Complexity of Financial Products: 
a Quantitative and Economic
Approach

Study on behalf of the
Deutscher Derivate Verband e.V. (DDV),
the German Derivatives Association
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Executive Summary

If a financial product is classified as complex according to MiFID II (Markets in Financial Instruments Directive), additional administrative effort is required for the assessment of suitability and appropriateness. Despite the extensive consequences of this classification, there are still barely any approaches for the measurement of complexity. In principle, there are different options for this task:

MiFID II evaluates complexity by means of the characteristics of the corresponding financial product. Similarly, the approach of the French Financial Markets Authority AMF (Autorité des Marchés Financiers), which was valid until the beginning of this year, combined the number of underlying asset classes with the number of calculation mechanisms. At first glance, those approaches allow for a simple and clear classification. However, there are reasonable doubts as to whether they really capture the crucial aspects of the financial product’s complexity.

In terms of valuation uncertainty, the AMF now cares about the observability of the underlying asset, the simplicity of the valuation method and the risk profile in a qualitative manner. With this new recommendation, they account for three out of seven facets of transparency that are relevant to the concept of complexity according to Becker/Döhrer/Johanning (2012). Based on these aspects, Koh/Koh/Chuen/Lim/Ng/Phoon (2015) developed a scoring model. Although those approaches reveal the derivation of complexity, they still suffer from subjectivity that exacerbates or even prevents practical application unless further guidance is given.

In this study, we take a different perspective and develop an economic approach to determining the complexity of financial instruments. This is based on a transparent notion of complexity from the investor’s perspective along with a clear method aligned with this notion. Every investor has an estimation of the value of a financial product in mind, which might differ from its true value. We define this difference as the “value surprise.” Hence, our model can be understood as a quantitative implementation of the recent AMF view.

We estimate value surprises using capital market data and techniques in order to translate them into a consistent complexity score. For illustration purposes, we consider German government bonds, a DAX futures contract, a bond fund, an equity fund, a discount certificate, a bonus certification, a credit-linked note, a non-traded corporate bond, and a life insurance policy.

Our proposed procedure is characterised by a disclosed notion of complexity that is objective and can therefore dispense with all forms of subjective weightings of individual characteristics.
The following table reports the resulting complexity scores and relates it to the current classification according to MiFID II. A higher number indicates a higher level of complexity.

<table>
<thead>
<tr>
<th>Financial Product</th>
<th>Complexity Score</th>
<th>MiFID II Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-year German government bond</td>
<td>0.53%</td>
<td>non-complex</td>
</tr>
<tr>
<td>Bond fund</td>
<td>1.63%</td>
<td>non-complex</td>
</tr>
<tr>
<td>DAX future</td>
<td>1.82%</td>
<td>complex</td>
</tr>
<tr>
<td>10-year German government bond</td>
<td>2.90%</td>
<td>non-complex</td>
</tr>
<tr>
<td>Credit-linked note</td>
<td>5.58%</td>
<td>complex</td>
</tr>
<tr>
<td>Discount certificate</td>
<td>7.29%</td>
<td>complex</td>
</tr>
<tr>
<td>Equity fund</td>
<td>8.97%</td>
<td>non-complex</td>
</tr>
<tr>
<td>Bonus certificate</td>
<td>9.72%</td>
<td>complex</td>
</tr>
<tr>
<td>Exemplary life insurance policy</td>
<td>11.73%</td>
<td>non-complex</td>
</tr>
<tr>
<td>Non-traded corporate bond (BBB)</td>
<td>31.59%</td>
<td>non-complex</td>
</tr>
</tbody>
</table>

This table of selected exemplary financial instruments shows that the calculated complexity score hardly coincides with the MiFID II classification.

In particular, our analysis reveals the following findings that contradict the current practice in terms of complexity classification:

- The complexity of financial products can strongly differ even within asset classes, e.g., in this selection of government bonds. As such, a reasonable classification should focus on individual product characteristics rather than those of the entire asset class itself.
- Both a transparent underlying portfolio composition and a liquid secondary market are crucial for a low complexity score. A lack of knowledge about the portfolio composition, such as in the case of investment funds or life insurance, and the absence of a secondary market results in a striking increase in the complexity score.
- On the other hand, supposedly challenging financial instruments with derivative features (like the DAX future) that are automatically classified as complex according to MiFID II can actually have very moderate complexity scores.
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1. Introduction

1.1 Regulatory Background for Complexity of Financial Products

Private investors cannot be equally familiar with all financial products. While the functioning of a money market account is easy to understand, other instruments such as home savings plans, life insurance, retail structured products, and actively managed funds impose minor or major challenges. Consequently, the determination of a product’s value is not always straightforward.

Since MiFID I (*Markets in Financial Instruments Directive*), there have been advisory and informational requirements to protect investors against potential difficulties associated with certain financial positions and to provide them with assistance. At the beginning of 2018, these rules were tightened by MiFID II. In particular, products classified as complex are subject to stricter treatment.

When providing investment advice, investment firms are responsible for conducting a suitability assessment for the investor and renewing this annually. For this reason, all necessary investment information in terms of knowledge and experience of the (potential) clients regarding the specific product type, their financial circumstances including their ability to bear losses, and their investment goals including their risk tolerance have to be collected. Moreover, the investor needs to be provided with adequate reports on the service provided. These reports consist of periodic communications taking into account the type and complexity of the financial instruments involved as well as the costs associated with the transactions and services undertaken on behalf of the client. According to Commission Delegated Regulation (EU) 2017/565, investment firms must ensure that they understand the type and features of the selected financial instruments and that they assess, while taking into account complexity and costs, whether equivalent investment services or financial instruments can meet their client's profile. If a product that is classified as complex is distributed on an execution-only basis, a one-off assessment of appropriateness is all that is required.

In principle, an effective investor protection requires that all financial products, which investors cannot handle on their own, need to be marked accordingly and that appropriate measures are taken. On the other hand, in order to avoid overregulation and excessive bureaucratic costs, we should not per se consider every instrument as suspicious but should examine them without preconceived notions as to the outcome.

MiFID II distinguishes between complex and non-complex financial instruments. Directive 2014/65/EU Article 24(4)(a) contains a non-exhaustive list of non-complex instruments. It comprises
shares, bonds, money-market instruments excluding those that embed a derivative, shares or units in specific investment funds, structured deposits and “other non-complex financial instruments” without naming them specifically. There is also a new, non-exhaustive list of complex financial products in MiFID II. There are four categories for these indicated in Guideline ESMA/2015/1787: debt instruments embedding a derivative, debt instruments incorporating a structure making it difficult for the client to understand the risk, structured deposits incorporating a structure making it difficult for the client to understand the risk of return, and structured deposits incorporating a structure making it difficult for the client to understand the cost of exiting before term.

