Financial Econometrics WS 08/09
Questions for review

Lecture 1

1. Show that the two period model where

$$\max_{\{\xi\}} \quad u(c_t, c_{t+1}) = u(c_t) + \beta\mathbb{E}[u(c_{t+1})]$$

$$\begin{aligned} \text{s.t.} \quad c_t &= e_t - \xi p_t \\ c_{t+1} &= e_{t+1} + \xi x_{t+1} \qquad (x_{t+1} = p_{t+1} + d_{t+1}) \end{aligned}$$

and the multiperiod model where the investor maximises

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j})$$

$$\begin{aligned} \text{s.t.} \quad c_t &= e_t - \xi p_t \\ c_{t+1} &= e_{t+1} + \xi d_{t+1} \\ c_{t+2} &= e_{t+2} + \xi d_{t+2} \\ &\vdots \end{aligned}$$

i.e. the investor can buy a dividend stream $\{d_{t+j}\}$ at price p_t
yield the same basic pricing equation

$$p_t = \mathbb{E}_t \left(\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right)$$

Cochrane (2005) p. 24 only sketches the derivation you need to fill the gaps!

2. Solve problem 1b in Cochrane (2005) p. 31!

3. Take the two period problem under uncertainty

$$\begin{aligned} u(c_t, c_{t+1}) &= u(c_t) + \beta\mathbb{E}(u(c_{t+1})) \\ \text{assume} \quad u(c_t) &= \frac{1}{1-\gamma} \quad c_t^{1-\gamma} \end{aligned}$$

In $t+1$ the economy can take only three states. The "recession" state occurs with probability p_1, the "normal" state with probability p_2 and the "boom" state with probability p_3.

In the recession state $c_{t+1} = c_1$ and the payoff of an asset is $x_{t+1} = x_1$. In the normal state we have $c_{t+1} = c_2$ and $x_{t+1} = x_2$ and in the boom state we have $c_{t+1} = c_3$ and $x_{t+1} = x_3$.

Derive the fundamental pricing equation in this special case:

$$p_t = \sum_{i=1}^{3} \beta \left(\frac{c_i}{c_t}\right)^{-\gamma} x_i \cdot p_i$$

4. What is an Arrow-Debreu security? Explain.

5. Suppose that there are two possible states of the nature in $t+1$ "good" and "bad". The probability for the good state is $p_1 = 0.6$ the one for the bad state is $p_2 = 1 - p_1 = 0.4$. The MRS $m_t + 1 = \beta u'(c_{t+1})/u'(c_t)$ in state 1 is 0.7 and 0.9 in state 2. Compute the price of the two Arrow-Debreu securites (i.e. an asset that deliver a payoff of 1 in the respective states of the nature).

6. Suppose you want to achieve an income (consumption) worth 1000 Euro regardless of the state of the nature in $t+1$. What do you have to do? And what does it cost you?

7. Suppose you can have iid draws from an exponential distribution $X \sim EXP(\lambda)$. Suggest two method of moments estimators for λ.

8. Why is the price of an excess return equal to zero? Explain.

9. The price of any (gross) return is equal to 1. Why? Explain.

10. Assuming power utility within the consumption based model one ties together two different aspects of investor/consumer behavior. Which two aspects are these and how are they stringed together?

11. Turning off uncertainty, the FOC for optimal investment read $-\frac{dc_t}{dc_{t+1}} = p_t/x_{t+1}$. I argued that p_t can be interpreted as the price of one unit consumption in $t+1$ and x_{t+1} is the price of one unit consumption in t. What is the economic explanation for this?

12. From state preference theory follows that we can write in an intertemporal model that assumes power utility

$$u(c_t) \quad = \quad \frac{1}{1-\gamma} \quad c_t^{1-\gamma}$$

the price of an Arrow-Debreu asset as follows:

$$p_i = \sum_i \pi_i \times hunger_i \times x_i$$

- What is an Arrow-Debreu asset?
- What is a state price and why is it also called state price density? What are its determinants?
- What is $hunger_i$? Explain why we call it "hunger".
- How does the price equation look like if the investor is risk neutral?

