t)|^2 = \frac{1}{g} (\mu - V(\vec{r}))
\]

\(ng \leftrightarrow \xi = \frac{1}{\sqrt{8\pi na}}\)

int. energy healing length

Expansion of condensates in the TF regime

**In the harmonic trap**

**Density profile**
parabolic, with TF radius

\[
R_i(0) = \sqrt{\frac{2\mu}{M\omega_0^2}}
\]

\[
b_\perp = \sqrt{1 + \tau^2}
\]

\[
b_\parallel = 1 + \lambda^2 \left( \tau \arctan(\tau) - \ln \sqrt{1 + \tau^2} \right)
\]

**Phase distribution**
uniform

**free expansion**

\[
R_i(t) = R_i(0) \cdot b_i(t)
\]

\[
\omega_i = \lambda \omega_\perp
\]

\[
\tau = t \cdot \omega_\perp
\]

scaling parameter

describing the size of the condensate at any time in units of the Thomas-Fermi radius

\[
\phi_i(x - x_0) = \frac{\alpha_i}{2} (x - x_0)^2
\]

\[
\alpha_i(t) = \frac{m}{\hbar} \frac{\dot{b}_i(t)}{b_i(t)} \quad \text{for large } t \quad \alpha_i(t) = \frac{m}{\hbar} \frac{1}{t}
\]

Interference of two condensates in the TF regime

\[ |\Psi_1 + \Psi_2|^2 = f_1^2 + f_2^2 + 2f_1f_2 \cos(\phi_1 - \phi_2) \]

Phase difference:

\[
\phi_1 - \phi_2 = \frac{\alpha_1}{2} (x - x_1)^2 - \frac{\alpha_2}{2} (x - x_2)^2 = \\
= \frac{\alpha}{2} x^2 - \frac{\alpha}{2} (x - \delta x)^2 = \\
= \alpha x \delta x + \frac{\alpha}{2} (\delta x)^2 \approx \\
\approx \left( \frac{m \delta x}{\hbar t} \right) x \\
\frac{2\pi}{\lambda_{\text{fringe}}} = \\
\text{fringe spacing}
\]
Force measurement

In general, the phase develops as the time integral of the Lagrangian.

Lagrange function: \[ L = E_{\text{kin}} - E_{\text{pot}} \]

Action: \[ S = S_0 + \int_{t_i}^{t_f} dt \left( E_{\text{kin}} - E_{\text{pot}} \right) \]

Phase: \[ \varphi = \frac{1}{\hbar} \left( S = \frac{1}{\hbar} \left[ S_0 + \int_{t_i}^{t_f} dt \left( E_{\text{kin}} - E_{\text{pot}} \right) \right] \right) \]

In case of a condensate, \( S_0 \) contains the parabolic phase profile, any initial centre of mass motion, and initial phase.

Principle of an atom interferometer:
Spatial phase

\[
spatial\ phase = \left( \frac{\Delta}{\lambda_{\text{fringe}}} \cdot 2\pi \mod 2\pi \right)
\]
Coherence is lost after breaking the tunnel coupling due to excitation of the condensate at the splitting point.
Coherence is preserved after the splitting for ~4 oscillation periods (2ms). Dephasing for longer observation times.
Magnetic trap as an avoided crossing

Hamiltonian for a spin $S=1/2$ system in a magnetic field

\[
H = -\mu \mathbf{B} \\
\mu = \mu_B g \frac{\mathbf{S}}{\hbar}
\]

\[
E_S \Phi = \frac{1}{2} \hbar \left[ \frac{e}{m} B_x \sigma_x + \frac{e}{m} B_y \sigma_y + \frac{e}{m} B_z \sigma_z \right] \Phi
\]

Static magnetic field

$B_x = \text{const.}$

$B_z \propto z$

Energy eigenvalues

\[
E_S = \pm \frac{1}{2} \hbar \sqrt{\omega_L^2 + \Omega^2}
\]

Adiabatic limit:

\[
\Gamma_{LZ} = 2\pi \frac{(\hbar \Omega)^2}{\hbar (dE/dt)} \gg 1
\]
**RF-induced adiabatic potentials**

Static + RF magnetic field

\[ B_x = B_0 + B_{RF} \cos(\omega_{RF} t) \]

\[ B_z \propto z \]

Energy eigenvalues

\[ \tilde{E}_S = \pm \frac{1}{2} \hbar \sqrt{\left(\omega - \omega_{RF}\right)^2 + \tilde{\Omega}^2} \]

\[ \tilde{\Omega} = \frac{1}{2} \frac{e}{m} B_{RF} \]

Reference:


Long phase coherence due to number squeezing

Expected coherence time for coherent states: $\tau_c \sim 20$ ms

Experiment: $\tau_c > 200$ ms

Splitting results in number squeezed states: lower phase diffusion rate for number squeezed states.