Indications of the potential motive of the differentiation between complex and non-complex can be found in Commission Delegated Regulation (EU) 2017/565 Article 57. This refers to special properties of non-complex financial products such as liquidity, costs, transparency, changes in risk profile and the derivative character.

When setting up a new and reasonable complexity measure, an approach that follows three successive steps would be desirable: (1) A distinct definition of the term “complexity” that accurately distinguishes what does and does not characterize complexity. Given the multitude of partly different and not always comprehensible views, this step is essential. Commission Delegated Regulation (EU) 2017/565 is rather vague in this point, only providing partial examples of financial products, and is therefore fully inappropriate with regard to a clear definition. (2) A method to determine complexity that corresponds with the definition chosen for it. (3) Consistent determination of a complexity measure that can be used to classify financial instruments.

With this ideal course of action in mind, the MiFID procedure is disappointing. Only the complexity classification in step (3) is mentioned; no reference is made to the prior steps concerning the definition and the applied method. The MiFID II complexity classification process has the character of a simple product catalogue reference. Since several products are automatically classified as complex for non-comprehensible reasons, this procedure is not very satisfying for investors. Moreover, concerns that the proposed classification is driven by an intended result rather than the real complexity cannot be addressed in a convincing way.

A further striking outcome of the complexity treatment in MiFID II is that all products are subject to binary classification: either an instrument is complex, resulting in additional administrative requirements, or it is non-complex and no further effort is required. Since the implementation of Regulation (EU) No 1286/2014 on Key Information Documents for packaged retail and insurance-based investment products (PRIIPs), investors have become used to multiple risk categories. Here,
there is not simply a classification into risky and non-risky, but seven classes that account for the broad range of financial products. A similar differentiation using multiple outcomes might also be desirable for a reasonable analysis of complexity. However, this property means that complexity should be captured by a measure with various magnitudes, rather than a binary classification.

1.2 Objective of our Study

We propose a distinct approach for the assessment of complexity and compare its outcome to the current MiFID II-compliant practice. In particular, our complexity measure should satisfy the following useful properties:

- General applicability to arbitrary financial products
- Individual, consistent and graduated result
- Clear applicability of the measure

In principle, every financial instrument can be assigned a certain complexity. Thus, we claim as a first characteristic that a reasonable complexity measure needs to be applicable to (almost) all financial products.

As a second property, an individual, consistent and graduated outcome is desirable. This condition means that the relevant product-specific features are taken into account rather than an automatic complexity classification that depends solely on its respective asset class. With regard to consistency, the approach should identify a product as being more complex than another whenever the product is indeed more challenging for an investor. In order to indicate the different complexity of financial products, a graduated outcome with an ordinal or even cardinal scale would be helpful. Finally, clear and (as far as possible) unambiguous applicability is required. These characteristics are satisfied if clear calculation steps are present without the possibility for individual discretion, such as with the subjective response to any questionnaire or weighting within a scoring model.

1.3 Appraisal of Complexity in the Literature

The literature has been dealing with the complexity of financial products for a long time. An essential trigger for this was the financial crisis in the middle of the 2000s and CDOs (Collateralized Debt Obligations), which played a crucial role thereby. These instruments are securitized tranches of an
underlying credit portfolio. The tranche itself determines the order of the potential loss that is transferred from the credit portfolio to the CDO tranche. Even at first glance, we can easily see that CDOs exhibit a substantial level of complexity due to an unobservable underlying value and a complicated loss distribution algorithm among the existing tranches.

In a seminal paper, Brunnermeier/Oehmke (2009) discuss the challenges for the determination of financial products’ complexity and refer to the difficulties of consistent treatment. For this purpose, they compare a Goldman Sachs stock to a CDO. While a CDO seems to be the more complex financial product due to an opaque underlying asset and complicated handling of portfolio losses, CDOs used to be part of the Goldman Sachs holdings at that time. Hence, CDOs are just one among various factors for the value and the properties of the Goldman Sachs stock. Consequently, one can perceive the stock rather than the CDO as the more complex product.

In line with Brunnermeier/Oehmke (2009), both Schwarcz (2009) and Omarova (2012) agree that a simple and clear definition of financial products’ complexity is not feasible. Schwarcz (2009) formulates three levels of complexity relating to the underlying asset, the product itself, and the financial market. Omarova (2012) refers to the increased pricing uncertainty due to complexity and indicates potential dangers such as uncertainty on capital markets, and a systemic risk increase.

These first papers on complexity focus on the challenges regarding a suitable definition. Nevertheless, they neither state a clear definition nor propose an actual measurement of complexity.

In a follow-up paper to Brunnermeier/Oehmke (2009), Arora/Barak/Brunnermeier/Ge (2011) address the issue with respect to the computational time when pricing financial instruments. The design of a CDO with numerous underlying borrowers and seniority tranches complicates the relation between the CDO return and the characteristics of the borrowers within the credit portfolio. According to their notion of computational complexity, a financial product is more complex if a pricing within an appropriate time span is impossible (i.e., the computational effort grows exponentially) despite an available pricing model. Although a general definition of complexity is hardly possible in accordance with Brunnermeier/Oehmke (2009), the computation time based approach by Arora/Barak/Brunnermeier/Ge (2011) addresses one out of several other complexity aspects in a very detailed manner.

Constructive approaches for a broader and more general treatment of complexity are proposed by Becker/Döhrer/Johanning (2012), the AMF, and Koh/Koh/Chuen/Lim/Ng/Phoon (2015).
Becker/Döhrer/Johanning (2012) stress the important role of transparency when dealing with complexity. In doing so, they divide transparency into the following seven subcategories: underlying asset, scenario, valuation, cost, risk, solvency, and liquidity. According to their argumentation, the complexity of financial instruments can be effectively countered by transparency in the interest of investor protection. Hence, they assess complexity by verifying the transparency of financial products.

The French Financial Markets Authority participate intensively in this discussion. In their published position on the direct marketing of complex products (AMF Position No 2010-05), they base their complexity classification on the number of related asset classes and the valuation mechanisms, among other aspects. In a press release on 23 February 2018, the AMF introduced a two-filter system. The first filter looks at the observability of the underlying asset and the second one at the valuation model to be used as well as the risk profile of the financial product. Hence, the AMF also accounts for three out of seven subcategories of transparency from Becker/Döhrer/Johanning (2012).

Koh/Koh/Chuen/Lim/Ng/Phoon (2015) propose a scoring model to determine complexity that consciously distinguishes between risk and complexity. Risk is captured quantitatively by the volatility of the return, liquidity, credit rating, duration, leverage, and level of diversification. Complexity is driven by the investors’ ability to understand the functioning and payoff structure of a financial product. For this purpose, Koh/Koh/Chuen/Lim/Ng/Phoon (2015) follow a qualitative approach based on five characteristics: the number of structural layers, expansiveness of derivatives used, availability and usage of known valuation model, number of scenarios determining return outcomes, and transparency/ease of understanding.

