Returns to Scale in Electricity Supply

Practical Example: Nerlove (1963)

17th January 2007
1 What’s it all about?

2 Task 1: Create Variables to Estimate the Cost Function.

3 Task 2: Estimate the cost equation and interpret.

4 Task 3: Are the Assumptions of the CLRM satisfied?

5 Task 4: Which restriction is implied by $\alpha_1 + \alpha_2 + \alpha_3 \equiv r$?

6 Task 5: Testing the homogeneity of the cost function.

7 Task 6: Reformulate and discuss.

8 Task 7: Testing constant returns to scale.

9 Task 8: Make a residual plot and interpret.
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Nerlove’s paper

- classic study of returns to scale in a regulated industry (1963)

Features of the electricity industry

- monopolies supply power
- electricity prices are set by the utility commission
- factor prices are given
- Cobb-Douglas production and cost function
- cost minimization of firms
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What's it all about?

The data

- cross-section data set of 145 firms in 44 states in the year 1955
- data on total costs, factor prices, output

Econometric approach

- OLS estimation of parameterized cost function
- assuming returns to scale to be constant (\(=r\))

Subjects of interest

- characteristics of the cost function (e.g. elasticities)
- analysis of returns to scale
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Task 1: Create Variables to Estimate the Cost Function.

Create the necessary variables to estimate the cost equation.

Linearized Cobb Douglas Cost Function

\[ \log(TC_i) = \beta_1 + \beta_2 \log(Q_i) + \beta_3 \log(p_{i1}) + \beta_4 \log(p_{i2}) + \beta_5 \log(p_{i3}) + \varepsilon_i \]

- \( TC_i \): total costs for firm \( i \)
- \( Q_i \): firm \( i \)'s output
- \( p_{i1} \): factor price for labor
- \( p_{i2} \): factor price for capital
- \( p_{i3} \): factor price for fuel
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- \(TC_i\): total costs for firm \(i\) \(\rightarrow \log(TC)\)
- \(Q_i\): firm \(i\)’s output \(\rightarrow \log(Q)\)
- \(p_{i1}\): factor price for labor \(\rightarrow \log(PL)\)
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Estimate the cost equation with OLS and interpret the results.

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Estimation in EViews

- Quick \(\rightarrow\) Estimate Equation (UMOD)
- Equation specification:
  \[ \log(TC) c \log(Q) \log(PL) \log(PK) \log(PF) \]
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Questions that should guide your interpretation

- Are the parameters significantly different from zero?
- What is the interpretation of the parameters?

Interpretation

- Coefficients in log-linear equations are elasticities
- \(\beta_3\): elasticity of total costs with respect to the wage rate, i.e. the percentage change in total costs when the wage rate changes by 1 percent.
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Task 3: Are the Assumptions of the CLRM satisfied?

Assumptions

1. **Linearity**
   
   \[ y_i = \log(TC_i), \quad x_i = (1, \log(Q_i), \log(p_{i1}), \log(p_{i2}), \log(p_{i3}))' \]

2. **Strict exogeneity**
   
   Regressors are independent of the firm’s efficiency \( \varepsilon_i \) because of some special features of the U.S. electricity industry in 1963.

3. **No multicollinearity**
   
   In Nerlove’s data set, \( \text{rank}(X) = 5 \) and \( n = 145 \).

4. **Spherical error variance**
   
   No correlation in the error term, but potential heteroskedasticity.
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8. Task 7: Testing constant returns to scale.
9. Task 8: Make a residual plot and interpret.
Task 4: Which restriction is implied by the notion that the degree of returns to scale is constant \((\alpha_1 + \alpha_2 + \alpha_3 \equiv r)\)?

Returns to Scale are constant

Remember that \(\beta_3 = \frac{\alpha_1}{r}, \beta_4 = \frac{\alpha_2}{r}, \beta_5 = \frac{\alpha_3}{r}\). Then it follows that

\[
\alpha_1 + \alpha_2 + \alpha_3 \equiv r \quad | \quad \text{substitute the } \alpha \\
\beta_3 + \beta_4 + \beta_5 \equiv r \quad | \quad : r \\
\beta_3 + \beta_4 + \beta_5 \equiv 1
\]

and thus the OLS equation is restricted by \(\beta_3 + \beta_4 + \beta_5 = 1\).