Lecture 2

1. Asset Pricing Playground (xls file)

State	Probability	Payoff x_{t+1}	$\frac{c_{t+1}}{c_t}$
1	0.1	100	1.02
2	0.3	200	0.97
3	0.2	300	1.03
4	0.3	10	0.92
5	0.1	600	1.05

Assume the basic two period model and

$$u(c) = \frac{1}{1-\gamma} \quad c^{1-\gamma}$$

and $\beta = 0.95$ and $\gamma = 0.8$

- Compute the value of the (shadow) risk free rate R^f_{t+1}
- Compute the expected payoff of the asset $\mathbb{E}(x_{t+1})$
- Compute the price of the asset p_t
- Compute the expected return of the asset $\mathbb{E}(R_{t+1})$
- Check your results with the help of the xls file.
- For what aspect of the investor's behavior does β account for? Attach to β in your xls file different values. How does the price change? Explain!
- Interpret γ! Attach to γ in your xls file different values. How does the price change? Explain!
- What happens to the marginal rate of substitution in state 2 and the price if the consumption growth in state 2 is reduced by 10 %? What happens with the price if the probability in State 2 is increased by 10 % while the probability in the first state is reduced by 10 %. Use the xls file and explain.
- What is a state price and why is it also called state price density? What are its determinants?
- What is $hunger_i$? Explain why we call it "hunger".
- How does the pricing equation look like if the investor is risk neutral?

2. What does the term stationarity state?

3. What determines the risk-adjustment in the price equation.

$$p_t = \frac{\mathbb{E}(x_{t+1})}{R^f} + cov(m_{t+1}, x_{t+1})$$

How does the equation look like if investors are risk neutral?

4. Explain why a negative excess return is not an anomaly of financial markets. Give an example of a negative excess return.

5. $$\mathbb{E}(R^i) = R^f + \beta_{R^i,m} \cdot \lambda_m$$

What is $\beta_{R^i,m}$? What is λ_m? Explain intuitively and by formulas. Which of the both is asset specific? Under what economic circumstances is λ_m high?

6. What is a martingale process? Explain.

7. What is a mean square error? What is the intuition behind squaring the prediction error?

8. Do prices follow martingales? What assumptions are necessary so that asset prices are a martingale? How can the price process be transformed that it follows a martingale?

9. In the factor pricing model context what is meant by "factor fishing"?

10. What sign do we expect for the slope coefficient b when we confront the capital asset pricing model (CAPM) with the data?

11. Which variables can be taken as factors in the international CAPM?

12. Describe briefly the idea behind the method of moments.

13. The Law of Total Expectations (also referred to as Law of Iterated Expectations) states that

a) $\mathbb{E}[\mathbb{E}(X|Y)] = \mathbb{E}(X)$
and
b) $\mathbb{E}[\mathbb{E}(X|Y,Z)] = \mathbb{E}(X)$
] Law of Total Expectations
and
c) $\mathbb{E}[\mathbb{E}(X|Y,Z)|Z] = \mathbb{E}(X|Z)$] Law of Iterated Expectations

Show (derive) these results for X, Y, Z continuous random variables with joint density $f_{XYZ}(x, y, z)$.

Hints:

$$\begin{aligned}
f_{X|Y,Z}(X|Y,Z) &= \frac{f_{XYZ}(x,y,z)}{f_{YZ}} \quad \text{(conditional density of } X|Y,Z) \\
\mathbb{E}(X|Y,Z) &= \int_{-\infty}^{\infty} x f_{X|Y,Z}(x|y,z)dx \quad \text{(conditional expectation)} \\
\mathbb{E}(g(X,Y)|X) &= \int_{-\infty}^{\infty} g(x,y) \cdot \frac{f_{XY}(x,y)}{f_X(x)} dy
\end{aligned}$$

14. Apply the law of total expectations to

$$\begin{aligned}
p_t &= \mathbb{E}(m_{t+1}x_{t+1}|I_t) \qquad \text{payoffs} \\
1 &= \mathbb{E}(m_{t+1}R_{t+1}|I_t) \qquad \text{returns} \\
0 &= \mathbb{E}(m_{t+1}R^e_{t+1}|I_t) \qquad \text{excess returns} \quad .
\end{aligned}$$

15. Why is it necessary to perform an "unconditioning" of the pricing equation $p_t = \mathbb{E}_t(m_{t+1}x_{t+1})$ when we want to estimate the unknown parameters by GMM ?

16. Why do we prefer to base the GMM estimation of the basic asset pricing equation on $1 = \mathbb{E}_t(m_{t+1}R_{t+1})$ or $0 = \mathbb{E}_t(m_{t+1}R^e_{t+1})$ instead of $p_t = \mathbb{E}_t(m_{t+1}x_{t+1})$?

17. Reminder: Practice your GAUSS skills!!! Go through the Gauss Introduction slides and repeat every practical exercise!