The advantage of the Koh/Koh/Chuen/Lim/Ng/Phoon (2015) approach is that all kinds of financial instruments are treated consistently in terms of risk and complexity using plausible criteria. Using the scoring-based process, products are classified into five different complexity ratings, which results in a purposeful and graduated outcome. The propositions of Becker/Döhrer/Johanning (2012) and Koh/Koh/Chuen/Lim/Ng/Phoon (2015) represent two initial approaches of note that, in contrast to MiFID II, come up with a statement of complexity based on a clearly explained process. However, the qualitative treatment of the characteristics, particularly of ease of understanding, complicates a clear measurement of complexity because subjective judgements are necessary.

To gain further insights about the discussion of complexity in the literature, we can examine the empirical and theoretical papers that focus on the economic consequences of complexity rather than its measurement. Despite the different perspectives of those papers, it easily becomes apparent which characteristics drive complexity.
Celerier/Vallee (2017) empirically evaluate the headline rate, i.e., the reported maximum return of a financial product, and its relation with complexity. In this case, complexity is determined by the number of product components, the number of payoff scenarios, and the length of the term sheet.

In an empirical study of mortgage backed securities, Ghent/Torous/Valkanov (2016) take into account similar properties to determine the product complexity. In particular, they make use of the number of collateral groups, the number of tranches, the number of pages of the term sheet, the number of pages of the collateral description, and the number of pages of the payoff allocation.

In an experimental study, Carlin/Kogan/Lowery (2013) understand complexity as a limited ability to correctly determine the value of an instrument. In particular, complexity of parameters and liquidity significantly affects the trading behaviour of the participants.

Carlin (2009) introduces complexity as a strategic option for banks within a competition model. In this particular case, high complexity comes from a very pronounced level of opacity. This lack of transparency is created by having a total value of the financial product that cannot be directly observed or compared, embedded hidden costs, or having different designations for identical products.

### 2. Derivation of an Appropriate Complexity Measure

#### 2.1 Economic Background for the Selected Complexity Measure

Even though the countless papers on complexity seem to have a different focus, they formulate the common concern of an accurate value estimation of a financial product. In particular, the characteristics mentioned in the Commission Delegated Regulation (EU) 2017/565 such as liquidity, costs, transparency, and changes in risk profile are also closely related to this issue. Summing up, all types of shortcomings and/or problems in terms of comprehensibility, knowledge of the underlying asset, the cost structure, the issuer solvency, the pricing model, and the existence of trading restrictions result in a more challenging estimation of the true value of a financial instrument.

**In line with this view, we define complexity as the challenge for a private investor to estimate the value of a financial product accurately. For quantification reasons, we introduce the term “value surprise,” which represents the difference between the estimated value and the real value.**

This approach ensures that every aspect addressed by the literature is also implicitly contained in our definition if it increases the value surprise. If a potential aspect of complexity has no impact on the
value surprise, it can consequently be disregarded.\footnote{An example of another approach that may result in a misleading complexity classification is a simple algorithmic approach that considers an instrument as complex when a minimum number of descriptions is exceeded. In this case, a further passage added to the term sheet of a financial product can switch the classification from non-complex to complex. If the extra description improves comprehensibility, this reclassification contradicts its purpose. In general, it is crucial whether something is helpful for an investor or not. Our notion of complexity takes the extra passage into account if and only if it effects the investor through a change in the value surprise.} Thus, we also provide a quantification for the AMF recommendation, which pursues a similar notion of complexity. Moreover, in contrast to scoring-based procedures, our approach is not dependent on any form of subjective weighting, but still captures the relevant effect of each particular aspect of complexity that drives the value surprise for an investor.

We note that our view of complexity is not necessarily a clear or straightforward version that everybody will automatically agree with. As every individual already has their own intuitive conception of complexity, we have conducted a simple experiment to shed light on the present notion. This is necessary, as a common understanding of complexity is ultimately an essential prerequisite for the development of an appropriate complexity measure for financial products.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{experiment_results}
\caption{Results of the Experiment}
\end{figure}

For the experiment, 51 participants were presented with a 45-year-old VW Golf I (made in 1974) as well as with a new VW Golf VII (made in 2018) without providing further information. At first, we
asked the participants which car they associated with a higher complexity from their personal perspective. As shown in Figure 1, about 92% of the participants assigned a higher complexity to the new car, and only 8% of the participants to the old one.

In a follow-up question, we concretise the question and asked the participants to judge the complexity of the cars from the user’s perspective. This time, slightly more than half of the participants initially opting for the new car changed their view and associated a higher complexity with the old one.

The experiment provides the following insights:

- **Individuals can have different intuitive conceptions of what complexity means.** An interpretation of complexity from the perspective of product development means something different than an interpretation from the perspective of the user. Apparently, both mutually exclusive views exist, albeit with a strong tendency towards the development perspective.

- **There are simultaneously different notions of complexity that can even change to another reasonable notion depending on the perspective taken.** Participants who initially took the development perspective associated the new car with a higher (developmental) complexity as they focused on its comparably higher technological level. Once individuals switched their viewpoint and took the perspective of the user, a majority changed their ranking of complexity. This is because the technical features of the new car can provide valuable support for the driver. This effect obviously results in a lower complexity of the new car compared to the old one.

- **Despite a clarification of the perspective of complexity to be taken, individuals can still have different attitudes regarding the classification of complexity.** Even after focusing on the user perspective, there were still two groups of similar size that had differing views of the relative complexity of the cars. If an individual is more familiar with the features of a modern car, it is more likely they will assign a lower complexity to the new car. On the other hand, individuals that are warier of modern technology may still consider the new one to be more complex.

As a consequence of these challenges with the notion of complexity, we first need to specify our chosen perspective on complexity. Our approach focuses exclusively on the user and consciously disregards all aspects concerning the development of a financial product.

In a second step, a clear method is desirable that excludes ambiguous outcomes but accounts for the view taken on complexity. For this purpose, we assume an investor who has access to all generally available information concerning their financial product, the capital market, and the established
valuation models for the determination of an accurate value estimate. Then, the size of the value surprise is relevant for the effective complexity.

Of course, not all investors can be equally well informed. In particular, investors might display differences depending on personal knowledge and financial skills that expose them to additional valuation uncertainty. However, if there is a higher value surprise for one product compared to another when a well-informed investor estimates their value, this ranking should also hold true for a less well-informed investor. Therefore, we assume a sufficiently well-informed investor, as the obtained results can be transferred to less-informed ones (e.g., private investors).

2.2 Quantitative Determination of a Complexity Measure

In order to translate the presented complexity notion into a quantitative measure, we need to differentiate two situations: firstly, one that is characterised by a capital market with available secondary market prices for relevant instruments. We note that these market prices do not necessarily have to be those of the corresponding financial products themselves, but can also refer to comparable instruments.

