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Helmholtz’s theory of space and its significance for Schlick

Matthias Neuber

Helmholtz’s theory of space had significant impact on Schlick’s early ‘critical realist’ point of view. However, it will be argued in this paper that Schlick’s appropriation of Helmholtz’s ideas eventually lead to a rather radical transformation of the original Helmholtzian position.

KEYWORDS: Helmholtz; Schlick; space; congruence; empiricism; critical realism

1. INTRODUCTION

Moritz Schlick, the founder of the famous ‘Vienna Circle’, had developed an interesting and highly original philosophical position before he actually founded the Circle in 1922. This early (and not very well known) philosophical position may be called ‘critical realism’.1 Still, it should be noted that this sort of critical realism had a strong empiricist foundation. Like other critical realists, namely – and especially – Alois Riehl, Wilhelm Wundt and Oswald Külp, Schlick was convinced that a stable form of ‘scientific philosophy’ can only be reached by accepting certain empiricist

1By ‘critical realism’, I roughly mean the view that the Kantian dualism between ‘appearances’ and ‘things-in-themselves’ can be modified in terms of the knowability of the latter. Herbert Feigl (who had written his dissertation (1927) under Schlick) comments on the early Schlick’s affiliation to the critical realist tradition very aptly as follows: ‘It is interesting to note that both Schlick (from 1910 to 1925) and Russell (by 1948, at any rate) were critical realists and thus had to come to grips with the problems of transcendence. And, while they differed sharply in their views on probability and induction, they argued essentially inductively for the existence of entities beyond the scope of the narrow domain of immediate experience. Both Schlick and Russell thus liberalized the radical empiricism of Hume, namely, by asserting the existence of a world of knowable things-in-themselves – be they such objects of common life as sticks or stones, rivers or mountains, or be they the fields and particles of modern physics. It was only under the impact of Carnap’s and Wittgenstein’s ideas and criticisms that Schlick withdrew to what he conceived of as a neutral, non-metaphysical position’. (Herbert Feigl, *Inquiries and Provocations: Selected Writings, 1929–1974*, edited by R.S. Cohen (Dordrecht: Reidel, 1981, 29).
theses. His theory of space (and spatial perception) is a good example in this respect.

1921, the year before Schlick came to Vienna, marked the 100th anniversary of the birth of Hermann von Helmholtz. It was on that occasion that Schlick (together with the physicist Paul Hertz) edited a collection of Helmholtz's *Schriften zur Erkenntnistheorie*, including, among other things, Helmholtz's seminal papers ‘Über den Ursprung und die Bedeutung der geometrischen Axiome’ (originally published in 1870) and ‘Die Tatsachen in der Wahrnehmung’ (originally published in 1878). Schlick contributed extensive explanatory notes to these two papers and, in doing so, he obviously tried to appropriate Helmholtz’s ideas on behalf of his own version of a critical realism with empiricist foundations. Much the same holds true for Schlick’s small paper ‘Helmholtz als Erkenntnistheoretiker’, which also appeared on the occasion of the Helmholtz centennial. In short, there seems to be an interesting relation between Helmholtz and Schlick.

What I want to show in the following is that Schlick’s attempt at an appropriation of Helmholtz’s ideas eventually lead to a position rather foreign to what Helmholtz himself had intended. In particular, Schlick’s comments on Helmholtz’s theory of space perception testify that there are several fundamental divergences between Helmholtz’s original position and Schlick’s account of it. Nevertheless it must be seen that Helmholtz’s theory played an instructive role for Schlick. It had significance for him in so far as what he called the ‘method of coincidences’ was foreshadowed in Helmholtz’s conception of congruence, which, for Helmholtz, formed the foundation of any theory of measurement. I will argue that it was exactly here that – in the case of early Schlick – realism and empiricism came together.

2. SCHLICK ON HELMHOLTZ THE EPISTEMOLOGIST

First of all it is important to realize that Schlick saw in Helmholtz one of the most influential figures of the rebirth of a rational, scientifically motivated understanding of philosophy. According to Schlick, the first half of the nineteenth century was dominated by a completely unacceptable form of philosophical obscurantism. Especially, the ambitions of

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speculative – post-Kantian – Naturphilosophie, as he could find it realized primarily in the works of Hegel and Schelling, appeared to him as pretentious and as an outcome of an irrational metaphysics. In his 1921 centennial lecture ‘Helmholtz als Erkenntnisteoretiker’, Schlick pointed out that Helmholtz rejected this kind of metaphysics for the benefit of a moderate (Kantian-inspired) ‘critique of knowledge’.4 ‘Helmholtz the philosopher’, as Schlick aptly put it, ‘is Helmholtz the theorist of knowledge’.5

