Knowledge Structures

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Introduction

The traditional approach to assessing human competence and knowledge is based on a numerical evaluation of some kind of ‘aptitude’ (like in intelligence tests, school grades, ...). There are serious problems with this approach:

- Is there any justification for representing ‘aptitude’ by real numbers, and which mathematical operations on these numbers are empirically meaningful?
- Dimensionality of the considered ‘aptitude’?

Knowledge structures provide
- precise, non-numerical characterization of the state of knowledge that captures the strengths and weaknesses in all parts of the knowledge domain,
- adaptive knowledge assessment procedures for efficiently identifying the individual state of knowledge as in an oral examination,
- information that can be fed into technology-enhanced learning systems for generating personalized learning paths.

Knowledge structures are formulated within a set-theoretical framework on a knowledge domain, which is a set $Q$ of (dictatorial) problems. Knowledge structures were first developed from a purely behavioralistic point of view, but the framework may be extended to include psychological constructs (e.g. competencies) underlying the observable behavior (Doignon & Falmagne, 1985; Falmagne, Koppen, Villano, Doignon & Johannesen, 1990; Doignon & Falmagne, 1999; Albert & Lukas, 1999). For a non-technical introduction see Falmagne, Cosyn, Doignon, and Thiby (2006, cf. http://www.alkeia.com/about.htm?Science_based_ALE.pdf).

Example

Basic Concepts

Precedence Relation

In general the solution behavior on a knowledge domain $Q$ will exhibit some dependencies. Some pieces of knowledge normally precede, in time, other pieces of knowledge. In our context, some algebra problem may be solvable by a student only if some other problems have already been mastered by that student. This may be because some prerequisites are required to master a problem, but may also be due to historical or other circumstances. A precedence relation is used to capture this information.

Probabilistic Knowledge Structure

In practical applications we cannot assume that an individual’s response to a problem is correct if and only if it is contained in the respective knowledge state. Moreover, knowledge states may not be equally likely. A probabilistic knowledge structure then is based on

- item-specific probabilities of a careless error,
- item-specific probabilities of a lucky guess,
- A probability distribution on the knowledge states.

Precedence relation on the complete set of beginning algebra problems (88 problem types, see http://www.alkeia.com)

Adaptive Knowledge Assessment

Knowledge assessment proceeds in an adaptive manner (taking into account previous answers) in order to assess an individual’s knowledge with asking a minimal number of questions. Guided by an underlying knowledge structure
- present problems only if they provide substantial information,
- avoid posing problems that are too difficult, or too easy for the individual.

At each step of the adaptive assessment procedure the current plausibility of the knowledge states is characterized by a likelihood function. After an appropriate problem is selected and the response is observed, the plausibility of the knowledge states is updated (e.g. Bayesian updating of the likelihood function).

Projects

Previous research dealt with theoretical developments as well as practical applications in the context of technology enhanced learning (Heller, 2004; Heller, Levine, Kenyon, Albert, & Hockemeyer, 2007; Heller, Steiner, Hockemeyer, & Albert, 2006; Heller & Reiprich, 2008).

The aim of various planned projects is to put the theory to work in a variety of domains and contexts.

- Representation and individual development of orthogonally knowledge in German with an implementation in a web-based tutorial system (together with K. Landers, U. Trautwein, and O. Luftik)
- Applying probabilistic knowledge structures to the design of a tutoring system in basic mathematics for students of psychology
- Using the framework as a theoretical basis for (adaptive) diagnostics and personalized intervention (e.g., in clinical settings)
- These research strands provide ample room for bachelor, master, diploma, and PhD theses.

References