Secondly, a situation where there are no meaningful prices available, so a different approach is required to determine the value surprise to which an investor is exposed. In the following, we show how a reasonable value surprise can be practically obtained in both situations, and therefore used as a measure for complexity.

Once market prices for a corresponding financial product are available, the value surprise at a given point in time is derived from the relative difference between the estimated value and the realized price at this date. For the estimated value, we need an appropriate pricing model that is implemented with both observable and unobservable input parameters. The unobserved parameter values are obtained from market prices over a prior estimation period comprising TS many periods. A detailed description of the technical framework can be found in Section A.1 of Appendix A. Within the observation period, we calculate the daily value surprises, which we aggregate in form of the square root of the mean squared relative values such as it is widely accepted for the determination of the standard deviation or volatility. We denote this measure as the complexity score.

Where only market prices of comparable products are available in the first situation, such as for non-traded corporate bonds, we can compute the yield to maturity for every appropriate comparable
instrument in a first step. Since it is unclear which comparable bond fits the non-traded bond best, we use a mean value as a proxy. This proxy is the average of the hypothetical bond values when considering each yield of the observed bonds. One potential value surprise for this non-traded corporate bond is the difference between the proxy and one hypothetical bond value. As we show analytically in Section A.1 of Appendix A, our measure makes use of all feasible value surprises and determines a complexity score as the square root of the mean squared relative differences.

In contrast to the first situation, where the presence of empirical data allows for multiple computations of value surprises and therefore a rather accurate complexity score, we estimate complexity in the second situation in terms of a minimum value. For this purpose, we take a modular approach and decompose the financial instrument into individual components for which the corresponding complexity scores are already known. As described in Section A.2 of Appendix A, the complexity is the weighted average of the individual complexity scores of the known components.

3. Analysis of Complexity of Selected Financial Products

3.1 Selection of Financial Products

To illustrate the resulting complexity score of the previously introduced approach, we consider various financial instruments belonging asset classes such as bonds, retail structured products, derivatives and investment funds. Table 1 provides an overview of the selected products.

The first instruments are two German government bonds with different expiration dates: one with a two-year time to expiration and another one that matures in about ten years. Since government bonds are considered to be relatively risk-free and to function in a simple manner, these two instruments represent examples for which we expect no remarkable complexity score.

As a further instrument, we consider an exemplary life insurance policy. Even if the components are not directly observable, the high proportion of fixed income securities suggests a moderately low complexity score.

To examine a classic derivative, we include a DAX future. Due to the sophisticated pricing and handling of futures contracts in general, a very high complexity score is expected for this instrument.
Table 1: Overview of Product Selection

<table>
<thead>
<tr>
<th>2-year German government bond</th>
<th>10-year German government bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISIN</td>
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</tr>
<tr>
<td>DE0001141711</td>
<td>DE0001135085</td>
</tr>
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<tr>
<td>17 April 2020</td>
<td>4 July 2028</td>
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<tr>
<td>Coupon</td>
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<tr>
<td>0.00%</td>
<td>4.75%</td>
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<td>Issuer</td>
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<tr>
<td>Federal Republic of Germany</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>Observation period</td>
<td>Observation period</td>
</tr>
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<td>11 July 2017 – 26 January 2018</td>
<td>11 July 2017 – 26 January 2018</td>
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<tr>
<td>TS</td>
<td>TS</td>
</tr>
<tr>
<td>20 days</td>
<td>20 days</td>
</tr>
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</table>

**Exemplary life insurance policy**

<table>
<thead>
<tr>
<th>Components</th>
<th>DAX future</th>
</tr>
</thead>
<tbody>
<tr>
<td>90% fixed income</td>
<td>ISIN</td>
</tr>
<tr>
<td>10% equities</td>
<td>DE0008469594</td>
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<td>Expiration date</td>
<td>Expiration date</td>
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<td></td>
<td>16 March 2018</td>
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<tr>
<td>Observation period</td>
<td>Observation period</td>
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<tr>
<td></td>
<td>2 January 2018 – 31 January 2018</td>
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<td>TS</td>
<td>TS</td>
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<td>20 days</td>
<td>5 days</td>
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**Bond fund**

<table>
<thead>
<tr>
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<tbody>
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<td>DE0009752501</td>
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<tr>
<td>Fund</td>
<td>Fund</td>
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<tr>
<td>UBS (D) RENT EURO - EUR ACC</td>
<td>DWS Aktien Strategie Deutschland IC</td>
</tr>
<tr>
<td>Benchmark</td>
<td>Benchmark</td>
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<tr>
<td>PAN-EURO GOVERNMENT INDEX (TRR, EUR)</td>
<td>HDAX</td>
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<tr>
<td>Observation period</td>
<td>Observation period</td>
</tr>
<tr>
<td>1 July 2016 – 7 January 2018</td>
<td>1 July 2016 – 17 January 2018</td>
</tr>
<tr>
<td>TS</td>
<td>TS</td>
</tr>
<tr>
<td>20 days</td>
<td>20 days</td>
</tr>
</tbody>
</table>

**Discount certificate**

<table>
<thead>
<tr>
<th>ISIN</th>
<th>ISIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE000CE9X9U5</td>
<td>DE000CV0VCH7</td>
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<tr>
<td>Expiration date</td>
<td>Expiration date</td>
</tr>
<tr>
<td>25 May 2018</td>
<td>15 June 2018</td>
</tr>
<tr>
<td>Cap</td>
<td>Bonus level</td>
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<tr>
<td>13550</td>
<td>13950</td>
</tr>
<tr>
<td>Issuer</td>
<td>Barrier</td>
</tr>
<tr>
<td>Commerzbank</td>
<td>11850</td>
</tr>
<tr>
<td>Underlying asset</td>
<td>Underlying asset</td>
</tr>
<tr>
<td>DAX</td>
<td>DAX</td>
</tr>
<tr>
<td>Observation period</td>
<td>Observation period</td>
</tr>
<tr>
<td>1 August 2017 – 26 March 2018</td>
<td>1 August 2017 – 2 March 2018</td>
</tr>
<tr>
<td>TS</td>
<td>TS</td>
</tr>
<tr>
<td>20 days</td>
<td>20 days</td>
</tr>
</tbody>
</table>

2 On 5 March 2018, the underlying asset hit the knock-out barrier and the bonus component ceased to exist.
In addition, we have two retail structured products: a discount certificate and a bonus certificate. Discount certificates are well-established instruments, which can be represented by an underlying asset and a call option short. Since the embedded option, which is driven by the implied volatility, imposes some pricing challenges, we expect a high complexity score. This is also the case for the bonus certificate, which can be replicated using a knock-out put option. It is well known that the pricing of a knock-out option can be especially demanding for underlying values close to its barrier, as the entire value is lost if the barrier is hit. For this reason, we expect an even higher complexity score for the bonus certificate than for the discount certificate. The bonus certificate was chosen particularly so that its underlying asset ranged close to the knock-out barrier during the observation period until it was knocked out on 5 March 2018.