So it was Helmholtz the epistemologist who set the stage for Schlick’s attempt at an appropriation of some significant Helmholtzian ideas. More exactly, it was – at least according to Schlick – Helmholtz the empiricist who had made important contributions both to philosophy and to the exact sciences:

As a physiologist he made sensation and perception the primary objects of his inquiry, and with the problem of perception he at once advanced to the epistemological issue; as a mathematician and mathematical physicist he made critical examination of abstract thinking in arithmetic, geometry and physics, and thus contributed fundamentally to the theory of conceptual knowledge. The findings he arrived at by both routes, from the perceptual and conceptual angles, were in perfect harmony with each other, and combined to form an empiricist theory of knowledge, that is, a philosophy which finds the source of all true knowledge in experience, and the sole warrant for the truth and validity of all knowledge of the real in perception.6

Going one step further, Schlick claimed that Helmholtz’s most important contribution in the field of epistemology was his theory of space. Though Helmholtz himself thought of this theory as a theory in the tradition of Kantian ‘apriorism’, Schlick appreciated it for its – undeniable – empiricist ingredients. For Schlick, the Kantian element in Helmholtz’s theory of space could be neglected. Or in his own words:

There have been efforts to interpret Helmholtz’s view as if he were in greater agreement with Kant than he thought himself to be – but they have not, in my opinion, had much success. Nor does it matter in the least how far Helmholtz departed from Kant, and to what extent he was aware of doing so. The one thing that does matter, is whether he is correct in his view of geometrical knowledge. And on this point there can nowadays be no doubt: Helmholtz’s greatest epistemological achievement, his theory of space, is quite certainly true.7

So, the question to be answered is whether the ‘truth’ of Helmholtz’s theory of space can really be separated from its belonging to the apriorist tradition. Only if this is possible can the Kantian element in Helmholtz’s

5Ibid.
6Ibid.
7Ibid., 340.
theory in fact be neglected and, consequently, be superseded by an empiricist account of space and geometrical knowledge.

3. SCHLICK ON HELMHOLTZ AND RIEMANN

One of the essential aspects of Schlick’s account of Helmholtz’s theory of space is that he is trying to assimilate this theory to the theory of space developed by Bernhard Riemann. As is well known, Riemann – in his 1854 Habilitationsvortrag – had worked out a theory of space according to which space was what he called an ‘n-dimensional manifold’. More exactly, space, for Riemann, constituted a special case of a ‘threefold extended quantity’. Schlick – in his seminal Raum und Zeit in der gegenwärtigen Physik (1917) – frequently referred to this theory of Riemann’s. At one place he stated:

Since the time of Riemann and Helmholtz we have been accustomed to talk of plane, spherical and other spaces, and discriminate from our observations to which of these classes our ‘real’ space belongs. We now understand how to interpret this: viz. not as if one of these can be predicated of space, without taking account of objects in it; but in the sense that experience teaches us only whether it is more practical to use Euclidean or non-Euclidean geometry for the physical description of nature. Riemann himself, and likewise Helmholtz, was quite clear about the question; but the results of both these investigators have often been misinterpreted […]

The particular views of Riemann and Helmholtz notwithstanding, Schlick is describing here a position which amounts to a ‘conventionalized’ form of spatial empiricism. This ‘conventionalized’ form of spatial empiricism has three important implications:

1. Space itself is dependent on matter.
2. Spatial perception reduces to experience and convention.
3. The axioms of geometry are not synthetic a priori.

This peculiar view is certainly Schlick’s. But is it also Helmholtz’s? Following the interpretation of Alberto Coffa, one is tempted to say ‘yes’.

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9For further details, see Roberto Torretti, Philosophy of Geometry from Riemann to Poincaré (Dordrecht: Reidel, 1978), p. 82ff.
10Schlick, Philosophical Papers, Vol. 1, 231.
11Coffa states quite generally that ‘Schlick was the first to attempt a systematic formulation of the picture of knowledge implicit in Helmholtz’s writings’ (Alberto Coffa, The Semantic Tradition from Kant to Carnap: To the Vienna Station, edited by L. Wessels (Cambridge: Cambridge...
However, in recent years Coffa’s interpretation has been contested. Michael Friedman, for example, thinks that Coffa and other commentators ‘have been far too quick […] simply to take Schlick’s attempt at appropriation [of Helmholtz’s ideas] at face value’. And Thomas Ryckman, to quote another example, is of the opinion that one must sharply distinguish between ‘Helmholtz and “Schlick”’s Helmholtz’. So who is right: Coffa or his critics? In order to answer this question, it is unavoidable to reflect on Helmholtz’s relation to Riemann. After that, we will be in a position to evaluate Schlick’s attempt at an appropriation.

