6th set assignments Introductory Econometrics

**Task 1**

Consider the following assumptions:

1. linearity
2. rank condition: $K \times K$ matrix $E(x_i x'_i) = \Sigma_{xx}$ is nonsingular
3. predetermined regressors: $E(g_i) = 0$ where $g_i = x_i \cdot \varepsilon_i$
4. $g_i$ is a martingale difference sequence with finite second moments

i) Show, that under those assumptions, the OLS estimator is distributed asymptotically normal:

$$\sqrt{n}(b - \beta) \rightarrow_d N(0, \Sigma_{xx}^{-1}E(\varepsilon_i^2 x_i x'_i)\Sigma_{xx}^{-1})$$

ii) Further, show that assumption 4 implies that the $\varepsilon_i$ are serially uncorrelated or $E(\varepsilon_i \varepsilon_{i-j}) = 0$.

**Task 2**

Show, that the test statistic

$$t_k \equiv \frac{\sqrt{n}(b_k - \beta_k)}{\sqrt{Avar(b_k)}} \rightarrow_d N(0, 1)$$

converges in distribution to a standard normal distribution. Note, that $b_k$ is the k-th element of $b$ and $Avar(b_k)$ is the (k,k) element of the $K \times K$ matrix $Avar(b)$. Use the facts, that $\sqrt{n}(b_k - \beta_k) \rightarrow_d N(0, Avar(b_k))$ and $Avar(b) \rightarrow_p Avar(b)$. Use Lemma 2.4(c) for argumentation.

**Task 3**

Show, that the test statistic

$$W \equiv (R \sqrt{n}b - r)'[R \hat{Avar}(b)R']^{-1}(R \sqrt{n}b - r) \rightarrow \chi^2(#r)$$

converges in distribution to a Chi-square with #r degrees of freedom. As a hint, rewrite the equation above as $W \equiv c'_n Q_n^{-1} c_n$. Use Lemma 2.4(d) and the footnote on page 41 for argumentation.