GMM estimation of basic asset pricing equation parameters

The basic pricing equation implies a set of CONDTIONAL moment restrictions

$$\begin{aligned} p_t &= \mathbb{E}_t\left(m_{t+1}x_{t+1}\right) \\ &= \mathbb{E}\left(m_{t+1}x_{t+1} \mid I_t\right) \end{aligned}$$

$\{m_t\}$ and $\{x_t\}$ non i.i.d. $\Rightarrow$ $\mathbb{E}_t(\cdot) \neq \mathbb{E}(\cdot)$

Information set (partially) not observed, conditional density not known, conditional expectation cannot be computed

Conditioning down to coarser information set

$$\begin{aligned} p_t &= \mathbb{E}_t\left(m_{t+1}x_{t+1}\right) \\ \mathbb{E}(p_t) &= \mathbb{E}\left(\mathbb{E}_t\left(m_{t+1}x_{t+1}\right)\right) \\ &= \mathbb{E}\left(m_{t+1}x_{t+1}\right) \end{aligned}$$

I.i.e.

Estimation and evaluation of asset pricing models (Basics)

Models contain **free parameters**

$$p_t = \mathbb{E}_t \left(\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} x_{t+1} \right)$$

- Estimation from data
- Testing hypotheses about parameters
- How good is the model?

Estimation and evaluation of asset pricing models (CBM)

$$p_t = \mathbb{E}_t(m_{t+1}\, x_{t+1}) \quad \text{or} \quad 1 = \mathbb{E}_t(m_{t+1}\, R_{t+1})$$

$$\uparrow f(data, parameters)$$

e.g. CBM with $u(c) = \frac{1}{1-\gamma}\, c^{1-\gamma} \Rightarrow m_{t+1} = \beta(\frac{c_{t+1}}{c_t})^{-\gamma}$

$\frac{c_{t+1}}{c_t}$: data (random variables)
$b = (\beta, \gamma)'$:free parameters

Assume model correct: "Best" choice for β, γ?
Best "fit", smallest (average) pricing errors

Estimation and evaluation of asset pricing models. The basic idea.

Estimates $\widehat{b}$ from data, distribution of $\widehat{b}$?

Average pricing errors:

$$\text{sample mean } \underbrace{\text{(observed price - predicted price)}}_{\text{should be close to zero}} = \alpha$$

$$p_t = \mathbb{E}_t\left(m_{t+1}(b) \cdot x_{t+1}\right) = \mathbb{E}\left(m_{t+1}(b) \cdot x_{t+1} | I_t\right)$$

$$\mathbb{E}(p_t) = \mathbb{E}[\mathbb{E}_t\left(m_{t+1}(b) \cdot x_{t+1}\right)] = \mathbb{E}[m_{t+1}(b) \cdot x_{t+1}]$$

Unconditional expectation: $\mathbb{E}[m_{t+1}(b)x_{t+1} - p_t] = 0$

Equivalently using returns:

$$1 = \mathbb{E}_t\left(m_{t+1}(b)R_{t+1}\right) \Rightarrow 0 = \mathbb{E}\left(m_{t+1}(b)R_{t+1} - 1\right)$$

Generalized Methods of Moments estimation is based on the WLLN

$$WLLN : \frac{1}{N}\sum_{i=1}^{N} y_i \quad \overrightarrow{p} \quad \mathbb{E}(Y)$$

sample average consistent estimate for population moment

$$\underbrace{\frac{1}{T}\sum_{t=1}^{T} p_t - \frac{1}{T}\sum_{i=1}^{T} m_{t+1}(b)x_{t+1}}_{\alpha} \approx 0$$

GMM basic idea(first step):
choose $\hat{b}$ to minimize α^2 (squared average pricing error) among set of test assets.

