Task 1

Show that these functions are Kernels:

\[ K(\psi) = (2\pi)^{-1/2} \exp(-0.5\psi^2) \]  
\[ K(\psi) = 1\{|\psi| \leq 1\}3/4 \cdot \{1 - \psi^2\} \]  
\[ K(\psi) = 1\{|\psi| \leq 1/2\} \]

Task 2

Show that the relation

\[ \int_{-\infty}^{+\infty} \psi^k K(\psi) d\psi = 0 \]  
holds, for \( k = 1, 3, 5, \ldots \)

Task 3

Proof the following Corollary:

**Corollary 1** Let assumptions DE I-IV hold. Then

\[ \hat{f}_x(x_0) \overset{p}{\rightarrow} f_x(x_0) \]  
\[ \text{plim} \hat{f}_x(x_0) \rightarrow f_x(x_0) \]
Task 4

Show that the following theorem holds:

**Theorem 1** Under ADE I’, ADE II, ADE III’ and ADE IV we obtain

\[
E[\hat{f}_x(x_0)] = f_x(x_0) + \frac{h^r}{r} \mu_r \partial_x f_x(x_0) + o(h^r) \quad (7)
\]

\[
\text{Var}[\hat{f}_x(x_0)] = (nh)^{-1} \kappa_0 f_x(x_0) + o((nh)^{-1}) \quad (8)
\]