Lecture 3

1. In the linear regression model

$$y_t = \beta_1 + \beta_2 x_t + \varepsilon_t$$

with X_t a scalar random variable we assume

$$\mathbb{E}(\varepsilon_t) = 0$$
$$\mathbb{E}(x_t \varepsilon_t) = 0$$

Show that the moment estimator for β_1 and β_2 that results from these unconditional moment restrictions is identical to the least squares estimator obtained by

$$\underset{\{\hat{\beta}_1, \hat{\beta}_2\}}{\text{argmin}} \sum_{t=1}^{T} (y_t - \beta_1 - \beta_2 x_t)^2$$

2. What are the consequences for the OLS estimator $\hat{\beta}$ if endogenous regressors are included in the CLRM? How can instruments help you out of the problem?

3. What essential properties should a 'good' instrument have?

4. The CAPM assumes $m_{t+1} = a + \tilde{b}\, R^m_{t+1}$
Write for this case $\mathbb{E}\Big(u_t(b, \underline{X}_t)\Big) = 0$
What is b? What is $\underline{X}_t$? What is $u_t(b, \underline{X}_t)$?
Derive a moment estimator for a and $\tilde{b}$. Use two asset returns R^a_{t+1} and R^b_{t+1} for which we have

$$\mathbb{E}_t\Big((a + \tilde{b}\, R^m_{t+1})\, R^a_{t+1}\Big) - 1 = 0$$
$$\mathbb{E}_t\Big((a + \tilde{b}\, R^m_{t+1})\, R^b_{t+1}\Big) - 1 = 0$$

and proceed as described in the lecture to derive the moment estimator
Solution:

$$\text{for convenience}: R^a_{t+1} = R^a, R^b_{t+1} = R^b, R^m_{t+1} = R^m \qquad \text{NOTE:} \quad \mathbb{E}_T = \frac{1}{T}\sum_{t=1}^{T}$$

$$\hat{a} = \frac{\mathbb{E}_T(R^m R^b) - \mathbb{E}_T(R^m R^a)}{\mathbb{E}_T(R^m R^b)\mathbb{E}_T(R^a) - \mathbb{E}_T(R^m R^a)\mathbb{E}_T(R^b)}$$

$$\hat{\tilde{b}} = \frac{\mathbb{E}_T(R^a) - \mathbb{E}_T(R^b)}{\mathbb{E}_T(R^m R^b)\mathbb{E}_T(R^a) - \mathbb{E}_T(R^m R^a)\mathbb{E}_T(R^b)}$$

5. How many moment conditions are necessary to estimate K parameters: a) in an exact identified case? b) in an overidentified case?

6. Why do you use a weighting matrix in the GMM objective function?

7. How does the weighting matrix of first stage GMM estimation look like?

8. In general, what restrictions are placed on the weighting matrix?

9. Why do we want efficient GMM estimators? How can we achieve a more efficient GMM estimator? How is the GMM estimator distributed?

10. What is an optimal weighting matrix? Why is $W = S^{-1}$ with S the variance-covariance matrix of the GMM residuals a sensible choice for the weighting matrix? Explain intuitively.

11. Write down the variance-covariance matrix of the GMM estimators if W is the optimal weighting matrix, i.e. $W = S^{-1}$.

12. In a correctly specified model, what would you require for the first and second moment of the GMM residual u_t?

Lecture 4

1. Explain the term "consistent estimator".

2. Explain the term "asymptotically normally distributed estimator".

3. Why are consistency and asymptotic normality desirable properties for an estimator?

4. We distinguish two classes of extremum estimators. Which are they? Explain the objective function to be maximized in each case. What does the objective function look like for nonlinear OLS? Give an example.

5. Consider a nonlinear regression equation $y_t = \phi(x;\beta) + \varepsilon$. A set of instruments is available for which $E(\varepsilon z) = 0$ where rows(z) > rows(x). Which extremum estimator would you suggest to estimate the parameter vector β? Write down the objective function.

6. From an applied perspective: Which advantages does applicability of the 2nd consistency theorem for Extremum Estimators offer compared to the 1st theorem?

7. Unfortunately, the second consistency theorem is rarely applicably in the context of empirical asset pricing based on the moment conditions $E(m(b)R^e) = 0$ or $E(m(b)R-1) = 0$. Why is that?

8. When we showed consistency of the OLS estimator, we had to rely on an assumption that also shows up when adapting the first consistency theorem to GMM. Which is it?

9. When the process that generates the data w_t is ergodic stationary then it is straightforward to show that $T^{-1}\sum g(w_t\ \theta)$ converges in probability to $E(g(w_t\ \theta)\forall\theta$, so we have point-wise convergence in probability of a random function. Why is uniform convergence a concept that is somewhat harder to grasp? Try to explain the meaning of uniform convergence of a random function to a non-stochastic function to a fellow student.