4. HELMHOLTZ VERSUS RIEMANN

Let us first come back to Riemann’s famous Habilitationsvortrag. What one can find there is – besides the original Riemannian theory of manifolds – a more or less explicit formulation of spatial realism. This means, in a nutshell, that Riemann is committed to the following claims:

1. Space itself is dependent on matter.
2. Spatial perception can be neglected.
3. The axioms of geometry are not synthetic a priori.

As can be easily seen, it is point (2) where Riemann is departing from the description of a ‘conventionalized’ form of spatial empiricism that we have given before. For Riemann, space is in the first place a matter of conceptual analysis, that is, of an abstract, completely unintuitive reflection on the general notion of manifold. He is definitely not concerned with the modalities (and peculiarities) of our perceptual apprehension of space. On the other hand, Riemann is not an advocate of spatial apriorism in the tradition of Kant. Quite the contrary is true: For Riemann, conceptual analysis of space (and spatial relations) must be attached directly to material reality as it is in itself. It is for this reason that Riemann’s account of physical geometry (and his view of nature in general) has been characterized as ‘conceptual realism’. This conceptual realism forms the philosophical background of Riemann’s famous assumption of a curved

University Press, (1991), 171–2). This – together with Schlick’s own comments on Helmholtz the epistemologist – suffices in order to see in Helmholtz’s theory of space an obvious case in point.


(instead of a flat or Euclidean) space. And it is exactly here where Riemann would turn out to be one of the most important precursors of Albert Einstein’s general theory of relativity.\textsuperscript{15}

It is beyond the shadow of a doubt that Helmholtz too played an important role in the prehistory of general relativity. Nevertheless, there are fundamental differences between his account of space and geometrical knowledge, on the one hand, and Riemann’s, on the other. Roughly put, Helmholtz’s account comprises the following claims:

1. Space itself is ‘transcendental’.
2. Spatial perception is fundamentally important.
3. The axioms of geometry are not synthetic a priori.

This is what Helmholtz is arguing for both in ‘Über den Ursprung und die Bedeutung der geometrischen Axiome’ and in his first paper on the foundations of geometry, entitled ‘Über die Tatsachen, die der Geometrie zum Grunde liegen’ (1868). The only indisputable agreement with Riemann is that Helmholtz is contesting the Kantian assumption that the axioms of (Euclidian) geometry are synthetic a priori. But what about the rest?

Let us begin with point (1). According to Helmholtz, one must distinguish categorically between space, on the one hand, and spatial measurement, on the other. Whereas space forms part of theoretical knowledge, spatial measurement forms part of the concrete practice of empirical science. The axioms of geometry, Helmholtz maintains, are embodied in the system of spatial measurement. Therefore, he thinks that they are not synthetic a priori. On the other hand, space itself is for Helmholtz the precondition (or, in Kantian terminology, the ‘condition of the possibility’) of all measurement and, consequently, of the axioms of geometry as well. Or as Helmholtz himself famously put it, ‘Space can be transcendental without the axioms being so’.\textsuperscript{16}

It is important to make clear what exactly is meant by this statement. To be sure, Helmholtz would not dispute the Riemannian assumption that the concrete spaces of measurement are dependent on matter. But he would add that those concrete spaces themselves are dependent on the general concept of space.\textsuperscript{17} Furthermore, Helmholtz would also add that this general

\textsuperscript{15}Concerning the philosophical background of Riemann’s account of physical geometry, it should be noted that he was influenced by the metaphysical realism of Johann Friedrich Herbart. For further details, see Erhard Scholz, ‘Herbart’s Influence on Bernhard Riemann’, Historia Mathematica, 9 (1982), 413–40. See also Erik C. Banks, ‘Kant, Herbart and Riemann’, Kant-Studien, 96 (2005), 208–34.

\textsuperscript{16}Helmholtz, Epistemological Writings, 149.

\textsuperscript{17}This line of argument reappeared prominently in the writings of the German Neo-Kantians. See, for example, Paul Natorp, Die logischen Grundlagen der exakten Wissenschaften (Leipzig: Teubner, 1910); Richard Högniswald, Zum Streit über die Grundlagen der Mathematik. Eine erkenntnistheoretische Untersuchung (Heidelberg: Winter, 1912); and Ernst Cassirer, Zur Einsteinschen Relativitätstheorie. Erkenntnistheoretische Betrachtungen (Berlin: Verlag Bruno Cassirer, 1921).
concept of space (‘space itself’) can be characterized by an overarching feature, namely by the free mobility of rigid bodies. Accordingly, the assumption of the free mobility of rigid bodies forms part of our concept of space as a precondition of all measurement. Space – via its overarching feature of free mobility – is for Helmholtz ‘a given form of intuition, possessed prior to all experience’, and therefore transcendental. Yet, the feature of free mobility itself is, as might be said, the ‘neuralgic point’ in Helmholtz’s whole account of space. Whereas Riemann allowed for the possibility of both constant and variable curvature of space (thereby paving the way for Einstein), Helmholtz’s account is restricted only to constant curvature, because free mobility of rigid bodies would be impossible under the assumption of a variable curvature. Concerning the axioms of geometry, Helmholtz – like Riemann – recognized the possibility of different (both Euclidean and non-Euclidean) systems of such axioms, thereby being an ‘empiricist’ in so far as he thought that the decision which system physically ‘fits’ is a matter of empirical discovery. Thus, it can be said that ‘Helmholtz argued against the Kantian philosophy of geometry while retaining an inherently Kantian theory of space’. 

