I. Principles of Financial Economics

Reference:
Cochrane (2001), Ch. 1 (without 1.5)
Empirical asset pricing - Introduction (1)

Asset pricing (Valuation of financial assets)

<table>
<thead>
<tr>
<th>delay of payoff</th>
<th>account for risk of payoff</th>
</tr>
</thead>
</table>

⇒ risk correction

50 years US stocks:
- 9% average return (real) p.a.
- 1% real interest rate p.a. (treasury bills)

8% premium earned for holding risk

What is the risk that is priced?

Asset pricing

normative

how should the world work?

are the prices "wrong"?

- trading opportunities?

- cost of capital

- non traded assets: "fair" price

positive

how does the world work?
Empirical asset pricing - Introduction (2)

Basic: Prices equal discounted expected payoff

What probability measure?

Absolute Asset Pricing

exposure to "fundamental" macroeconomic risk

Asset priced given other asset prices (e.g. option pricing)

Relative Asset Pricing

e.g. CAPM:

\[ \mathbb{E}(R^i) = R^f + \beta_i \left( \mathbb{E}(R^m) - R^f \right) \]

\[ \beta_i = \frac{\text{cov}(R^i, R^m)}{\text{var}(R^m)} \]

Market price of risk (factor) risk premium not explained
Empirical asset pricing - Introduction (3)

Basic pricing equation

\[ p_t = \mathbb{E}_t(m_{t+1}x_{t+1}) \]

- asset price at \( t \)
- stochastic discount factor (r.v.)
- payoff (r.v.)

\[ m_{t+1} = f(\text{data, parameters}) \]

the model

Moment condition:

\[ \mathbb{E}_t(m_{t+1}x_{t+1}) - p_t = 0 \]

use \( \frac{1}{n} \sum \rightarrow \mathbb{E}(\cdot) \) WLLN

Generalized Method of Moments (GMM) to estimate parameters
Empirical asset pricing - Introduction (4)

Portfolio theory
Mean-Variance frontier
CAPM
APT
Option pricing
contingent claims state preference
consumption-based model
stochastic discount factor

Cochrane’s approach
From an utility maximising investor's first order conditions we obtain the basic asset pricing formula (1)

Basic objective: find \( p_t \), the present value of stream of uncertain payoff \( x_{t+1} \)

\[
x_{t+1} = p_{t+1} + d_{t+1}
\]

Utility function

\[
U(c_t, c_{t+1}) = u(c_t) + \beta \mathbb{E}_t [u(c_{t+1})]
\]

Random variables: \( p_{t+1}, d_{t+1}, x_{t+1}, e_{t+1}, c_{t+1}, u(c_{t+1}) \) \hspace{1cm} \( \mathbb{E}_t [\cdot] \triangleq \mathbb{E} [\cdot | \mathcal{F}_t] \)
From an utility maximising investor’s first order conditions we obtain the basic asset pricing formula (2)

\[
\max_{\xi} \left[ U(c_t, c_{t+1}) \right] \text{ s.t.}
\]

\[
c_t = e_t - p_t \xi; \quad c_{t+1} = e_{t+1} + x_{t+1} \xi
\]

\[
\max_{\xi} \{ u(e_t - p_t \xi) + \beta E_t [u(e_{t+1} + x_{t+1} \xi)] \}
\]

\[
-p_t \cdot u'(c_t) + \beta \cdot E_t \left[ u'(c_{t+1}) \cdot x_{t+1} \right] = 0
\]

utility loss if investor buys another unit of the asset

\[
p_t u'(c_t) = E_t \left[ \beta u'(c_{t+1}) x_{t+1} \right]
\]

discounted expected utility increase from extra payoff

\[
p_t = E_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right]
\]

Investor continues to buy or sell the asset until marginal loss equals marginal gain.