Finally, we consider two financial instruments that are exposed to credit risk. This source of risk is also crucial for CDOs. The experiences with these products during the financial crisis triggered the debate on complexity in the first place. For this purpose, we chose a credit-linked note, whose issuer is obliged to service promised payments unless an underlying reference entity defaults. This product category was temporarily subject to an intensive product intervention debate initiated by the regulator in Germany. The arguments against credit-linked notes related to both the comprehensibility and the treatment of credit risk. In light of this background, we expect a very high complexity score for this instrument. In addition, we have included a non-traded corporate bond. In contrast to the credit-linked note, a plain vanilla bond does not have an additional issuer in the middle, which should obviously result in a simpler payoff structure. Therefore, we expect a slightly lower complexity score than for the credit-linked note.
### 3.2 Insights about Complexity

Table 2 contains the complexity score resulting from our approach for the products presented in Section 3.1. The applied valuation formulae can be found in Appendix B.

The two-year German government bond has a complexity score of 0.53%, making it the least complex product within our product selection by far. In contrast, its ten-year counterpart revealed a comparably high complexity with a score of 2.90%. This initially surprising result results from a higher duration of the long-term bond, which amplifies differences between the estimated yield curve and its realisation. Thus, product structure can be seen as a major source of the value surprises that determine the complexity of a financial instrument in our approach.

Looking at the life insurance policy, described in Section B.7 of Appendix B, and the DAX future we observe results that defy our previous expectations. With a score of 1.82%, the DAX future appears to be even less complex than the ten-year German government bond, and the life insurance policy has a notably high complexity score of 11.73%. Obviously, the value of the DAX future can be estimated quite reliably using standard models with observable market information. The resulting moderate value surprise emphasises that derivative components within financial products do not per se increase the complexity of these products.

**In consequence, it is necessary to shift the focus from product categories towards individual product characteristics within an asset class. In particular, a longer time to expiration of a bond increases the value surprise and thus its complexity score.**

Comparing the complexity score of the equity fund to the bond fund, we can see a clear difference. The 1.63% complexity score of the bond fund is the second lowest of all products examined. The equity fund, on the other hand, is considerably more complex with a score of 8.97%. Given that the duration of 2.5 years of the bond fund is comparable to the two-year German government bond, the threefold difference in complexity scores seems quite large. The crucial difference between the two fixed income products lies in the lack of information regarding the portfolio composition of the fund, which obviously increases the complexity score.

With a score of 7.29%, the discount certificate is less complex than the equity fund. This result supports the view of a well-functioning secondary market for retail structured products, which allows a fairly accurate value estimation that again translates into a lower complexity score. This argument is also reflected in the very complicated (in terms of valuation) bonus certificate. With a score of 9.72%, it appears to be only moderately more complex than the discount certificate.
Table 2: Resulting Complexity Score

<table>
<thead>
<tr>
<th>Financial Product</th>
<th>Complexity Score K</th>
<th>MiFID II Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-year German government bond</td>
<td>0.53%</td>
<td>non-complex</td>
</tr>
<tr>
<td>10-year German government bond</td>
<td>2.90%</td>
<td>non-complex</td>
</tr>
<tr>
<td>Exemplary life insurance policy</td>
<td>11.73%</td>
<td>non-complex</td>
</tr>
<tr>
<td>DAX future</td>
<td>1.82%</td>
<td>complex</td>
</tr>
<tr>
<td>Bond fund</td>
<td>1.63%</td>
<td>non-complex</td>
</tr>
<tr>
<td>Equity fund</td>
<td>8.97%</td>
<td>non-complex</td>
</tr>
<tr>
<td>Discount certificate</td>
<td>7.29%</td>
<td>complex</td>
</tr>
<tr>
<td>Bonus certificate</td>
<td>9.72%</td>
<td>complex</td>
</tr>
<tr>
<td>Credit-linked note</td>
<td>5.58%</td>
<td>complex</td>
</tr>
<tr>
<td>Non-traded corporate bond (BBB)</td>
<td>31.59%</td>
<td>non-complex</td>
</tr>
</tbody>
</table>

The importance of a well-functioning and liquid secondary market is additionally stressed by those products exposed to credit risk, e.g., the credit-linked note and the non-traded corporate bond. While the credit-linked note’s complexity score of 5.58% ranks below the complexity of the retail structured products examined, the non-traded corporate bond’s complexity score of 31.59% makes it the most complex product in our selection by far. This result is not in line with our prior expectations. For an accurate valuation of a corporate bond, an investor needs information regarding the specific credit risk. Due to the absence of a secondary market, the value surprise is particularly amplified by the rough value estimate. Despite its straightforward structure compared to the credit-linked note, these two factors make the non-traded corporate bond to the most complex instrument in our product selection.

Thus, a clear and transparent portfolio composition as well as a liquid secondary market are fundamental for a low complexity score. A lack of transparency with respect to the portfolio composition, as with the two investment funds, and a lack of a secondary market enormously increase the complexity of financial products. On the other hand, a well-functioning secondary market can limit the complexity of financial products that appear challenging at first glance.

The lack of information on portfolio composition as well as an illiquid or even non-existent secondary market increase the complexity of the life insurance policy in particular. In this case, the additional consideration of guarantees or performance-related participation in surplus are expected to amplify the complexity score even more due to their option-like nature. This is in line with the complexity score.
of the retail structured products with embedded options exhibiting a higher complexity than the DAX future.

4. Results and Discussion

In this study, we demonstrate that there has been neither a clear and generally accepted notion for the complexity of financial products nor a practical approach to measuring it thus far. Starting from scratch, underestimating the need for a precise definition of complexity is an initial problem when seeking an accurate complexity score. This is as most people feel sufficiently convinced by their own definition of complexity and that further specification is not required. With a simple experiment, we show that people can simultaneously have different notions of complexity that can indeed result in a different assessment of the same situation.

We propose a precise definition of complexity by taking the investor’s perspective. In brief, we allocate a higher complexity score to a financial product where value surprises are more pronounced. In our approach, we implicitly account for all aspects that make it more complicated for the investor to estimate the value of a financial instrument precisely. This clear definition allows us to apply empirical methods for translating the value surprise into a complexity score. Our approach provides the following findings for the selected products, which diverge from the MiFID II classification of complexity:

- The complexity of a financial instrument should not be assessed based its product category alone. The specific characteristics within an asset class can lead to strong variations of the complexity score. On the one hand, supposedly non-complex products, (e.g., life insurance policies and investment funds), can turn out to have an unexpectedly high complexity score. On the other hand, products that are classified as complex in accordance with MiFID II can have a quite low complexity score.
- A lack of transparency concerning portfolio composition, such as is the case with investment funds or life insurance policies, is a major reason for a higher complexity score.
- Another reason for an increased complexity score is the absence of a secondary market.