This brings us to point (2). For Helmholtz’s ‘inherently Kantian theory of space’ is nothing but a theory of space perception. As is well known, it was the central aim of Helmholtz’s 1868 paper on the ‘facts which lie at the basis of geometry’ to give a psycho-physiological foundation for Riemann’s conceptual theory of space. According to Helmholtz, it is possible to derive the fundamental assumption of Riemann’s theory – namely that the line-element is of Pythagorean (or infinitesimally Euclidean) form – from psycho-physiologically determinable ‘facts’ by which our perceptual...
intuition of space is generated. These ‘facts’ are for Helmholtz related to our experience of bodily motion. They are, Helmholtz maintains, derivable from our experience of moving in arbitrary directions: up to, away from and around the objects ‘in’ space. And it is exactly at this point where the condition of free mobility comes into play. For it is only under this condition that arbitrary continuous motions of rigid bodies are possible, and it is from this very condition that one can derive the Pythagorean form of the line-element.\textsuperscript{23} Moreover, Helmholtz thinks it legitimate to infer from this that space – via the condition of free mobility – is the ‘necessary form of outer intuition’.\textsuperscript{24}

The contrast with Riemann is rather obvious now. It is not only the case that Helmholtz is concerned with perception and Riemann is not. It is also the case that Helmholtz thinks that he can give a psycho-physiological ‘justification’ for his un-Riemannian assumption that space itself is transcendental. Free mobility of rigid bodies is, in Helmholtz’s theory, the ‘guarantor’ that enables concrete measurement. Whereas Riemann merely talked of the ‘hypotheses’ which lie at the basis of geometry, Helmholtz substituted ‘hypotheses’ with ‘facts,’ because he thought that via the condition of free mobility the merely hypothetical character that the Pythagorean metric actually had in Riemann’s theory was eliminated for the benefit of \textit{necessity}.\textsuperscript{25} It is quite plausible to interpret this as the most distinct expression of Helmholtz’s ‘inherently Kantian theory of space’.

In its concrete form (and application), this theory amounts to a conception of measurement according to which rigidity has the status of a transcendental condition: The notion of a rigid body is constitutive for the notion of \textit{congruence}, and the notion of congruence in turn is (supposed to be) the enabling element of geometrical measurement. So, Helmholtz concludes, ‘all our geometrical measurements rest upon the presupposition that the measuring instruments which we take to be rigid, actually are bodies of unchanging form’.\textsuperscript{26}

\textsuperscript{23}The mathematical formulation of the condition of free mobility was given by Sophus Lie in 1890. For further details, see Torretti, \textit{Philosophy of Geometry}, 171ff.

\textsuperscript{24}Helmholtz, \textit{Epistemological Writings}, 124.

\textsuperscript{25}It should be noted that under the assumption of constant curvature there are still three possibilities of geometrical systems: Euclidean, hyperbolic, and elliptic. For further details, see Friedman, ‘Geometry as a Branch of Physics’, 198.

\textsuperscript{26}Helmholtz, \textit{Epistemological Writings}, 15. According to DiSalle (2006, 76–9), to Helmholtz our (even primitive) experiences with light rays provide us with the concept of straight line, while the (equally primitive) willful actions of moving of our bodies around in space suggest to us the group structure of spatial displacement, and so the notion of congruence. DiSalle concludes that Helmholtz’s theory of space is empiricist in the sense that geometry (or rather its possibility) is rooted in these primitive regularities in our experience of space, experiences that duly inform us of something of the character of real physical space. However, this seems to be a rather abridged, ‘radical empiricist’, version of Helmholtz’s theory, since it completely neglects the transcendental status that Helmholtz ascribes to the \textit{condition} of free mobility. Nevertheless I would agree with DiSalle’s opinion that it was only Riemann who ‘imagined that physics itself
5. SCHLICK VERSUS HELMHOLTZ

