1st set GAUSS assignments Financial Econometrics

The following assignment is based on Hamilton(1994) pp. 409-411

1. Create a t-distributed random variable

Write a procedure that returns a t-distributed random variable Y with ν degrees of freedom:

$$Y = \frac{X}{\sqrt{U/\nu}}$$

where X is a standard normally distributed random variable and U is a χ^2_ν-distributed random variable. U can be generated from a standard normally distributed variable Z as: $U = \sum_{i=1}^{\nu} Z_i^2$ (X and Z have to be independent). As input arguments use the degrees of freedom (ν) of the t-distributed variable (Y) and the sample size (n) of the normally distributed random variables (Z and X). Hint: to create those variables use the command `rndn` (look it up in the help reference for details).

2. Create an objective function to estimate the parameter ν

The density function of a t_ν- distributed r.v. is:

$$f_Y(y;\nu) = \frac{\Gamma[(\nu+1)/2]}{(\pi\nu)^{1/2}\Gamma(\nu/2)}[1+(y^2/\nu)]^{-(\nu+1)/2}$$

The unconditional second and forth moments are:

$$\begin{aligned} \mu_2 \equiv E(Y^2) &= \nu/(\nu-2) \\ \mu_4 \equiv E(Y^4) &= \frac{3v^2}{(v-2)(v-4)} \end{aligned}$$

The sample second and forth moments are:

$$\begin{aligned} \hat{\mu}_2 &= (1/N)\sum_{i=1}^{N} y_i^2 \\ \hat{\mu}_4 &= (1/N)\sum_{i=1}^{N} y_i^4 \end{aligned}$$

Write a procedure which returns the objective function

$$Q(\nu) \equiv g'Wg$$

where

$$g = \begin{bmatrix} \left\{\hat{\mu}_2 - \frac{\nu}{\nu-2}\right\} \\ \left\{\hat{\mu}_4 - \frac{3v^2}{(v-2)(v-4)}\right\} \end{bmatrix}$$

As input arguments use a data set (i.e. this will later on be the data generated in 1.) and the degrees of freedom (ν). The procedure returns $Q(\nu)$. Moreover use the identity matrix as weighting matrix W.

3. Evaluating the objective function

Grid search: Write a procedure that calls on the procedure from 2. and evaluates the objective function for different values of ν (use a `for` loop that loops over a range of values for ν and for each value calls on the procedure from 2.). ν has to be larger than 4. Use a if condition that stops the program and prints a warning into the output window if this condition is not fullfiled.
Inside the procedure also save the values of ν and the corresponding value of the objective function into a vector. To do so use an `if` condition. Define a scalar that takes on the value from the first iteration and by vertical concatenation the other values are added. Still inside the procedure, plot the values of the objective function and the corresponding degrees of freedom into a graph. Label the axis of the graph and use a title. The input arguments are the data set generated in 1. as well as the stop/start/step values for the `for` loop.

Finally call your data generating procedure from 1. (for n=10000, v=7) and use the output as your data set. Call on the grid search procedure from 3.. Use a `for` loop that runs from 5 to 15 in steps of 0.05. Interpret the resulting graph.

This assignment can be handed in for grading until 11th Nov. 2008.
If you want to hand in this assignment for grading, include a pdf file (beside your program code) that shortly describes the procedures. Include the graph generated by Gauss and explain and interpret it. Moreover, shortly outline the general idea behind GMM. What is the meaning of the weighting matrix W? We use two moment conditions in this exercise, could we use more or less than two and what are the implications for GMM estimation?
Send your program code and the pdf file to franziska-julia.peter@uni-tuebingen.de