The two asset, two parameter case

$$\mathbb{E}\left(m_{t+1}(\beta,\gamma)\,x^1_{t+1} - p^1_t\right) = 0$$
$$\mathbb{E}\left(m_{t+1}(\beta,\gamma)\,x^2_{t+1} - p^2_t\right) = 0$$

$$\mathbb{E}\left(m_{t+1}(\beta,\gamma)\,R^1_{t+1} - 1\right) = 0$$
$$\mathbb{E}\left(m_{t+1}(\beta,\gamma)\,R^2_{t+1} - 1\right) = 0$$

$$\frac{1}{T}\sum_{t=1}^{T} m_{t+1}(\beta,\gamma)\,R^1_{t+1} - 1 = 0$$
$$\frac{1}{T}\sum_{t=1}^{T} m_{t+1}(\beta,\gamma)\,R^2_{t+1} - 1 = 0$$

solve equations for $\beta,\ \gamma \Rightarrow \widehat{\beta},\ \widehat{\gamma} \Rightarrow$

To apply GMM data have to be generated by stationary (and ergodic) processes (not necessarily i.i.d.)

Problem: WLLN works for **stationary data**:

(Weakly) stationary process: $\{Y_t\}_{t=-\infty}^{\infty}$

$\{\ldots, y_0, y_1, \ldots, y_5, \ldots\}$

$\mathbb{E}(Y_t) = u$

$var(Y_t) = \sigma^2$

$cov(Y_t, Y_{t-j}) = \gamma_j$

Solution: $\Rightarrow$ We use:

$1 = \mathbb{E}\left(m_{t+1}(b) \cdot R_{t+1}\right)$ instead of $\mathbb{E}(p_t) = \mathbb{E}\left(m_{t+1}(b) \cdot x_{t+1}\right)$

$0 = \mathbb{E}\left(m_{t+1}(b) \cdot R_{t+1} - 1\right)$

We define the GMM residual or “pricing error“

Define GMM residual: object whose mean should be zero

$$u_{t+1}(b) = m_{t+1}(b)R_{t+1} - 1$$

$$\mathbb{E}(u_{t+1}(b)) = 0$$

$$\mathbb{E}_T[u_t(b)] = \frac{1}{T}\sum_{t=1}^{T} u_t(b) \approx 0$$

Notational convenience (Hansen’s notation, sometimes causing confusion)

$$\mathbb{E}_T(\cdot) = \frac{1}{T}\sum_{t=1}^{T}(\cdot)$$

We have more assets than unknown model parameters

For GMM parameter estimation: Select N test assets
$R_t{}^1, R_t{}^2, \cdots, R_t{}^N \quad t = 1, \cdots, T$

$$\begin{bmatrix} \mathbb{E}_T[u_t^1(b)] \\ \mathbb{E}_T[u_t^2(b)] \\ \vdots \\ \vdots \\ \mathbb{E}_T[u_t^N(b)] \end{bmatrix} = g_T(b) \quad N \times 1 \quad \text{vector}$$

If $\sharp$ assets $= \sharp$ parameters b can be chosen such that average pricing errors are zero usually $\sharp$ assets $> \sharp$ parameters.

The GMM objective function

$$\hat{b} = \underset{\{b\}}{\text{argmin}} \quad g_T'(b) \cdot I_N \cdot g_T(b) \qquad \text{first step } \textbf{GMM estimate}$$

$$= \underset{\{b\}}{\text{argmin}} \quad \left[\mathbb{E}_T[u_{t+1}^1(b)]\right]^2 + \left[\mathbb{E}_T[u_{t+1}^2(b)]\right]^2 + \ldots + \left[\mathbb{E}_T[u_{t+1}^N(b)]\right]^2$$

$\Rightarrow$ minimize sum of squared average (pricing)errors

equal weight for all test assets $1, \ldots, N$

Alternatively other weight matrix

$$\hat{b} = \underset{\{b\}}{\text{argmin}} \quad g_T'(b) \, W \, g_T(b) \quad \text{e. g. } W = \begin{bmatrix} 1 & 0 & & \\ 0 & 2 & & \\ & & 100 & \ddots \\ 0 & & & \end{bmatrix}$$

GMM estimators have desirable properties

GMM estimators consistent:
Bias and variance of estimator go to zero asymptotically $\widehat{b} \overrightarrow{p} b$