We are now in a position to discuss what Schlick made of all this. Here it is important first to make clear that Schlick did not at all embrace the Helmholtzian theory in an unqualified manner. On the contrary, Schlick was rather harsh in his critique of Helmholtz’s account of rigidity. In one of his comments on Helmholtz’s 1870 geometrical paper Schlick pointed out:

What kind of sense is there in saying a body is actually rigid? According to Helmholtz’s definition of a fixed body [...], this would presuppose that one could speak of the distance between points ‘of space’ without regard to bodies; but it is beyond doubt that without such bodies one cannot ascertain and measure the distance in any way. [...] If the content of the concept ‘actually’ is to be such that it can be empirically tested and ascertained, then there remains only the expedient [...] to declare those bodies to be ‘rigid’ which, when used as measuring rods, lead to the simplest physics. [...] Thus, what has to count as ‘actually’ rigid is then not determined by a logical necessity of thought or intuition, but by a convention, a definition.²⁷

What Schlick is attempting to argue for in this passage is that any form of Kantian apriorism – be it ‘logical necessity’ of thought or, as in the case of Helmholtz, of intuition – can be easily substituted by some sort of Poincaréan conventionalism. As he did in his critique of the early Hans Reichenbach’s Kantian-inspired book _Relativitätstheorie und Erkenntnis a priori_ (1920),²⁸ Schlick denies the existence of principles (or statements) which are synthetic and at the same time _a priori_. According to Schlick, conventions – and with them the criterion of simplicity – suffice in order to make sense of the presuppositions of measurement. But conventions are _not_ – as the Kantian doctrine would require – synthetic and at the same time _a priori_.

So it is Helmholtz’s idea of an ‘a priori-rigidity’²⁹ which serves as the main target of Schlick’s critique. Helmholtz’s account of rigidity is, as Schlick maintains, not in accord with scientific practice. Even worse: according to Schlick, Helmholtz’s _definition_ of a rigid body is bluntly circular. Schlick expresses this objection as follows:

This definition reduces congruence (the equality of two tracts) to the coincidence of point pairs in rigid bodies ‘with the same fixed point pairs in

²⁷Schlick in Helmholtz, _Epistemological Writings_, 34.
²⁸See Schlick, _Philosophical Papers_, vol. 1, 333. See also the discussion of Schlick’s critique in Michael Friedman, _Reconsidering Logical Positivism_ (Cambridge: Cambridge University Press, 1999), ch. 3.
space’ and thus presupposes that ‘points in space’ can be distinguished and held fixed. This presupposition was explicitly made by Helmholtz [...], but for this he had to presuppose in turn the existence of ‘certain spatial structures which are regarded as unchangeable and rigid’. Unalterability and rigidity [...] cannot for its own part again be specified with the help of that definition of congruence, for one would otherwise clearly go round in a circle. For this reason the definition seems not to be logically satisfactory. 30

One escapes the circle, Schlick continues, ‘only by stipulating by convention that certain bodies are to be regarded as rigid, and one chooses these bodies such that the choice leads to the simplest system of describing nature.’ 31 Here again, it is conventions that provide the alternative to Helmholtz’s allegedly misguided account of rigidity.

Nevertheless, it is more than obvious that Schlick appreciated Helmholtz’s theory of space for what he called its ‘truth’. It is this aspect of his approach towards Helmholtz the epistemologist to which we shall turn now.

6. HELMHOLTZ MODIFIED

Despite the fact that Schlick sought to substitute for Helmholtz’s ‘a priori-rigidity’ an epistemologically weaker, conventionalist conception of rigidity, there are still a number of points where he thought himself in complete agreement with Helmholtz. The first point worth mentioning is that Helmholtz, according to Schlick, did not commit the Kantian ‘mistake’ of confusing spatial intuition with the concept of space. What this means becomes clear from the following quote:

Helmholtz is entirely correct to align the spatial intuition he describes with the sensory qualities, for it is a matter in both cases of subjective, psychical contents. What he describes, is psychological space [...], not physical-geometrical space. 32

30 Schlick in Helmholtz, Epistemological Writings, 31. Schlick is referring here to the following passage from Helmholtz’s 1870 geometry paper: ‘[T]he definition of a rigid body can [...] only be given by the following characteristic: Between the coordinates of any two points belonging to a rigid body, an equation must exist that expresses an unchanged spatial relation between the two points (which finally turns out to be their separation) for any motion of the body, and one which is the same for congruent point pairs. Such point pairs however are congruent, which can succinctly coincide with the same fixed point pairs in space.’ (Helmholtz, Epistemological Writings, 15)

