Financial Econometrics

Second set of assignments:

1. The Law of Total Expectations (also referred to as Law of Iterated Expectations) states that

   \[ a) \quad \mathbb{E}[\mathbb{E}(X|Y)] = \mathbb{E}(X) \]
   
   and

   \[ b) \quad \mathbb{E}[\mathbb{E}(X|Y, Z)] = \mathbb{E}(X) \]

   \[ c) \quad \mathbb{E}[\mathbb{E}(X|Y, Z)|Z] = \mathbb{E}(X|Z) \]

   Law of Total Expectations

   Law of Iterated Expectations

   Show (derive) these results for \( X, Y, Z \) continuous random variables with joint density \( f_{XYZ}(x, y, z) \).

   Hints:

   \[ f_{X|Y,Z}(X|Y, Z) = \frac{f_{XYZ}(x, y, z)}{f_{YZ}} \quad \text{(conditional density of } X|Y, Z) \]

   \[ \mathbb{E}(X|Y, Z) = \int_{-\infty}^{\infty} xf_{X|Y,Z}(x|y, z)dx \quad \text{(conditional expectation)} \]

   \[ \mathbb{E}(g(X, Y)|X) = \int_{-\infty}^{\infty} g(x, y) \cdot \frac{f_{XY}(x, y)}{f_X(x)} dy \]

2. What does the ergodic theorem state?

3. What does the multivariate central limit theorem (CLT) for i.i.d. (independent, identically distributed) samples state?

4. Do we require data to be generated by i.i.d. processes to apply a CLT?

5. Describe the fundamental differences between the two theories of scientific discovery referred to as "inductivism" and "critical rationalism".

6. Apply the law of total expectations to

\[ p_t = \mathbb{E}(m_{t+1}x_{t+1}|F_t) \quad \text{payoffs} \]
\[ 1 = \mathbb{E}(m_{t+1}R_{t+1}|F_t) \quad \text{returns} \]
\[ 0 = \mathbb{E}(m_{t+1}R_{t+1}^e|F_t) \quad \text{excess returns} \]

7. Why is it necessary to perform an "unconditioning" of the pricing equation

\[ p_t = \mathbb{E}_t(m_{t+1}x_{t+1}) \] when we want to estimate the unknown parameters by GMM?

8. Why do we prefer to base the GMM estimation of the basic asset pricing equation on

\[ 1 = \mathbb{E}_t(m_{t+1}R_{t+1}) \quad \text{or} \quad 0 = \mathbb{E}_t(m_{t+1}R_{t+1}^e) \] instead of \[ p_t = \mathbb{E}_t(m_{t+1}x_{t+1}) \]?