GMM estimators asymptotically normal. Required for inference:

$$var(\widehat{b}) = \begin{pmatrix} var(\widehat{b}_1) & \cdots & \\ cov(\widehat{b}_1, \widehat{b}_2) & var(\widehat{b}_2) & \\ \vdots & \vdots & \\ cov(\widehat{b}_1, \widehat{b}_k) & \cdots & var(\widehat{b}_k) \end{pmatrix}$$

To conduct $t-$test: $\frac{\widehat{b}_k}{\widehat{\sigma} k} \overset{a}{\sim} N(0,1)$

There exists an optimal weighting matrix

Optimal weighting matrix
(and GMM parameter standard errors): use consistent estimate $\widehat{S}$ of S in minimization:

$$\hat{b} = \underset{\{b\}}{argmin} \quad g_T(b)' \, \widehat{S}^{-1} \, g_T(b)$$

write $u_t(b) = \begin{pmatrix} u_t^1(b) \\ \vdots \\ u_t^N(b) \end{pmatrix} \qquad \left(u_t^i(b) = m_{t+1}(b) x_{t+1}^i - p_t^i \right)$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad i{=}assets$

Recall: $\mathbb{E}(u_t^i) = 0 \quad \Rightarrow \quad \mathbb{E}(u_t(b)) = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$

The optimal weighing matrix takes into account variances and covariances of pricing errors across assets

$$S = \mathbb{E}\left[u_t(b) \cdot u_t'(b)\right] = \begin{bmatrix} \mathbb{E}\left([u_t^1(b)]^2\right) \cdots & & \\ & \ddots & \\ \mathbb{E}\left[u_t^1(b) u_t^2(b)\right] & & \\ \vdots & & \mathbb{E}\left([u_t^N(b)]^2\right) \end{bmatrix}$$

$S =$ variance covariance matrix of pricing errors

$$= \begin{bmatrix} var\left(u_t^1(b)\right) \cdots & & \\ cov\left(u_t^1(b) u_t^2(b)\right) & var\left(u_t^2(b)\right) \ddots & \\ \vdots & & \\ & & var\left(u_t^N(b)\right) \end{bmatrix}$$

Estimate $\widehat{S}$: Replace $\mathbb{E}$ by $\frac{1}{N}\sum$ using $\widehat{b}$ obtained with weighting matrix $I_N \Rightarrow \widehat{S}$.

Steps of GMM estimation

1) $\widehat{b}^1 = \underset{\{b\}}{argmin} \quad g_T(b)' \, I_N \, g_T(b) \Rightarrow$