- OLS equation is overidentified
- Cost function is linearly homogeneous in factor prices
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\]

\[
r\beta_3 + r\beta_4 + r\beta_5 = r \quad | : r
\]

\[
\beta_3 + \beta_4 + \beta_5 = 1
\]

and thus the OLS equation is restricted by \(\beta_3 + \beta_4 + \beta_5 = 1\).

- OLS equation is **overidentified**
- cost function is **linearly homogeneous** in factor prices
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Test with **two alternative procedures** if the above restriction \((\beta_3 + \beta_4 + \beta_5 = 1)\) can be rejected.

Wald Coefficient Test (**alternative 1**)

- Estimate the unrestricted model (UMOD):
  \[
  \log(TC_i) = \beta_1 + \beta_2 \log(Q_i) + \beta_3 \log(p_{i1}) + \beta_4 \log(p_{i2}) + \beta_5 \log(p_{i3}) + \epsilon_i
  \]
- View \(\rightarrow\) Coefficient Test \(\rightarrow\) Wald-Coefficient Restrictions

F-ratio \[
\frac{(SSR_R - SSR_U)/\#r}{SSR_U/(n-K)}\] (**alternative 2**)

- Estimate the restricted model (RMOD):
  \[
  \log \left( \frac{TC_i}{p_{i3}} \right) = \beta_1 + \beta_2 \log(Q_i) + \beta_3 \log \left( \frac{p_{i1}}{p_{i3}} \right) + \beta_4 \log \left( \frac{p_{i2}}{p_{i3}} \right) + \epsilon_i
  \]
- Calculate the F-ratio via the SSR of UMOD and RMOD
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Task 6: Reformulate the model to get the average costs as the dependent variable. What happens to the parameter $\beta_2$ in this new setting?

Reformulated model

$$\log(TC_i) = \beta_1 + \beta_2 \log(Q_i) + \beta_3 \log(p_{i1}) + \beta_4 \log(p_{i2}) + \beta_5 \log(p_{i3}) + \varepsilon_i$$

$$\log \left( \frac{TC_i}{Q_i} \right) = \beta_1 + (\beta_2 - 1) \log(Q_i) + \beta_3 \log(p_{i1}) + \beta_4 \log(p_{i2}) + \beta_5 \log(p_{i3}) + \varepsilon_i$$

Estimation in EViews

• Quick $\rightarrow$ Estimate Equation (UMOD_average)

• Equation specification:

  $$\log(TC/Q) \; c \; \log(Q) \; \log(PL) \; \log(PK) \; \log(PF)$$
Task 6:

Reformulate the model to get the average costs as the dependent variable. What happens to the parameter $\beta_2$ in this new setting?

**Reformulated model**

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**Estimation in EViews**

- Quick → Estimate Equation (UMOD_average)
- Equation specification:
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  \log(TC/Q) c \log(Q) \log(PL) \log(PK) \log(PF)
  \]
Task 6:

Discuss the difference in the two values for $R^2$ obtained in the total cost equation (1) and the average cost equation (2).

Does the higher $R^2$ make (1) preferable to (2)?

- (1) total cost equation: 0.926
- (2) average cost equation: 0.695

⇒ nonsense, because (1) and (2) represent the same model
⇒ high $R^2$ in (1) comes from the scale effect that total costs increase with firm size
⇒ equations must share the same dependent variable in order to compare their fit via $R^2$
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Task 7: Testing constant returns to scale.

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Task 7: Testing constant returns to scale.

Test the hypothesis that there exist CRS \((H_0 : r = 1)\).

Consider the restricted model

\[
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\]

then \(r = 1\) if and only if the coefficient \(\beta_2\) equals 1 since \(\beta_2 = \frac{1}{r}\).

Application of the \(t\)-test

1. Calculate the \(t\)-ratio for the hypothesis \((H_0 : \beta_2 = 1)\).
2. Look for the critical value in the \(t\)-distribution \(t(n - K)\) \((K = 4\) and \(n = 145)\).
3. Conclude.
Task 7: Testing constant returns to scale.

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Task 8:

Make a residual series from your restricted model. Plot the residuals against log($Q_i$). Are you disturbed after looking at the plot?

Plot in EViews

- Object → New Object → Graph (Scatter diagram)

Notice from the Residual Plot

- as output increases, the residuals first tend to be positive, then negative and again positive → degree of returns to scale is not constant as assumed in the log-linear specification
- residuals are more widely scattered for lower output → failure of the homoskedasticity assumption
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Thank you for your attention and have a nice week!