31 Ibid.

32 Schlick in Helmholtz, Epistemological Writings, 167. In Raum und Zeit in der gegenwärtigen Physik, Schlick explicitly identifies what he regards as Kant’s ‘mistake’ as follows: ‘The space of sensory objects is identical, for Kant, with the space of geometry and physics; he regards it as something intuitive, but contrasted, as “pure” intuition, with the “empirical intuition” of the particular senses: it is thus taken to be the space neither of sight, nor of touch, nor of the motor senses alone, and yet in a certain respect to be all of these at once. In contrast to this Kantian
This passage strongly confirms the idea that a sharp distinction should be drawn between the intuitive space of psychology, on the one hand, and the conceptual space of geometry and physics, on the other. Whereas intuitive space is purely subjective, physical-geometrical space is – at least on Schlick’s account – objective. More exactly speaking, Schlick assumes that the subjectivity of intuitive space is testified by the fact that our different sense-organs provide us with different ‘modalities’ of spatial intuition. Visual spatial perception, for instance, has a different quality than haptic spatial perception. They are not even similar to each other. Accordingly, Schlick thinks it legitimate to infer from this qualitative diversity of the different intuitive spaces the subjectivity of spatial intuition. He even goes so far as to deny that intuition in general has something to do with knowledge. Or as he puts it in his magnum opus, *Allgemeine Erkenntnislehre* (1918):

The intuition of things is not knowing and also not a precondition of knowing. The objects of knowledge must be thinkable without contradiction, that is, allow of a univocal designation via concepts, but they do not need to be intuitively representable.\(^{33}\)

The opposition to the *Grundidee* of Kant’s *Critique of Pure Reason* – especially to the ‘Transcendental Aesthetic’ – could not be stated more clearly.

This last point is centrally important, for it makes it clear that physical-geometrical space, for Schlick, must be something purely conceptual. And indeed, in one of his comments on Helmholtz’s writings, Schlick explicitly states:

'[Physical-geometrical space] is a non-qualitative, formal, conceptual construction; [psychological space], as an intuitive given, is clothed, according to Helmholtz’s words, with the qualities of sensation, and is just as subjective as they are.'\(^{34}\)

So, if physical-geometrical space is – in terms of epistemology – purely conceptual and therefore freed from the subjective elements of spatial intuition, then it appears highly plausible to assume that physical-geometrical space is – in terms of ontology – the *objective ordering of transcendent material points*. For only by the exclusion of the ‘anthropomorphic distortions’ of our perceptual system a direct coordination of

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\(^{34}\)Schlick in Helmholtz, *Epistemological Writings*, 167.
concepts and mind-independent facts can be achieved. It is exactly here where the realism of the early Schlick comes into play: Physical-geometrical space serves as his paradigm case for the coordination of concepts and mind-independent facts.\textsuperscript{35}

However, there remains the question of how we actually arrive from our individual perceptual spaces of intuition at the ‘conceptual construction’ of physical-geometrical space. Indeed, initially we are all ‘imprisoned’ in subjectivity – all we can rely on in our epistemic \textit{Urzustand} is a purely qualitative, unstructured stream of consciousness. Hence, the question to be answered is how spatial objectivity is constituted or, in other words, by which means space as a ‘conceptual construction’ is effected.

It is this very question which opens the final chapter of Schlick’s \textit{Raum und Zeit in der gegenwärtigen Physik}. There, he puts the whole issue thus:

\begin{quotation}
It is scarcely necessary to mention that the words space and time in the preceding sections have been used only in the ‘objective’ sense in which these conceptions occur in natural science. ‘Subjective’ psychological experience of extension in space and order in time is quite distinct from these. Ordinarily there is nothing to induce us to analyse this difference in detail; the physicist does not need to concern himself in the slightest with the investigations of the psychologist into spatial perception. But when we wish to form a clear picture of the ultimate epistemological foundations of natural science, it becomes necessary to give an adequate account of the relationship between these two points of view. This is the task of the philosopher; for it is generally accepted that it is for philosophy to reveal the fundamental assumptions of the separate sciences, and bring them into harmony with each other.\textsuperscript{36}
\end{quotation}

In his \textit{Allgemeine Erkenntnislehre}, Schlick explains how both areas – the psychological description of subjective spatial perception and the physical description of objective spatial structures – are to be brought together:

\begin{quotation}
The ordering of our contents of consciousness in space and time is likewise the means by which we learn to determine the transcendent order of things outside our consciousness, and the latter order is the most important step towards their cognition. The problem now is to become clear on how one proceeds from the intuitive spatiotemporal order to the construction of the transcendent order. This always occurs by the same method, which we can designate as the method of coincidences. It is epistemologically of the very highest importance.\textsuperscript{37}
\end{quotation}


\textsuperscript{36}Schlick, \textit{Philosophical Papers}, Vol. 1, 259.