10. We use the ULLN to prove uniform convergence. What are the necessary conditions to apply the ULLN? Explain.

11. Where do the conditions for ULLN show up in the conditions for consistency of the (nonlinear) GMM estimator?

12. Adapting the first consistency theorem to GMM estimators, two assumptions prevail which are in a way crucial for applied work as they are both testable and have (at least the second) a clear economic meaning. Which are these? And what is the meaning of these assumptions (especially one of them) within the context of estimating the parameters of an asset pricing model using the SDF approach?

Lecture 5

1. Using the results from the lecture show that:

   $\sqrt{n}\ g_n(\hat{\theta}) = \sqrt{n}[\frac{1}{n}\sum g(w_t,\hat{\theta})] \underset{d}{\longrightarrow} MVN(0, Avar(g_n(\hat{\theta})),$
   where
   $\text{Avar}(g(\hat{\theta})) = [I - G(G'W'G)^{-1}G'W)\ \text{S}\ (I - G(G'W'G)^{-1}G'W]'$

   (This is the result from p. 53 in the script which is crucial for construction of the J-statistic.)

   Hint: Start at the GMM F.O.C. and combine them with the MVT application on $g(\hat{\theta})$ and the expression we found for $\sqrt{n}(\hat{\theta} - \theta_0)$. Note that $\sqrt{n}g_n(\theta_0) \underset{d}{\longrightarrow} MVN(0, S)$.

2. Why is $\text{Avar}(g_n(\hat{\theta}))$ not of full rank?

3. Compare $\text{Avar}(g_n(\hat{\theta}))$ with $\text{Avar}(g_n(\theta_0))$. Is $\text{Avar}(g_n(\theta_0))$ of full rank?
   With w_t iid, what is $\text{Avar}(g_n(\theta_0))$?

   Hint: The Lindeberg-Levy CLT (Hayashi, p.96) states:
   If z_t is an iid random vector with $\text{E}(z_t)=\mu$ and $\text{Var}(z_t)=\text{E}[(z_t-\mu)(z_t-\mu)']$ then
   $\sqrt{n}(\bar{z}_n - \mu) = \sqrt{n}(\frac{1}{n}\sum_{t=1}^{n}(z_t - \mu)) \underset{d}{\longrightarrow} MVN(0, \Sigma)$.

4. Explain the "Wald test flavor" of the J-statistic.

5. What is the null hypothesis that motivates the construction of the J-test?

6. Where does the null hypothesis show up (or better: does not show up) in the J-statistic?

7. In which way does the less than full rank of $\text{Avar}(g_n(\hat{\theta}))$ complicate the computation of the J-statistic?

8. For the GMM estimator $\hat{b}_{GMM}$ resulting from

   $$\underset{\{\hat{b}\}}{argmin}\ g_T(\hat{b})'W g_T(\hat{b})$$

   We have $\hat{b}_{GMM} \underset{p}{\longrightarrow} b$ and

   $\sqrt{T}(\hat{b}_{GMM} - b) \underset{d}{\longrightarrow} N(0, \widehat{Avar(\hat{b}_{GMM}})))$

Where $\widehat{Avar(\hat{b}_{GMM})}$ denotes the asymptotic variance covariance matrix.
In a finite sample we use the approximation

$$\hat{b}_{GMM} \overset{a}{\sim} N(b, \frac{\widehat{Avar(\hat{b}_{GMM})}}{T})$$

to test hypotheses about b.

We have $Avar(\hat{b}_{GMM}) = (\hat{d}'W\hat{d})^{-1}\hat{d}'W\hat{S}W\hat{d}(\hat{d}'W\hat{d})^{-1}$.

To compute $Avar(\hat{b}_{GMM})$ you need to write

$$\hat{d} = \frac{\partial g_T(b)}{\partial b}|_{\hat{b}}$$

$g_T(b)$ is a vector valued function, i.e. it returns, for a given parameter vector $b = (b_1, b_2, ..., b_K)'$, the vector of sample moments:

$$\begin{pmatrix} \mathbb{E}_T(u_t^1(b)) \\ \vdots \\ \mathbb{E}_T(u_t^N(b)) \end{pmatrix} = \begin{pmatrix} \frac{1}{T}\sum_{t=1}^T u_t^1(b) \\ \vdots \\ \frac{1}{T}\sum_{t=1}^T u_t^N(b) \end{pmatrix}$$

then

$$\frac{\partial g_T(b)}{\partial b} = \begin{pmatrix} \frac{\partial \mathbb{E}_T(u_t^1(b))}{\partial b_1} & \cdots & \frac{\partial \mathbb{E}_T(u_t^1(b))}{\partial b_K} \\ \vdots & \ddots & \vdots \\ \frac{\partial \mathbb{E}_T(u_t^N(b))}{\partial b_1} & \cdots & \frac{\partial \mathbb{E}_T(u_t^N(b))}{\partial b_K} \end{pmatrix} N \times K$$