No complete solution: endogenous variables
Turning off uncertainty we are in the standard two-goods case (1)

\[
\max [u(c_t) + \beta u(c_{t+1})] \ \text{s.t.} \ c_t = e_t - p_t \cdot \xi, \ c_{t+1} = e_{t+1} + x_{t+1} \cdot \xi
\]

\[
\frac{\partial U(c_t, c_{t+1})}{\partial \xi} = -p_t \cdot \frac{\partial u(c_t)}{\partial c_t} + \beta \cdot x_{t+1} \cdot \frac{\partial u(c_{t+1})}{\partial c_{t+1}} = 0
\]

- \[
p_t \cdot u'(c_t) = x_{t+1} \cdot \beta u'(c_{t+1})
\]
- \[
p_t = x_{t+1} \cdot \frac{\beta u'(c_{t+1})}{u'(c_t)}
\]

\[
\frac{dc_t}{dc_{t+1}} = \frac{\beta \cdot u'(c_{t+1})}{u'(c_t)} = \frac{p_t}{x_{t+1}}
\]

opportunity cost to transfer consumption from t to t+1

\[
p_t u'(c_t) = \mathbb{E}_t \left[ \beta u'(c_{t+1}) x_{t+1} \right]
\]

\[
p_t = \mathbb{E}_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right]
\]
We often use a convenient power utility function (1)

\[ u(c_t) = \frac{1}{1 - \gamma} c_t^{1-\gamma} \]

\[ u'(c_t) = c_t^{-\gamma} \]

\[ \lim_{\gamma \to 1} \left( \frac{1}{1 - \gamma} c_t^{1-\gamma} \right) = \ln (c_t) \]

\[ \frac{dc_t}{dc_{t+1}} = \beta \frac{u'(c_{t+1})}{u'(c_t)} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \]

utility \( u(c_t) \)

parameter \( \gamma \):

0.3  0.5  0.8

increasing concavity of utility function

We often use a convenient power utility function (1)
Prices, payoffs, excess returns

<table>
<thead>
<tr>
<th>Price $p_t$</th>
<th>Payoff $x_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>stock</td>
<td>$p_t$</td>
</tr>
<tr>
<td>return</td>
<td>1</td>
</tr>
<tr>
<td>excess return</td>
<td>0</td>
</tr>
<tr>
<td>one $$ one period discount bond</td>
<td>$p_t$</td>
</tr>
<tr>
<td>risk-free rate</td>
<td>1</td>
</tr>
</tbody>
</table>

Payoff $x_{t+1}$ divided by price $p_t$ ⇒ gross return $R_{t+1} = \frac{x_{t+1}}{p_t}$

Return: payoff with price one

$$1 = \mathbb{E}_t (m_{t+1} \cdot R_{t+1})$$

Zero-cost portfolio:
Short selling one stock, investing proceeds in another stock ⇒ excess return $R^e$

Example: Borrow 1$ at $R^f$, invest it in risky asset with return $R$. Pay no money out of the pocket today → get payoff $R^e = R - R^f$.

Zero price does not imply zero payoff.
The covariance of the payoff with the discount factor rather than its variance determines the risk-adjustment

\[ \text{cov}(m_{t+1}, x_{t+1}) = \mathbb{E}(m_{t+1} \cdot x_{t+1}) - \mathbb{E}(m_{t+1}) \mathbb{E}(x_{t+1}) \]

\[ p_t = \frac{\mathbb{E}(m_{t+1} \cdot x_{t+1})}{\mathbb{E}(x_{t+1})} + \text{cov}(m_{t+1}, x_{t+1}) \]

\[ R^f_t = \frac{1}{\mathbb{E}(m_{t+1})} \]

\[ p_t = \frac{\mathbb{E}(x_{t+1})}{R^f_t} + \text{cov}(m_{t+1}, x_{t+1}) \]

\[ p_t = \frac{\mathbb{E}(x_{t+1})}{R^f_t} + \beta \frac{\text{cov}(u'(c_{t+1}), x_{t+1})}{u'(c_t)} \]

Marginal utility declines as consumption rises.

Price is lowered if payoff covaries positively with consumption. (makes consumption stream more volatile)

Price is increased if payoff covaries negatively with consumption. (smoothens consumption) Insurance!

Investor does not care about volatility of an individual asset, if he can keep a steady consumption.
All assets have an expected return equal to the risk-free rate, plus risk adjustment

\[ 1 = \mathbb{E}(m_{t+1} \cdot R_{t+1}^i) \]

\[ 1 = \mathbb{E}(m_{t+1}) \mathbb{E}(R_{t+1}^i) + \text{cov}(m_{t+1}, R_{t+1}^i) \]

\[ R_f = \frac{1}{\mathbb{E}(m_{t+1})}; 1 - \frac{1}{R_f} \mathbb{E}(R_{t+1}^i) = \text{cov}(m_{t+1}, R_{t+1}^i) \]