Considering all our findings, it is obvious that the MiFID II classification via product categories is not an appropriate method because it does not capture all relevant aspects of complexity. Therefore, we strongly recommend the implementation of a quantitative and economic approach such as proposed in this study.
A generally accepted complexity score can provide all market participants, including investors, issuers and regulators, with answers to the following questions:

- What makes a financial product complex from an investor’s perspective?
- How can issuers positively influence the complexity of a financial product?
- Which regulatory interventions lead to a reduction of financial products’ complexity and which only increase the regulatory burden?

From an **investor**’s perspective, a complexity score can be used to measure the effective complexity of a financial product. Moreover, with a regular backtesting routine it is possible to review the complexity score and to reclassify the product itself if necessary. An approach of this kind additionally provides incentives **for issuers** to reduce the complexity of financial products and can be used by the **regulator** to test whether its measures for complexity reduction are effective or not. If the additional regulatory burden associated with the assessment of suitability and appropriateness according to MiFID II is truly useful for reducing complexity, the complexity score of the regarding financial product should reflect this in the near future.
A Technical Framework: Determination of Complexity

A.1 Complexity Score with Secondary Market Data

The first step for the determination of a financial products’ effective complexity is the decision upon the pricing model $F(\Omega, \Theta)$ for the product at hand. For this purpose, we need to choose an appropriate formula. The function $F(\Omega, \Theta)$ depends on the observable parameters $\Omega$ as well as unobservable parameters $\Theta$, where $\Omega$ and $\Theta$ both denote vectors that can individually be of arbitrary, non-negative and integer dimension.

In a next step, we focus on the market data for a suitable historical period where we have to distinguish between two cases, A and B, depending on the availability of prices: in the ideal case (case A) there is an observation period $BZ$ with a time series of $TB$ many periodic market prices $P_t$ of the considered product. The set $BZ = \{t| t=1 (1) TB\}$ contains all instants of time within this time period.

In case B, there is no secondary market for the considered product, but market prices for a list of sufficiently comparable products. Out of this list, we use the prices of the cross-section with the largest possible extent. At the valuation date, there are $N$ many comparable instruments $i = 1 (1) N$, whose prices are denoted by $V_i$.

Figure 2 illustrates the procedure for case A. At time $t$, an investor attaches the value $F(\Omega_t, \Theta)$ to the financial product that depends on the observable parameters $\Omega_t$ at time $t$ as well as the best guess for the unobservable parameters $\Theta$. The best guess for the unobservable parameters $\Theta^*_t$ is derived throughout an estimation period $SZ_t$ with $TS$ many instants of time directly before date $t$. Hereby, it is necessary that the valuation date $t$ is chosen such that the observation period $BZ$ contains the estimation period, i.e., $SZ_t = \{t| t=-TS+t (1) t-1\} \subseteq BZ$.

![Figure 2: Time Structure in Case A](image-url)
The best guess $\Theta^*_t$ at time $t$ is obtained by minimizing the sum of squared value surprises within the estimation period $SZ_t$:

$$\Theta^*_t = \arg \min_{\Theta} \sum_{s=-TS+t}^{t-1} (P_s - F(\Omega_s, \Theta))^2$$

At time $t$, $F(\Omega_t, \Theta^*_t)$ represents the investor’s best possible value estimate, since it results from the valuation formula with the best possible parameters, i.e., the observed market prices $\Omega_t$ and the estimate $\Theta^*_t$ for the unobservable parameters $\Theta$. The value surprise (in percentage) for an investor follows from the difference between the calculated value $F(\Omega_t, \Theta^*_t)$ and the realized price $P_t$ relative to the price:

$$\frac{P_t - F(\Omega_t, \Theta^*_t)}{P_t}$$

Intuitively, a lower ability or possibility to understand the product causes a higher value surprise.

Similar to the standard practice for the computation of the standard deviation, we calculate the complexity score by taking the square root of the average squared relative price deviations between the value estimates and the realized market prices. For sake of comparability, we annualise the complexity score by multiplying the daily score with the square root of the annual trading days, i.e., $\sqrt{250}$. In case A, the complexity score $K$ is given by:

$$K = \sqrt{\frac{1}{TB - TS} \sum_{t=TS+1}^{TB} \left( \frac{P_t - F(\Omega_t, \Theta^*_t)}{P_t} \right)^2 \cdot 250}$$

As illustrated in Figure 2, our complexity score follows an in-the-sample versus out-of-the-sample view. The relative value surprise $\frac{P_t - F(\Omega_t, \Theta^*_t)}{P_t}$ at $t$, is calculated using the unobservable parameters $\Theta^*_t$, which are estimated from the prior in-the-sample estimation period $SZ_t = \{ t | t = -TS+t (1) t-1 \}$. The value surprise can be understood as an out-of-the-sample score, since both the unobservable estimated parameter $\Theta^*_t$ and the observable market prices $\Omega_t$ are known at $t$. Thus, from the observation period $BZ$ we can calculate out-of-the-sample complexity scores at the dates $t = TS + 1 (1) TB$ as we need $TS$ instants of time for the in-the-sample estimation of the unobservable parameters to obtain one value estimation $F(\Omega_t, \Theta^*_t)$. 
Case B follows from the idea of deriving the value surprises of a product using a list of comparable instruments. At time \( t \), we define a valuation model of the investor as a function that depends only on the (possibly aggregated) unobservable parameter \( \theta \), for example, the yield of a non-traded corporate bond. In this case, the valuation model reads \( F(\Omega_t, \theta) \) at time \( t \). For each product \( i \) out of the list of comparable instruments, the yield \( \theta_i \) is extracted from the corresponding prices \( V_i \). In a next step, these yields are used to calculate hypothetical bond values \( F(\Omega_t, \theta_i) \) that correspond to the non-traded corporate bond. All implicit parameters \( \theta_i \) are contained in the set \( \Pi \), which is given by:

\[
\Pi = \{ \theta_i | F(\Omega_t, \theta_i) = V_i, \forall i = 1(1)N \}
\]

Due to the restricted availability of data, we use these unobservable parameters to calculate a proxy by taking the average across all hypothetical values. This mean value can be obtained at \( t \) using:

\[
\bar{F} = \frac{1}{N} \sum_{i=1}^{N} F(\Omega_t, \theta_i)
\]

In a next step we calculate the relative deviation of the hypothetical value \( F(\Omega_t, \theta_i) \) from the best guess \( \bar{F} \) for each component \( i \) of the list of comparable instruments:

\[
\frac{F(\Omega_t, \theta_i) - \bar{F}}{F(\Omega_t, \theta_i)}
\]