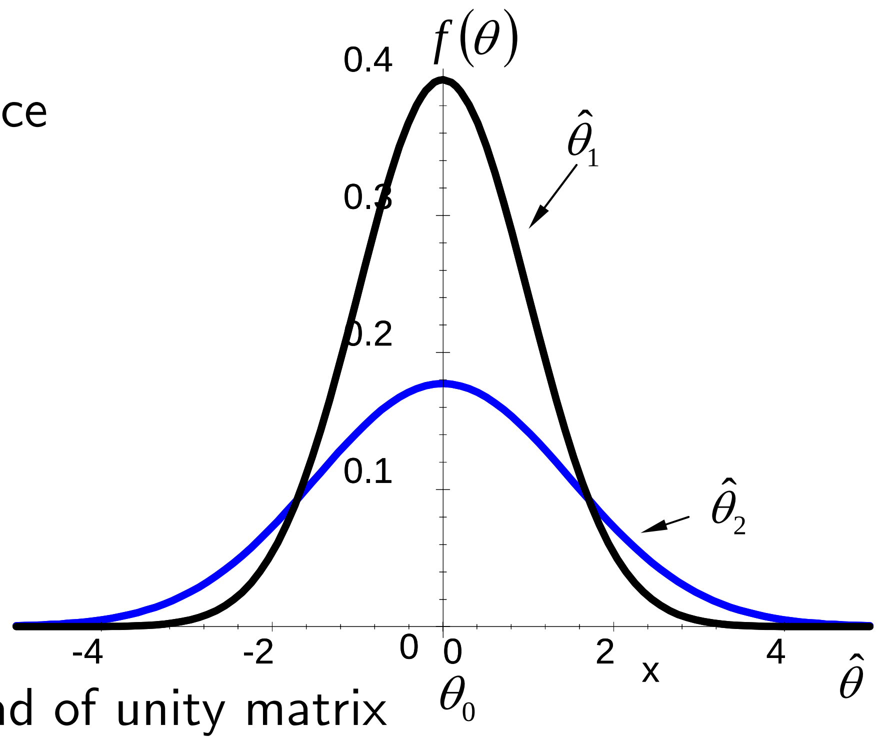
2) $\widehat{S} \Rightarrow$

3) $\widehat{b}^2 = \underset{\{b\}}{argmin} \quad \mathsf{g}_T(b)' \widehat{S}^{-1} \, g_T(b)$

...repeat... ...

Another look at the optimal weighting matrix

Efficiency: Smallest asymptotic variance among GMM esimators



Efficient estimator: employ S^{-1} instead of unity matrix

$$S = \mathbb{E}\left[u_t(b) \cdot u_t'(b)\right] \text{ resp. } \sum_{j=-\infty}^{\infty} \mathbb{E}\left[u_t(b) \cdot u_{t-j}'(b)\right]$$

variance-covariance matrix of moments conditions!

when no serial correlation in moment conditions

with serial correlation in moment conditions

Some intuition behind optimal weighting matrix (1)

Intuition behind GMM weighting matrix

Example

$N = 2$, $cov(u_t^1(b),\, u_t^2(b)) = 0$ [zero covariance of pricing errors]

$$S = \begin{bmatrix} var[u_t^1(b)] & 0 \\ 0 & var[u_t^2(b)] \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} \frac{1}{var[u_t^1(b)]} & 0 \\ 0 & \frac{1}{var[u_t^2(b)]} \end{bmatrix} = \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix}$$

Example $S = \begin{pmatrix} 10 & 0 \\ 0 & 0.1 \end{pmatrix}$

Some intuition behind optimal weighting matrix (2)

GMM objective $g_T(b)' S^{-1} g_T(b)$ becomes

$$\underset{\{b\}}{argmin}\ \mathbb{E}_T \left[u_t^1(b)\right]^2 \cdot W_1 + \mathbb{E}_T \left[u^2(b)\right]^2 \cdot W_2$$

Example
$W_1:\ 0.1 \Rightarrow var\left(u_t^1(b)\right) = 10$
$W_2:\ 10 \Rightarrow var\left(u_t^2(b)\right) = 0.1$

$\Rightarrow$ Asset (1) gets less weight in minimization
"Model imprecise" for asset 1, more precise for asset 2.

Some more intuition behind optimal weighting matrix: Correlations across pricing errors (1)

Another example: Correlations between asset returns: Two "similar" assets (high correlation of pricing errors) are downweighted. Count more like **one** asset.

Example $S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.999 \\ 0 & 0.999 & 1 \end{pmatrix}$ $\quad cov(u_t^2, u_t^3) = 0.999$

$$corr(u_t^2, u_t^3) \approx 1 = \frac{0.999}{\sqrt{1}\sqrt{1}}$$

$$\underset{\{b\}}{argmin} \left[\mathbb{E}_T(u_t^1(b)),\ \mathbb{E}_T(u_t^2(b)), \mathbb{E}_T(u_t^3(b)) \right] \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.99 \\ 0 & 0.99 & 1 \end{bmatrix}^{-1} \times$$

$$\begin{bmatrix} \mathbb{E}_T(u_t^1(b)) \\ \mathbb{E}_T(u_t^2(b)) \\ \mathbb{E}_T(u_t^3(b)) \end{bmatrix}$$