\[ \mathbb{E}(R_{t+1}^i) - R_f = -R_f \cdot \text{cov}(m_{t+1}, R_{t+1}^i) \]

\[ \mathbb{E}(R_{t+1}^i) - R_f = -\frac{1}{\mathbb{E}(\beta \frac{u'(c_{t+1})}{u'(c_t)})} \cdot \text{cov}\left(\beta \frac{u'(c_{t+1})}{u'(c_t)}, R_{t+1}^i\right) \]

excess return

\[ \mathbb{E}(R_{t+1}^i) - R_f = -\frac{\text{cov}(u'(c_{t+1}), R_{t+1}^i)}{\mathbb{E}(u'(c_{t+1}))} \]

Investors demand higher excess returns for assets that covary positively with consumption. Investors may accept expected returns below the risk-free rate. Insurance!
The basic pricing equation has an expected return-beta representation

\[
\mathbb{E}(R_{t+1}^i) - R^f = -R^f \cdot \text{cov}(R_{t+1}^i, m_{t+1})
\]

\[
\mathbb{E}(R_{t+1}^i) - R^f = -\frac{\text{cov}(R_{t+1}^i, m_{t+1}) \cdot \text{Var}(m_{t+1})}{\text{Var}(m_{t+1})} \cdot \mathbb{E}(m_{t+1})
\]

\[
\mathbb{E}(R_{t+1}^i) = R^f - \left( \frac{\text{cov}(R_{t+1}^i, m_{t+1})}{\text{Var}(m_{t+1})} \right) \cdot \left( \frac{\text{Var}(m_{t+1})}{\mathbb{E}(m_{t+1})} \right)
\]

asset specific quantity of risk \quad \uparrow \quad \downarrow \quad \text{price of risk for all assets}

Beta-pricing model:

\[
\mathbb{E}(R^i) = R^f + \beta_{R^i,m} \cdot \lambda_m
\]

With \( m = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \) and lognormal consumption growth \( \frac{c_{t+1}}{c_t} \)

\[
\mathbb{E}(R^i) = R^f + \beta_{R^i,\Delta c} \cdot \lambda_{\Delta c}
\]

\( \lambda_{\Delta c} \approx \gamma \cdot \text{Var}(\Delta \ln c) \)

The more risk averse the investors or the riskier the environment, the larger the expected return premium for risky (high-beta) assets.
Marginal utility weighted prices follow martingales (1)

Basic first order condition:

\[ p_t u'(c_t) = \mathbb{E}_t \left( x_{t+1} \left( \beta \left( u'(c_{t+1}) \right) \right) (p_{t+1} + d_t) \right) \]

Market efficiency \( \iff \) Prices follow martingales (random walks)? \( \text{NO!} \)

Risk neutral investors \( u'(\cdot) = \text{const.} \) or no variation in consumption

Required:

\( \beta = 1 \iff \text{OK short time horizon} \)

\( \text{no dividends} \)

Then:

\[ p_t = \mathbb{E}(p_{t+1}) \]

\[ p_{t+1} = p_t + \varepsilon_{t+1} \]

if \( \sigma^2(\varepsilon_{t+1}) = \sigma^2 = \text{Random Walk} \)

\( \Rightarrow \) Returns are not predictable \( \mathbb{E} \left( \frac{p_{t+1}}{p_t} \right) = 1 \)
Marginal utility weighted prices follow martingales (2)

With risk aversion (but no dividends) and $\beta=1$

$$\tilde{p}_t = \mathbb{E}(\tilde{p}_{t+1})$$

$$\tilde{p}_t = p_t \cdot u'(c_t)$$

Scale prices by marginal utility, correct for dividends and apply risk neutral valuation formulas

Predictability in the short horizon?

consumption risk aversion \{ does not change day by day

$\Rightarrow$ Random Walks successful $\Rightarrow$ Predictability of asset returns (day by day)?

Technical analysis, media reports...
Some popular linear factor models

Factor pricing models

**CAPM**:

\[ m_{t+1} = a + b R^w_{t+1} \]

Free parameters

Compatible with utility maximisation?

**ICAPM**:

\[ m_{t+1} = a + b' f_{t+1} \]

factors (macro, term spread, price-earnings ratio help forecast conditional distribution of future asset returns)

**APT**:

similar, but factors determined by principal component analysis of payoff covariance matrix

Practice: just test \( m = b' f \) and don’t worry about derivations