The complexity score \( K \) is given by the annualised standard deviation of these relative deviations across the list of \( N \) comparable instruments:

\[
K = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left( \frac{F(\Omega_t, \theta_i) - \bar{F}}{F(\Omega_t, \theta_i)} \right)^2} \cdot 250
\]

In both cases, A and B, the primary objective of the introduced complexity score is to obtain a measure for the appropriate ranking of financial products according to their complexity. In accordance with our view on complexity, a higher score \( K \) indicates a high complexity of the product as it results from the fact that it is harder to correctly assess the value of a financial product.
A.2 Complexity Score in the Absence of Secondary Market Data

In a situation in which we cannot resort to secondary market data when determining the complexity of a financial product, we analyse the product following a modular approach. In particular, we disassemble the financial product into its parts for which the complexity score can be determined individually. The value of a financial product $FP$ consists of $M$ parts, where each part $i$ adds a value of $\pi_i$. The total value of each part $i$ corresponds to a fraction $x_i$ of the total value $FP$:

$$x_i \cdot FP = \pi_i$$

The portfolio that follows from these fractions $x_i$ allows us to formulate a duplicate portfolio of the financial product at hand. It becomes apparent that this modular approach is impacted by many sources of additional noise, since, depending on the product structure, the portfolio fractions can be observed or estimated to greater or lesser extent. We account for this by stating a minimum complexity in this case. The actual complexity might be even substantially higher. Still, the minimum complexity can be regarded as a rough estimate and excludes all possible scores below that threshold.

We define the minimum complexity $MK$ in situations without a secondary market as the sum of the $M$ individual parts’ complexity scores $K_i$ weighted by the value fractions $x_i$:

$$MK = \sum_{i=1}^{M} x_i \cdot K_i$$
B Technical Appendix: Overview of Applied Valuation Approaches

B.1 Valuation of German Government Bonds

To value German government bonds, we apply the Svensson approach. The interest rate at time $t$ for an arbitrary maturity time $s+t$ can be derived according to the following relation:

$$ y_s(t_0,t_1,t_2,t_3,t_1,t_2,t) = \beta_{0,t} + \beta_{1,t} \cdot \left( \frac{1-\exp\left(-\frac{s}{\tau_{1,t}}\right)}{\tau_{1,t}} \right) $$

$$ + \beta_{2,t} \cdot \left( \frac{1-\exp\left(-\frac{s}{\tau_{2,t}}\right)}{\tau_{2,t}} - \exp\left(-\frac{s}{\tau_{2,t}}\right) \right) + \beta_{3,t} \cdot \left( \frac{1-\exp\left(-\frac{s}{\tau_{2,t}}\right)}{\tau_{2,t}} - \exp\left(-\frac{s}{\tau_{2,t}}\right) \right) $$

for given observable parameters $\Omega_t = (\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \beta_{3,t}, \tau_{1,t}, \tau_{2,t})$. As an unobservable quantity $\Theta_t$, a deviation $\epsilon_t$ from the bond price is allowed. Therefore, the valuation formula for a bond at time $t$ with $n$ payments of $CF_{s_i}$ at times $t+s_i$, respectively, is given by:

$$ F(\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \beta_{3,t}, \tau_{1,t}, \tau_{2,t}, \epsilon_t) = \sum_{i=1}^{n} \frac{CF_{s_i}}{(1+y_{s_i})^{s_i}} + \epsilon_t $$

B.2 Valuation of Investment Funds

The formula for the value of an investment fund is captured by a linear combination of a constant and a benchmark. The benchmark $B_t$ is available at any point in time, as the observable quantity $\Omega_t = B_t$. Unobservable quantities $\Theta_t = (a_t, b_t)$ are the coefficients of the linear combinations. As a result, the valuation formula of an investment fund is given by:

$$ F(B_t, a_t, b_t) = a_t + b_t \cdot B_t $$

B.3 Valuation of DAX Futures

To value a DAX future, we apply the typical cost-of-carry formula, where we estimate the underlying interest rate implicitly. The value of the DAX is the observable parameter $\Omega_t = Dax_t$ and the implied
The interest rate of the DAX future is the unobservable parameter $\Theta_t = r_t$. The valuation formula of the DAX future with a remaining maturity of $T$ years is:

$$F(Dax_t, r_t) = Dax_t \cdot (1 + r_t \cdot T)$$

### B.4 Valuation of Retail Structured Products

For the valuation of discount certificates, we apply the typical Black-Scholes formula extended by the default risk of the issuer. By including the risk-neutral hazard rate $\lambda_t$, the survival probability with respect to a maturity of $T$ years (from $t$ to $t+T$) amounting to $e^{-\lambda_t T}$ can be calculated. Empirically, we extract $\lambda_t$ from the CDS spread of the issuer at the respective point in time by dividing the spread by an assumed Loss Given Default of 0.6. The certificate is assumed to expire worthless if the issuer defaults. The required rate of interest $r_t$ is obtained from the yield curve of German government bonds. Moreover, the price of the underlying asset $S_t$ of the retail structured product also belongs to the group of observable quantities $\Omega_t$. The implied volatility $\sigma_t$ of the return of the underlying asset belongs to the unobservable quantities $\Theta_t$. The resulting valuation formula of a discount certificate is therefore given as:

$$F(S_t, r_t, \lambda_t, \sigma_t) = b \cdot e^{-\lambda_t T} \cdot \left[ S_t - (S_t \cdot N(d_2) - X \cdot e^{-r_t T} \cdot N(d_2)) \right]$$

$$d_{1,2} = \frac{\log \left( \frac{S_t}{X} \right) + \left( r_t \pm \frac{1}{2} \cdot \sigma^2 \right) \cdot T}{\sigma_t \cdot \sqrt{T}}$$

Here, $b$ represents the subscription ratio of the certificate and $X$ the determined cap.

Accordingly, the valuation formula for a bonus certificate can be written as:

$$F(S_t, r_t, \lambda_t, \sigma_t) = b \cdot e^{-\lambda_t T}$$

$$\cdot \left( S_t + S_t \cdot (N(d_1) - N(d_3)) - X \cdot e^{-r_t T} \cdot (N(d_2) - N(d_4)) - S_t \right)$$

$$\cdot \left( (N(d_5) - N(d_7)) \cdot \left( \frac{H}{S_t} \right)^{2r \sigma^2 - 1} + X \cdot e^{-r_t T} \cdot (N(d_6) - N(d_8)) \right)$$
\[ d_{3,4} = \frac{\log\left(\frac{S_t}{H}\right) + \left(\tau_t \pm \frac{1}{2} \cdot \sigma^2\right) \cdot T}{\sigma_t \cdot \sqrt{T}} \]

\[ d_{5,6} = \frac{\log\left(\frac{H}{S_t}\right) + \left(\tau_t \pm \frac{1}{2} \cdot \sigma^2\right) \cdot T}{\sigma_t \cdot \sqrt{T}} \]

\[ d_{7,8} = \frac{\log\left(\frac{H^2}{S_t \cdot X}\right) + \left(\tau_t \pm \frac{1}{2} \cdot \sigma^2\right) \cdot T}{\sigma_t \cdot \sqrt{T}} \]

Where \(X\) represents the bonus level and \(H\) the security level. The variables \(d_1\) and \(d_2\) are calculated using the corresponding equation above above.