Number squeezing

Phase diffusion rate in a condensate:

\[ R = \frac{2\pi}{h} \left( \frac{d\mu}{dN_i} \right)_{N_i=N/2} \Delta N_r \]

Derivative of the chemical potential

Standard deviation of the relative atom number

Coherent state: \( \Delta N_r = \sqrt{N} \)

Squeezed state: \( \Delta N_r = \frac{\sqrt{N}}{s} \)

s: squeezing parameter

Phase sensitive recombination of BECs

Excitation energy: 

$$N\hbar \omega$$

Adiabatic merging

$$\Delta \phi = \pi$$

Sudden removal of a thin “phase membrane”

$$\Delta \phi = 0$$
Phase sensitive recombination of BECs

The excitation due to the phase difference decays and heats the cloud:

→ loss of condensate fraction

Jo et al., PRL 98, 180401 (2007)
Oscillation of the condensed fraction as a function of the relative phase and the recombination time

Hold time in the trap determines the relative phase

Recombination time

“slow”

“sudden”

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Jo et al., PRL 98, 180401 (2007)
Bragg diffraction

Two photon transition between two momentum states of the ground state:

\[ \vec{p}_{\text{final}} = \vec{p}_{\text{initial}} + \hbar (\vec{k}_1 - \vec{k}_2) \]

With \( k_1 \approx k_2 (\equiv k) \) and \( \hbar \delta = E_{\text{final}} - E_{\text{initial}} = \frac{2}{m} (\hbar k p_{\text{initial}} + \hbar^2 k^2) \)

\[ |g, p_{\text{initial}}\rangle \rightarrow |g, p_{\text{final}} = p_{\text{initial}} + 2\hbar k\rangle \]
Bragg diffraction

Oscillatory behavior with the two photon Rabi-frequency:

\[ \Omega_1 = \frac{\Omega_{0,\text{ laser}1} \Omega_{0,\text{ laser}2}}{2\Delta} \]

with the resonant single photon Rabi-frequencies

\[ \Omega_0^2 = \frac{\Gamma^2}{2} \frac{I}{I_{\text{sat}}} \]

and

\[ I_{\text{sat}} = \frac{2\pi^2 \hbar c}{3\lambda^3 \tau} \quad \Gamma = 1/\tau \]
Bragg pulse interferometer

First pulse split the condensate into equal populations with $p=0$ and $p=2\hbar k$.

Second pulse inverts the states.

Third pulse recombines the wavepackets,

Output ports:
1. $P_1$ population in $p=0$
2. $P_2$ population in $p=2\hbar k$

If there is no phase shift, the populations $P_1$ and $P_2$ are equal.

This is a single particle interferometer scheme. BEC just makes the detection easier.

Bragg pulse interferometer

measuring the phase of a Bose-Einstein condensate wave function

FIG. 2. (a)–(e) One of the two output ports of the interferometer with $T_0 = 4$ ms and $\delta x$ as indicated. (f) A plot of the density along the $x$ direction of (d).

J. E. Simsarian et al., PRL 85, 2040-2043 (2000)
Atom Michelson interferometer based on Bragg pulses

FIG. 1 (color online). (a) Schematic drawing of the atom chip (not to scale). The dimensions of the chip are 5 cm × 2 cm. (b) A photo image of the atom chip glued onto a copper holder.

Wang et al., PRL 94, 090405 (2005)

FIG. 3. Interference pattern of (a) phase shift = 2nπ and (b) phase shift = (2n + 1)π. The absorption images are taken 10 ms after the recombining pulse.

FIG. 4. Interference fringes after 1 ms propagation time in the waveguide. The magnetic gradient is turned on for 500 μs and the average separation of clouds during the magnetic pulse is 8.82 μm.
Magnetic lattice

Produced by: thin film evaporation (300nm thick gold), and dry etching. **Substrate:** Si, covered by SiO$_2$ isolation layer.

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On-chip atom interferometry
**Diffraction from a lattice**

1. BEC prepared 30µm below the lattice
2. BEC forced to oscillate towards the lattice
3. Observation after 20ms TOF: **diffraction & interference**

Günther et al., *PRL* 95, 170405 (2005)

Günther et al., *PRL* 98, 140403 (2007)
Diffraction from the lattice

Displacement $d$ ($\mu$m)

Phase modulation index $S$

Displacement $d$ ($\mu$m)

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Phase modulation

Periodic potential of the meander:

\[ U = u_0 e^{-kz} \cos kx \]
\[ k = \frac{2\pi}{a} \]
\[ u_0 = 2\mu_0 \mu I / a \]

Wave function directly after phase imprinting

\[ \psi(x, y, z) = \sqrt{n(x, y, z)} \exp \left( -\frac{i}{\hbar} \int_0^t U(t) dt \right) \]
\[ = \sqrt{n(x, y, z)} \exp (-i S(d) \cos kx) \]

Phase modulation index

\[ S(d) := \int_0^t \frac{u_0 e^{-kz(t)}}{\hbar} dt \]

Expansion in momentum eigenfunctions of the axial motion (Bessel functions of first kind):

\[ \psi = \sqrt{n(x, y, z)} \cdot \sum_n (-i)^n J_n(S) e^{i nkx} \]

Number of atoms in the \( n \text{th} \) diffraction order proportional to:

\[ N_n \sim |J_n(S)|^2 \]

Ref: C. Henkel, J.-Y. Courtois, and A. Aspect
Phase modulation

Expansion in momentum eigenfunctions of the axial motion (Bessel functions of first kind):

$$\Psi = \sqrt{n(x, y, z)} \cdot \sum_{n} (-i)^n J_n(S) e^{in k x}$$

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Diffraction and interference

\[ n(S, x) = \Psi(S, x) \cdot \Psi^*(S, x) - \sum_{n=-\infty}^{\infty} \Psi_n(S, x) \cdot \Psi_n^*(S, x) \]

Günther et al., *PRL* 95, 170405 (2005)
Günther et al., *PRL* 98, 140403 (2007)
Reproducible phase

- New type of atom interferometer based on a single diffraction pulse
- Lattice constant defined by the fabricated conductor geometry

\[
\sigma = 21^\circ = \frac{\pi}{8}
\]
Single atom manipulation & detection

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