Some more intuition behind optimal weighting matrix: Correlations across pricing errors (2)

$$S^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 500.25 & -499.75 \\ 0 & -499.75 & 500.25 \end{bmatrix}$$

$$\underset{\{b\}}{argmin}\; g_T(b)'\, S^{-1} g_T(b) =$$

$$\Big[\mathbb{E}_T\left(u_t^1(b)\right), \mathbb{E}_T\left(u_t^2(b)\right)\cdot 500.25 - \mathbb{E}_T\left(u_t^3(b)\right)\cdot 499.75,$$

$$\mathbb{E}_T\left(u_t^3(b)\right)\cdot 500.75 - \mathbb{E}_T\left(u_t^2(b)\right)\cdot 499.75\Big] \times \begin{bmatrix} \mathbb{E}_T(u_t^1(b)) \\ \mathbb{E}_T(u_t^2(b)) \\ \mathbb{E}_T(u_t^3(b)) \end{bmatrix}$$

Some more intuition behind optimal weighting matrix: Correlations of pricing errors (3)

$$\underset{\{b\}}{argmin}\ g_T(b)'\, S^{-1} g_T(b) =$$

$$\mathbb{E}_T\left(u_t^1(b)\right)^2 + \mathbb{E}_T\left(u_t^2(b)\right)^2 \cdot 500.25 + \mathbb{E}_T\left(u_t^3(b)\right)^2 \cdot 500.25 -$$
$$2 \cdot \mathbb{E}_T\left(u_t^2(b)\right)\mathbb{E}_T\left(u_t^3(b)\right) \cdot 499.75$$

$$\approx \mathbb{E}_T\left(u_t^1(b)\right)^2 + 0.5\,\mathbb{E}_T\left(u_t^2(b)\right)^2 + 0.5\,\mathbb{E}_T\left(u_t^3(b)\right)^2$$

since

$$\mathrm{E}_T\left(u_t^2(b)\right) \approx \mathbb{E}_T\left(u_t^3(b)\right)$$

To test hypotheses about our models we need the distribution of the GMM estimates

Standard errors of GMM estimates

We want:

$$var(\widehat{b}) = \begin{pmatrix} var(\widehat{b}_1) & cov(\widehat{b}_1, \widehat{b}_2) \cdots & cov(\widehat{b}_1, \widehat{b}_k) \\ cov(\widehat{b}_1, \widehat{b}_2) & var(\widehat{b}_2) & \\ & & \ddots \\ cov(\widehat{b}_1, \widehat{b}_k) & \cdots & var(\widehat{b}_k) \\ & & \end{pmatrix} (K \times K)$$

$$b = (b_0, b_1, \cdots, b_k)$$

$t = \frac{\widehat{b}_k - 0}{\sqrt{var(\widehat{b}_k)}} \overset{a}{\sim} N(0,1)$ under $H_0: \ b_k = 0$

The central limit theorem plus an application of the delta method gives the asymptotic variance covariance matrix of estimated parameters

Application of Delta-Method

C.L.T. + delta method gives:

$$\sqrt{T} \cdot (\hat{b} - b) \overset{a}{\backsim} N\left(0, (d'S^{-1}d)^{-1}\right)$$

$$\underbrace{\widehat{var}(\hat{b})}_{\text{asymptotic VC matrix}} = \frac{1}{T}(d'S^{-1}d)^{-1} \quad d = \frac{\partial g_T(b)}{\partial b}\Big|_{\hat{b}}$$

(Note: asymptotic variances $T \to \infty$)

Some details of the asymptotic variance covariance matrix (1)

Some more details:

a) In application: replace S^{-1} by consistent estimate $\widehat{S}^{-1}$

b) Recall

$$g_T(b) = \begin{bmatrix} \frac{1}{T}\sum u_t^1(b) \\ \vdots \\ \frac{1}{T}\sum u_t^N(b) \end{bmatrix} = \begin{bmatrix} \frac{1}{T}\sum m_t(b)R_t^1 - 1 \\ \vdots \\ \frac{1}{T}\sum m_t(b)R_t^N - 1 \end{bmatrix}$$

$$\frac{\partial g_T(b)}{\partial b} = \underset{[N \times k]}{\begin{bmatrix} \frac{1}{T}\sum \frac{\partial u_t^1(b)}{\partial b_1} \; \frac{1}{T}\sum \frac{\partial u_t^1(b)}{\partial b_2} \cdots \frac{1}{T}\sum \frac{\partial u_t^1(b)}{\partial b_k} \\ \vdots \\ \frac{1}{T}\sum \frac{\partial u_t^N(b)}{\partial b_1} \; \frac{1}{T}\sum \frac{\partial u_t^N(b)}{\partial b_2} \cdots \frac{1}{T}\sum \frac{\partial u_t^N(b)}{\partial b_k} \end{bmatrix}}$$