\textsuperscript{37}Schlick, \textit{General Theory of Knowledge}, 272.
To be sure, the construction of the objective spatiotemporal order is based upon the intuitive – subjective – order. But by the method of coincidences it becomes possible to realize the ‘escape from perspective’ and thus to attain objective knowledge. It is for this reason that Schlick thinks the method of coincidences is ‘epistemologically of the very highest importance’.

This is not the place to reconstruct the method of coincidences in detail. In a nutshell, the assumption of this method is motivated by the following considerations. If, as Schlick maintains, concepts are strictly separated from intuition, then the question must be answered how conceptual thought acquires empirical content. In the case of geometry, in particular, the question to be answered is how the implicitly defined axioms of geometrical systems – whether Euclidean or non-Euclidean – can get a factual interpretation. Following Riemann, one can arrive at those different geometrical systems by way of conceptual analysis. Thus, the conception of an ‘n-dimensional manifold’ leaves open a whole range of different geometrical systems, and it must be somehow determined how geometrical axioms and physical facts can be coordinated. The method of coincidences is Schlick’s answer to this problem. Indeed, it is this very method by which the gap between the conceptual and the intuitive is supposed to be bridged. According to Schlick, we construct the transcendent ordering on the basis of singularities in our various intuitive spaces. These singularities are nothing else but concrete sections of the whole perceptual situation. To use one of Schlick’s examples: When I see the tip of my pencil touch my finger, I have two perceptual singularities at the same time – one in my visual field and one in my tactile field. Both fields have a completely different intuitive spatiality – my visual perception has no intuitive relation to my tactile perception. By the method of coincidences I bring both fields into relation: I construct a single, nonintuitive spatial ordering which contains both the pencil and my finger. Completely abstracting from the qualitative peculiarities of my various intuitive spaces I thereby generate a single point in objective – physical – space. Or, as Schlick puts in his Raum und Zeit in der gegenwärtigen Physik:

In order to fix a point in space, we must in some way or other, directly or indirectly, point to it: we must make the point of a pair of compasses, or a finger, or the intersection of cross-wires, coincide with it (i.e. bring about a time-space coincidence of two elements which are usually apart). Now these coincidences always occur consistently for all the intuitional spaces of the various senses and for various individuals. It is just on account of this that a ‘point’ is defined which is objective, i.e. independent of individual experiences and valid for all. [...] Upon close investigation, we find that we arrive at the construction of physical space and time by just this method of coincidences.

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39 For a detailed (and well-informed) reconstruction, see Friedman, ‘Helmholtz’s Zeichentheorie and Schlick’s Allgemeine Erkenntnisslehre’, 25–8.
and by no other process. The space-time manifold is neither more nor less than
the quintessence of objective elements as defined by this method. The fact of its
being a four-dimensional manifold follows from experience in the application
of the method itself.\textsuperscript{40}

The last sentence indicates that Schlick sees a close connection between
his method of coincidences and Einstein’s general theory of relativity.
Hence, he can conclude that by taking seriously his own analysis of space
and time one will encounter ‘just that significance of space and time which
Einstein has recognized to be essential and unique for physics, where he has
established it in its full right’.\textsuperscript{41}

This, in turn, brings us back to Helmholtz. For, on the interpretation
given by Schlick, it was Helmholtz who first recognized the importance
of coincidence for any theory of measurement. Helmholtz’s ‘principle of
congruence’, which Helmholtz had introduced in his 1870 geometrical
paper, is, according to Schlick, the clearest expression of this insight. Thus,
he writes in one of his comments on the 1870 geometrical paper:

Congruence is established through observation of the coming together of
material points. All physical measurement can be reduced to this same
principle. […] Helmholtz’s proposition [that ‘we may not forget here that all
geometrical measurements rest finally on the principle of congruence’] can
therefore be extended to the truth that, in general, no other events are
physically ascertainable than meetings of points, and Einstein has logically
drawn the conclusion from this that all physical laws may in principle contain
only assertions about such coincidences.\textsuperscript{42}

In this way, the assimilation of Helmholtz’s theory of space to the most
fundamental idea of Einstein’s general theory of relativity is made perfectly
explicit.\textsuperscript{43}

\textsuperscript{40}Schlick, \textit{Philosophical Papers}, Vol. 1, 262–3.

\textsuperscript{41}Ibid., 263. For the details of the connection between Schlick’s method of coincidences and
Einstein’s general theory of relativity, see Don Howard, ‘Point Coincidences and Pointer
Coincidences: Einstein on Invariant Structure in Spacetime Theories’, in \textit{The History of General
Relativity IV: The Expanding Worlds of General Relativity}, edited by H. Goenner, J. Renn, J.
Ritter and T. Sauer (Boston, 1999), 463–500. See also Thomas Ryckman, ‘P(oint)-C(oincidence)
Thinking: The Ironical Attachment of Logical Empiricism to General Relativity (and Some

\textsuperscript{42}Schlick in Helmholtz, \textit{Epistemological Writings}, 33–4.