Write $\frac{\partial g_T(b)}{\partial b}$ in detail for the GMM estimation framework of the consumption based model where

$$m_{t+1} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}.$$

Use two moment restrictions for two asset returns $R^a_{t+1}$and R^b_{t+1}:

$$\mathbb{E}(m_{t+1}R^a_{t+1} - 1) = 0$$

$$\mathbb{E}(m_{t+1}R^b_{t+1} - 1) = 0$$

What is b?
What is $u_t(b)$?
What is $\mathbb{E}_t(u_t(b))$ and $g_T(b)$?
What is $\frac{\partial g_T(b)}{\partial b}$?

Write all in greatest detail!

You have succeeded in computing a consistent estimate of $Avar(\hat{b}_{GMM})$ for your GMM application.

$$\widehat{Avar(\hat{b}_{GMM})} = \begin{pmatrix} 5 & 0.3 \\ 0.3 & 10 \end{pmatrix}$$

You have used $T = 100$ observations. Your GMM estimates are given by

$$\hat{\beta}_{GMM} = 0.8 \quad \hat{\gamma}_{GMM} = 0.1$$

Compute an estimate of $Var(\hat{\beta}_{GMM})$ and $Var(\hat{\gamma}_{GMM})$.
Test the hypotheses

$$\begin{array}{ccc} H_0: \quad \beta = 1 & & H_0: \quad \gamma = 0 \\ \text{versus} & & \text{versus} \\ H_A: \quad \beta \neq 1 & & H_A: \quad \gamma \neq 0 \end{array}$$

using the t-statistics

$$t_1 : \frac{\hat{\beta}_{GMM} - 1}{\sqrt{\widehat{Var}(\hat{\beta}_{GMM})}} \qquad t_2 : \frac{\hat{\gamma}_{GMM}}{\sqrt{\widehat{Var}(\hat{\gamma}_{GMM})}}$$

t_1 and t_2 are approximately N(0,1) under the respective Null-Hypothesis.

9. What is the asymptotic variance covariance matrix of the GMM estimator, $Avar(\hat{b}_{GMM}) = (d'Wd)^{-1}d'WSWd(d'Wd)^{-1}$, if the weighting matrix is the inverse of the variance covariance matrix of the pricing errors, i.e. $W = S^{-1}$

10. Ten test assets are used to estimate the unknown parameters of the consumption based model using GMM. How much weight is put on each test asset if an identity matrix I is used as a weighting matrix W in the objective function.

11. Why would you like to use an optimal weighting matrix? In what terms is the resulting estimator an efficient estimator?

12. Explain the iterated GMM procedure.

13. Three uncorrelated test assets are used to estimate the unknown parameters of the consumption based model using two step GMM. Assume that in the second step the variance of the pricing errors of the first test asset is higher than the variance of the pricing errors of the second test asset. How are the pricing errors of the three test assets weighted in the objective function?

14. Three test assets are used to estimate the unknown parameters of the consumption based model using iterated GMM. Assume that the returns of two test assets are highly correlated. How are the pricing errors of these two test assets weighted in the GMM objective function?

15. If your model is correct, i.e. the expected pricing error is equal to zero, how should the GMM estimator and its standard error change when you compare the first step results with the iterated GMM results? How would the t- statistic of the estimator change?

16. Discuss the drawbacks of using an optimal weighting matrix with respect to

- comparing models
- estimating the variance covariance matrix of the pricing errors in a finite sample.

Lecture 6

1. There are different measures to evaluate the performance of an asset pricing model.

 - Using graphs that plot the predicted (excess) return against the observed (excess) return. Describe how these plots look like and how they are produced.

 - Using J- statistics. What is the null hypothesis of the J-statistic? Describe intuitively the test construction.

2. Assume the p-value of a J-statistic of an estimated asset pricing model is 0.07. Interpret the result.

3. To estimate the unknown parameters of the consumption based model with GMM, return of tests assets have to be defined. What is the motivation of using 10 NYSE stock portfolios sorted by marked value (size). (Hint: See Cochrane 1996 JPE paper p. 587)

4. How is the J-statistic computed when

   a) the optimal weighting matrix, $W = S^{-1}$,

   b) a weighting matrix, W,
   is used.

