First set of assignments

1. Cochrane Ch. 1: Read!

2. Show that the two period model where

$$\max_{\{\xi\}} \quad u(c_t, c_{t+1}) = u(c_t) + \beta \mathbb{E}[u(c_{t+1})]$$

$$\begin{aligned} \text{s.t.} \quad c_t &= e_t - \xi p_t \\ c_{t+1} &= e_{t+1} + \xi x_{t+1} \qquad (x_{t+1} = p_{t+1} + d_{t+1}) \end{aligned}$$

   and the multiperiod model where the investor maximises

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j})$$

$$\begin{aligned} \text{s.t.} \quad c_t &= e_t - \xi p_t \\ c_{t+1} &= e_{t+1} + \xi d_{t+1} \\ c_{t+2} &= e_{t+2} + \xi d_{t+2} \\ &\vdots \end{aligned}$$

   i.e. the investor can buy a dividend stream $\{d_{t+j}\}$ at price p_t
   yield the same basic pricing equation

$$p_t = \mathbb{E}_t \left(\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right)$$

   Cochrane (2005) p. 24 only sketches the derivation you need to fill the gaps!

3. Solve problem 1b in Cochrane (2005) p. 31!

4. Take the two period problem under uncertainty

$$\begin{aligned} u(c_t, c_{t+1}) &= u(c_t) + \beta \mathbb{E}(u(c_{t+1})) \\ \text{assume} \quad u(c_t) &= \frac{1}{1-\gamma} \quad c_t^{1-\gamma} \end{aligned}$$

   In $t+1$ the economy can take only three states. The "recession" state occurs with probability p_1, the "normal" state with probability p_2 and the "boom" state with probability p_3.
   In the recession state $c_{t+1} = c_1$ and the payoff of an asset is $x_{t+1} = x_1$. In the normal state we have $c_{t+1} = c_2$ and $x_{t+1} = x_2$ and in the boom state we have $c_{t+1} = c_3$ and $x_{t+1} = x_3$.

   Derive the fundamental pricing equation in this special case:

$$p_t = \sum_{i=1}^{3} \beta \left(\frac{c_i}{c_t} \right)^{-\gamma} x_i \cdot p_i$$

5.

State	Probability	Payoff x_{t+1}	$\frac{c_{t+1}}{c_t}$
1	0.1	100	1.02
2	0.3	200	0.97
3	0.2	300	1.03
4	0.3	10	0.92
5	0.1	600	1.05

Assume the basic two period model and

$$u(c) = \frac{1}{1-\gamma} \quad c^{1-\gamma}$$

and $\beta = 0.95$ and $\gamma = 0.8$

5a) Compute the value of the (shadow) risk free rate R^f_{t+1}

5b) Compute the expected payoff of the asset $\mathbb{E}(x_{t+1})$

5c) Compute the price of the asset p_t

5d) Compute the expected return of the asset $\mathbb{E}(R_{t+1})$