\textsuperscript{43}Einstein’s own accentuation of the importance of coincidence can be found in his 1916 seminal
paper ‘Die Grundlage der allgemeinen Relativitätstheorie’. There he writes: ‘All our space-time
verifications invariably amount to a determination of space-time coincidences. If, for example,
events consisted merely in the motion of material points, then ultimately nothing would be
observable but the meeting of the material points of our measuring instruments with other
material points, coincidences between the hands of a clock and points on the clock dial, and
observed point-events happening at the same place at the same time.’ (Albert Einstein, ‘The
Original Memoirs on the Special and General Theory of Relativity}, translated by W. Perrett and
However, as we have seen before, Schlick does not agree with Helmholtz’s definition of a rigid body. Since this definition fundamentally contains the assumption of congruence, the essential core of the Helmholtzian theory of space gets modified by Schlick’s ‘conventionalist expedient’. In the last analysis, this modification leads to a mixture of ideas delivered by Riemann, Poincaré and Helmholtz, respectively. From Riemann, Schlick takes over the idea that physical-geometrical space is purely conceptual rather than intuitive. This, at the same time, sets the stage for the realist component in Schlick’s theory of space. But realism alone would not suffice – Schlick is convinced that the assumption of the ‘transcendent reality’ of physical space must be supplemented by the assumption of its empirical accessibility. And here he thinks it legitimate to install the Helmholtzian conception of congruence in such a way that the method of coincidences – and with it the empirical content of the axioms of geometry – will result. Thus, empiricism plays an essential role in Schlick’s account of space. On the other hand, it must be seen that the empiricist component is significantly qualified by the integration of conventions in the sense of Poincaré. In order to determine the rigidity of bodies, we must, according to Schlick, rely on conventions. But then rigidity – as well as congruence – is dependent on what we are doing (and not on what there is). Whether this is compatible with the overarching realism Schlick intends to establish may be doubted.

7. CONCLUDING REMARKS

Let us finally come back to the question whether what Schlick regarded as the ‘truth’ of Helmholtz’s theory of space can really be separated from the Kantian – apriorist – tradition. As we have seen, in some sense it can be separated. But the crucial point is that, for Helmholtz, the assumption of a rigid body conditioned the transcendental status of ‘space itself’. In the hands of Schlick rigidity gets conventionalized and ‘space itself’ consequently loses its transcendental status. Thus, it can be clearly concluded that Schlick’s ‘modified Helmholtz’ is rather incompatible with the original, Kantian-inspired Helmholtz.

G.B. Jeffery (New York: Dover, 1923) 117) For an elaboration of this assertion, see Howard, ‘Point-Coincidences and Pointer-Coincidences’.

See footnote 31, above.

It should be noted (but cannot be elaborated here) that Schlick was heavily influenced in this regard not only by Riemann but also by David Hilbert from whom he took over the idea that geometrical concepts are not gained from intuition but are defined implicitly, i.e. through each other in the context of an axiomatic system. For further details, see Schlick, General Theory of Knowledge, § 7. See also Friedman, ‘Geometry as a Branch of Physics’, 203-4 and Friedman, ‘Helmholtz’s Zeichentheorie and Schlick’s Allgemeine Erkenntnislehre’, 22-4.
But there is a further divergence that needs to be mentioned. As both Michael Friedman\(^\text{46}\) and Helmut Pulte\(^\text{47}\) have pointed out, Helmholtz would not have accepted Schlick’s rigid separation between intuition and conceptual knowledge. For Helmholtz:

[from the first step on, when we perceive the objects lingering before us distributed in space, this perception is the recognition of a lawlike connection between our motions and the sensations occurring thereby. Thus, the first elementary representations already contain thinking and proceed in accordance with the laws of thinking.\(^\text{48}\)]

Therefore, Helmholtz – like Kant – does not distinguish between psychological and physical-geometrical space. For him psychological – intuitive – space is indeed identical with the physical-geometrical system of ‘lawlike’ relations among our sensory experiences. And this conception as well is, as Friedman has rightly diagnosed, ‘entirely alien to Schlick’s way of thinking’.\(^\text{49}\)

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\(^{46}\)See Friedman, ‘Helmholtz’s \textit{Zeichentheorie} and Schlick’s \textit{Allgemeine Erkenntnislehre}’, 38–43.


\(^{48}\)Helmholtz, \textit{Epistemological Writings}, 138.

\(^{49}\)Friedman, ‘Helmholtz’s \textit{Zeichentheorie} and Schlick’s \textit{Allgemeine Erkenntnislehre}’, 41.


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