5. Derive from the general asset pricing equation

$$p_t = \mathbb{E}(m_{t+1}x_{t+1})$$

   the specific return and excess return equations:

$$\begin{aligned} 1 &= \mathbb{E}(m_{t+1}R_{t+1}) && \text{returns} \\ 0 &= \mathbb{E}(m_{t+1}R^e_{t+1}) && \text{excess returns} \end{aligned} \quad .$$

6. Plot the consumption data and return series of the GAUSS exercise. Describe the properties of these time series. Argue why the CBM is estimated with consumption growth rate and not with the level of consumption.

7. Explain in your own words the meaning of the weak law of large numbers (WLLN) and the central limit theorem (CLT).

8. One assumption in deriving the distribution of the GMM estimator is that the model is correct. Explain how this assumption enters in the asymptotic result:

$$\sqrt(T)g_T(\hat{b}) \underset{d}{\rightarrow} N(0, Avar(g_T(\hat{b})))$$

9. Why does the general form of the J-statistic uses a pseudo inverse of the estimated $Avar(g_T(\hat{b}))$?

10. Explain what it means if a test "overrejects". Some researchers have shown that the J-statistic overrejects. What are the consequences for testing asset pricing models?

11. How is the theorem from multivariate statistics $(\underline{X} - \underline{\mu})'\underline{\Sigma}^{-1}(\underline{X} - \underline{\mu}) \sim \chi^2(N - K)$ with $\underline{X} \sim MVN(\underline{\mu}, \underline{\Sigma})$ used to construct the J-statistic. Explain.

12. Compare the Fama French model and the CBM. Discuss the drawbacks and the advantages in both cases. Describe how the fundamental risk factor is constructed to explain the data. How successful are these models empirically? Compare and interpret the graphs that plot the predicted (excess) return against the observed (excess) return. Compare and interpret J-statistics of the CBM and CAPM (see script p.56 and p.58).

Lecture 7

1. Explain the economic meaning of the parameters γ and ϕ in the utility function of Garcia, Renault and Semenov (2003).

2. How can you test in the Garcia, Renault and Semenov (2003) model if habit matters?

3. How can you test in the Garcia, Renault and Semenov (2003) model if people use power utility to maximize their intertemporal utility under their budget constraint.

4. The habit level is unobserved. How can this problem be solved?

   - Describe a two step method that generates first a habit time series and then optimizes the GMM objective function using the estimated habit time series.

   - Describe an alternative approach when the habit process is estimated simultaneously with the pricing errors applying the GMM.

   Argue which approach would you prefer and why?

5. Compare the empirical performance of the habit model with the Fama French model and the CAPM. Discuss and describe the economic theory underlying these models.

6. How is consumption in the Yogo (2006) paper disaggregated?

7. Explain the economic meaning of the parameters γ, β, ρ and σ in the utility function of Yogo (2006).

8. Explain how the consumption based model (CBM) is nested in the Yogo (2006) model. How are Yogo's parameters γ and σ related in the CBM framework?

9. Discuss Yogo's results on the script page 83. Discuss the economic plausibility of the parameters and their statistical significance.

10. Describe the moment conditions that are used in Yogo (2006). From which investment decision does he derive an additional moment condition?

11. When do pricing errors follow a martingale difference sequence?

12. How can time-t information in testing the implication of asset pricing theory. Explain in that context the term "managed portfolios". What are, generally spoken, meaningful instruments?

Lecture 8

1. What are the dual purposes to use time t available variables in GMM based empirical asset pricing?
2. Why is the use of "managed portfolios" a sensible strategy to test the asset pricing equation in its original form (i.e. the "conditional implications" of the model)?
3. We use unconditional moment conditions to test the conditional implications of an asset pricing model. What is the basic idea behind this (Hint: Think of the properties of a martingale difference sequence)?
4. Show that from $E(Y_t \mid X_t) = 0$ follows that $E(Y_t X_t) = 0$.
5. The Hansen/Richard critique states that the conditional asset pricing equation is not testable at all. Show how the use of instruments can alleviate the problem.
6. Show that accounting for time variable parameters in a linear factor model $m = b'f$ amounts to "blowing up" the number of factors or "scaling the factors".
7. Consider a linear factor model with $m = \tilde{a} + b\tilde{f}$. Discuss alternative methods to test the model based on excess returns as test assets.
8. Show the equivalence of $E(mR^e) = 0$ and $E(R^e) = \lambda'\beta$ in the linear factor case with K factors (i.e. $m = \tilde{a} + b'\tilde{f}$ where $\tilde{f}$ is a $K \times 1$ vector of factors).
   What is β?
   What is λ?
   What is λ if $\tilde{f}$ is a vector of excess returns with $p(\tilde{f}) = 0$?
9. Using excess returns as test assets to test a linear factor model $m = \tilde{a} + b'\tilde{f}$ entails an identification problem. Why? Explain.
10. Try estimating and testing the CAPM using the excess return of the market as factor and the excess returns of the ten deciles as test assets, i.e. $m = a + bR^{ew}$ and the standard GMM setting. Use the risk free rate proxy to compute the excess returns. Interpret your results! You are in trouble. How could you resolve your problem. Try your suggestion!
11. Why does the above problem not occur if you use returns instead of excess returns as test assets?
12. Try estimating the CBM with $m = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}$ using the excess returns of the ten size deciles. Again you have a problem. Discuss two solutions!