### B.5 Valuation of Credit-Linked Notes

For the valuation of credit-linked notes, we rely on the yield curve that is given by the Svensson parameters \(\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \beta_{3,t}, \tau_{1,t}\) and \(\tau_{2,t}\) observable at time \(t\) as we did for the valuation of German government bonds. Since the issuer as well as the underlying reference entity might default, we also employ the CDS spread of the issuer and the reference entity and transfer them to their respective hazard rates \(\lambda_{E,t}\) and \(\lambda_{A,t}\) in order to determine the respective probabilities of default. For valuation purposes, we need the probability \(pE(i)\) that the issuer defaults in the \(i\)-th of \(n\) coupon periods. Thus, for \(i > 1\) the issuer as well as the reference entity survive the first \(i-1\) coupon periods and the insolvency of the issuer occurs before the \(i\)-th date. In case \(i=1\), the probability refers to a default before the first coupon date. In addition, the probability \(pA(i)\) is needed for the reference entity which implies for \(i > 1\) that the issuer as well as the reference entity survive the first \(i-1\) coupon dates and only the reference entity but not the issuer defaults before the \(i\)-th coupon date. For \(i=1\), the probability of default refers to a default of the reference entity but not of the issuer before the first coupon date.

The annualized conditional probabilities \(p_{0,0}, p_{1,0}, p_{0,1}, p_{1,1}\) can be obtained from the hazard rates by means of the default correlation \(\rho\). Here \(p_{0,0}\) refers to the probability that neither of them defaults, \(p_{1,0}\) to the probability that solely the reference entity but not the issuer defaults, \(p_{0,1}\) to the probability that solely the issuer but not the reference entity defaults, and \(p_{1,1}\) to the probability that both default in the respective year:
The required probabilities $pA(i)$, and $pE(i)$, then emerge as:

$$pA(i)_t = \begin{cases} 
  p_{i,0} \cdot s_1 & , if \ i = 1 \\
  (1 - (1 - p_{0,0}) \cdot s_1) \cdot (1 - (1 - p_{0,0}))^{i-2} \cdot p_{1,0} & , else 
\end{cases}$$

$$pE(i)_t = \begin{cases} 
  (p_{0,1} + p_{1,1}) \cdot s_1 & , if \ i = 1 \\
  (1 - (1 - p_{0,0}) \cdot s_1) \cdot (1 - (1 - p_{0,0}))^{i-2} \cdot (p_{0,1} + p_{1,1}) & , else 
\end{cases}$$

The credit-linked note expires if the reference entity defaults with repayment of a residual value that has to be determined. This is the reason why we distinguish between an insolvency of the issuer and an insolvency of the reference entity while the issuer is still solvent. If only the reference entity defaults, the residual value which is paid out is calculated as the nominal value reduced by a Loss Given Default of $100-LGD$. Contrary, if the issuer becomes insolvent, we assume a total cessation of all payments.
In this case, the group of unobservable quantities consists of the amount of the Loss Given Default \( LGD \) as well as the default correlation \( \rho \). All other parameters can be obtained from observable market prices. The valuation formula results in:

\[
F(\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \beta_{3,t}, \tau_{1,t}, \tau_{2,t}, \lambda_{E,t}, \lambda_{A,t}, LGD, \rho) = \\
\sum_{i=1}^{n} p_A(i)_t \cdot \left( \frac{100 - LGD}{1 + \tau_{s_i}} \right) + \sum_{j=1}^{i} \frac{CF_j}{(1 + \tau_{s_j})^{s_j}} + \sum_{i=1}^{n} p_E(i)_t \cdot \left( \frac{\sum_{j=1}^{i} CF_j}{(1 + \tau_{s_j})^{s_j}} \right) \\
+ \left( 1 - \sum_{i=1}^{n} p_A(i)_t - \sum_{i=1}^{n} p_E(i)_t \right) \cdot \sum_{i=1}^{n} \frac{CF_i}{(1 + \tau_{s_i})^{s_i}}
\]

**B.6 Valuation of Non-Traded Corporate Bonds**

The corporate bond is valued by discounting the payments \( CF_{s_i} \) by the yield to maturity \( \theta \) in the \( n \)-many coupon dates \( s_i \) for \( i = 1 \) \( (1) \) \( n \):

\[
F(\theta) = \sum_{i=1}^{n} \frac{CF_i}{(1 + \theta)^{s_i}}
\]

**B.7 Valuation of an Exemplary Life Insurance Policy**

Life insurance policies are financial products with numerous facets. A description of their main characteristics can be found in Graf/Kling/Ruß (2011). Essentially, a life insurance policy is a long-term investment combined with insurance coverage. The regularly premium paid is therefore composed of the savings contribution, the risk contribution, and the cost contribution. While the risk contribution serves as compensation for the insurance coverage received and the cost contribution serves as a compensation for expenses of the insurance provider (e.g., acquisition and administrative activities), the savings contribution is intended for capital accumulation. The regular savings contributions can be invested in equities, bonds, real estate, and various other illiquid asset classes. However, to avoid the risk of insolvency, the insurance provider needs to ensure that the accounted cover pool on the asset side of the balance sheet equals the aggregated policy reserves on the liability side, which stem from the aggregated savings contributions compounded by the guaranteed minimum interest rate. The selected investment policy therefore needs to be verified by a trustee. According to
the German Insurance Supervision Act (Versicherungsaufsichtsgesetz, VAG), the buyer of a life insurance policy is entitled to performance-related participation in capital gains actually attained that exceed the guaranteed minimum interest rate. Generally, the policyholder participates in 90% of surpluses achieved. The cover pool assets are typically invested in equities, bonds, private debt and additional products that are not traded on capital markets.

In order to derive the complexity score of an exemplary life insurance policy, we assume a typical composition of the cover pool assets: 90% invested in fixed income products and 10% in equities. The fixed income portfolio is divided equally into long-term German government bonds (30%), a short-term bond fund (30%) as well as non-traded corporate bonds, i.e., private debt (30%). The equity position of 10% is invested in an equity fund. We do not consider additional aspects like guarantees, profit participation or transparency issues regarding the portfolio composition of the equity and the bond funds.

Based on the complexity scores in Table 2, a minimum complexity $M_K$ is calculated as a weighted average of the complexity scores of the included positions. In this illustrative example, the complexity score amounts to:

$$M_K = 30\% \cdot 2.90\% + 30\% \cdot 1.63\% + 30\% \cdot 31.59\% + 10\% \cdot 8.97\% = 11.73\%.$$
References