13. Consider a time series regression test of a linear asset pricing model. Set up the moment conditions implied by the orthogonality condition $E(\varepsilon_t^i) = 0$ and $E(\varepsilon_t^i \tilde{f}_t) = 0$ where $\varepsilon_t^i = R_t^{ei} - \beta \tilde{f}_t$ (as shown in the lecture) in the GMM toolbox. Use the excess returns of the ten size deciles and the excess return of the market as $\tilde{f}$. Conduct a Wald test to test the joint significance of the intercepts $\alpha_1, \cdots, \alpha_{10}$. Interpret your results!

14. Consider the expected return-beta representation $E(R_t^{ei}) = \beta_i' \cdot \lambda + \alpha_i$.

    How can λ be interpreted economically?

    Describe the estimation procedure of λ when $\tilde{f}$ in $m = \tilde{a} + b\tilde{f}$ is a) in excess return and b) something else. One model implication is that $\alpha_i = 0$. If you want to do proper inference in a two step OLS approach, i.e. a proper test that $\alpha_i = 0 \quad \forall i$, what aspects need to be taken into account and what problems do you face? How can you overcome the problems and how does the variance covariance matrix of $\hat{\alpha}$ look like?

    Describe how you can estimate λ along with the β using a GMM approach. Write down the moment conditions. Show how a GMM "J-test-type" is constructed from the variance covariance of $\hat{\alpha}_i = \hat{\alpha}_i ... \hat{\alpha_N}$, where $\hat{\alpha}_i = \frac{1}{T} \sum R_t^{ei} - \hat{\beta}\hat{\lambda}$.

15. Describe the Fama McBeth procedure and contrast it with the two step regression approach. What is identical and what is different?

Lecture 9

1. What are the general aims of Market Microstructure theory?
2. Explain the terms, bid/ask price, midquote and spread to an uninitiated person.
3. Name two time series features of the transaction price series.
4. As an liquidity supplier, how would you change your quoted prices (bid/ask) if you want to reduce your inventory?
5. Why can a spread between the bid and ask price exist? Discuss the components of the spread. What prevents the liquidity supplier to increase the spread in excess over these costs?
6. Describe the timing process of quote setting and price formation in the Huang Stoll model (1997).
7. Write down the base model of Huang Stoll (1997). Explain the meaning of the structural parameters.

Lecture 10

1. Explain the δ-Method to a second year statistic student.
2. Solve the following assignemnet and check if you really understood the δ-Method:

   Suppose you have obtained GMM estimates for $b = \begin{bmatrix} \theta \\ \phi \end{bmatrix}$ *i.e.* $\hat{b} = \begin{bmatrix} \hat{\theta} \\ \hat{\phi} \end{bmatrix}$.

   We have
   $$\sqrt{T}(\hat{b} - b) \underset{d}{\longrightarrow} N(0, \Sigma)$$
   where Σ is the asymptotic variance covariance matrix.

   A consistent estimate of Σ, denoted $\widehat{\Sigma}$, is given by
   $$\widehat{\Sigma} = \begin{pmatrix} 2 & 0.2 \\ 0.2 & 3 \end{pmatrix}$$

   The sample has $T = 100$ observations.
   Provide estimates of $Var(\hat{\theta})$ and $Var(\hat{\phi})$ using this information. The GMM estimates are $\hat{\theta} = 0.6$ and $\hat{\phi} = 0.4$

   You are interested in testing whether
   $$r = \frac{\phi}{\phi + \theta} = 0.5$$

   Construct a suitable test statistic. For this purpose compute an estimate of the variance of $\hat{r} = \frac{\hat{\phi}}{\hat{\phi}+\hat{\theta}}$, $Var(\hat{r})$, by using the δ-method.

   Hints:
   $$a(b) = \frac{\phi}{\phi + \theta} = r$$
   $$\hat{r} = a(\hat{b}) \underset{p}{\longrightarrow} a(b)$$
   $$\sqrt{T}(a(\hat{b}) - a(b)) \underset{d}{\longrightarrow} N(0, A(b)\Sigma A(b)')$$
   where $A(b) = \frac{\partial a(b)}{\partial b'} = \left(\frac{\partial a(b)}{\partial \phi}, \frac{\partial a(b)}{\partial \theta}\right)$

The test statistic is

$$t = \frac{\hat{r} - 0.5}{\sqrt{\widehat{Var(\hat{r})}}}$$

t is approximately N(0,1) under the Null Hypothesis that $r = 0.5$.

Hints: Application of the δ-method

$$\begin{aligned}
Var(\hat{\theta}) &= 0.02 \\
Var(\hat{\phi}) &= 0.03 \\
\hat{r} &= 0.4 \\
A(\hat{b}) &= (-0.4, 0.6) \\
A(\hat{b}) \left(\frac{1}{100} \widehat{\Sigma} \right) A(\hat{b})' &= 0.0130 \\
t &= -0.8757 \\
\rightarrow & \quad \text{we can not reject the null hypothesis:} \qquad r = 0.5
\end{aligned}$$

3. How can the parameters of the Huang Stoll (1997) base model be bestimated? Write down the moment conditions for the GMM estimation. Describe how the model could be estimated by OLS. What is the advantage of using GMM?

4. Referring to the results of the Huang Stoll's (1997) base model, should market makers be worried about informed traders? How much of the spread is attributable to adverse selection and inventory holding? (See script p. 176) Are the parameters estimates economically plausible? Are the parameters estimates statistically significantly different from zero?

5. Huang and Stoll (1997) cluster trades in trade size categories and estimate the base model. How does this clustering affect the results? Compare the adverse selection and inventory holding estimate across stocks. What do you observe? (See script p. 181)

6. Explain the Huang Stoll (1997) extended model that disentangles the adverse selection component and inventory holding component. Show how the following conditional expectation, $E(Q_{t-1}|Q_{t-2}) = (1 - 2\pi)Q_{t-2}$, can be derived.

7. How are the parameters of the Huang Stoll (1997) extended model estimated? Write down the moment conditions for GMM estimation.

8. Discuss the results of the Huang Stoll (1997) extended model (See script p. 186). What parameters would be expected by economic theory? Discuss the parameter estimates.

9. Huang Stoll (1997) bunch the trade data and estimate the extended model. How does the bunching affect the results? Discuss the parameter estimates (See script p. 187).

Lecture 11+12

1. Explain the aim and the procedure of an event study to an uninitiated person.

2. What is abnormal about an "abnormal return". Explain this term.

3. A abnormal return is defined as: $\epsilon^*_{it} = R_{it} - E(R_{it}|X_t)$. Explain the compontents of the equation.

4. What assumptions are made about the serial correlation and the cross sectional correlation of $(N \times 1)$ vector of asset returns in a classical event study. What distributional assumptions are made about the asset returns?

5. Write down the market model that gives you the conditional expectation $E(R_{it}|X_t)$. What is X_t in the market model?

6. Reminder: X and Y are jointly normally distributed
   $\begin{pmatrix} X \\ Y \end{pmatrix} \sim BVN(\mu_X, \mu_Y, \sigma^2_X, \sigma^2_Y, \rho_{XY})$.
   What is the relation of parameters and moments?
   $X \sim$
   $Y \sim$
   $X|(Y = y)$
   $Y|(X = x)$
   $E(X|Y = y) =$
   $Var(X|Y = y) =$

7. Reminder: X, Y and Z are normally distributed.
   $W = a * X + b * Y + c * Z \sim$
   How is W distributed?

8. You conduct an event study. You use the market model. What practical problems could you face if you want to add more factors?

9. You conduct an event study. You use the market model. Write down the conditional variance of the asset return. Would you prefer the correlation between the asset return and the market return to be large or small? Argue why!

10. Explain briefly the time line of an event study. Explain how you choose the time windows in an event study.

11. Describe step by step the steps that have to be undertaken to conduct an event study after you have chosen the time windows.

12. What is the null hypothesis that is tested in an event study?

13. Write down the properties (expectation, variance, distribution) of the abnormal returns in the estimation period.
14. Write down the properties (expectation, variance, distribution) of the abnormal returns in the event period for a) a one day event window, b) a multiple event window.
15. Write down the properties (expectation, variance, distribution) of the cumulative abnormal returns in the event period.
16. Write down different test statistics that test the null hypothesis of 12). How are these test statistics distributed?