Some details of the asymptotic variance covariance matrix (2)

$$\frac{\partial g_T(b)}{\partial b} = \begin{bmatrix} \frac{1}{T}\sum_{t=1}^{T} & \frac{\partial m_t(b)}{\partial b_1} R_t, \frac{\cdots}{\partial b_2} \cdots \\ \downarrow & \longrightarrow \\ & \text{Parameters} \\ N & \end{bmatrix}$$

For power utility

$$m_{t+1}(b) = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}$$

$$b = \beta, \gamma$$

Linear factor models $m_{t+1} = b' f_{t+1} \quad b \neq 0$?

Risk factor?

$$\frac{\partial m_{t+1}(b)}{\partial b_1} = \text{?}$$

We employ the estimated variance covariance matrix to test hypotheses about the model

$var(\widehat{b})$ used for testing hypotheses:

$H_0: \quad b_k = 0$

t-statistic: $\frac{\widehat{b}_k - 0}{\sqrt{var(\widehat{b}_k)}} \overset{a}{\sim} N(0,1) \widehat{=}$ Standard t-test.

joint significance:

$$H_0 : \underbrace{(b_{j1} = b_{j2} = \quad = b_{jN}}_{\text{some subset of } b} = \ 0) \text{ or } \underset{J\times 1}{b_J} = 0$$

$$\widehat{b}_j' \left[\underbrace{var(\widehat{b})_J}_{\text{appropriate subset of } var(\widehat{b})} \right]^{-1} \widehat{b}_j \overset{a}{\sim} \chi^2(J) \Bigg] \widehat{=} \text{ Standard } F\text{-test}$$

One can test the validity of the model (the moment conditions) using the J-test

$\{R_t, \Delta c_t, \ldots\}$ data is a random sample $\Rightarrow \widehat{b}$ is a random variable $\Rightarrow$

$u_t(b)$ is a random variable $\Rightarrow \mathbb{E}_T\left(u_t(b)\right) = \frac{1}{N}\sum\cdots$ is a random variable

pricing errors too large to be explained by random sampling?

$\Leftrightarrow$ Is the model in correct?

is a random vector

$$T \cdot J_T = T \cdot \underbrace{\left[g_T(\widehat{b})' \, \widehat{S}^{-1} \, g_T(\widehat{b})\right]}_{\text{objective function at minimum}} \overset{a}{\sim} \chi^2\left(\begin{array}{l}\text{no. moment conditions}\\ \text{- no. of parameters.}\end{array}\right)$$

is a random variable, too

$\Rightarrow$ Reject or accept model (resp. moment conditions) at given specificance level

Example: no. of moment conditions: 10, no. parameters: 2,

$TJ_T = 7.9,$

Some important remarks

Inference is different if other weighting matrix than optimal weighting matrix is used

- different formula for parameter standard errors

- different formula for J-statistic

When comparing alternative models (e.g. parameter restrictions) use the same weighting matrix (weighting matrix depends on unknown parameters)

Performance comparison (1)

Problems using J-statistic

Popular measure

Compare observed average return with $\mathbb{E}(R)$ predicted by model

From

$$1 = \mathbb{E}(mR)$$

$$1 = \mathbb{E}(m)\mathbb{E}(R) + cov(m, R)$$

$$\mathbb{E}(R) = \frac{1}{\mathbb{E}(m)} - \frac{cov(m,R)}{\mathbb{E}(m)}$$

Use as predictor

$$\widehat{\mathbb{E}(R)} = \frac{1}{\frac{1}{T}\sum_{t=1}^{T} m_t} - \frac{\frac{1}{T}\sum_{t=1}^{T} m_t R_t - \frac{1}{T}\sum_{t=1}^{T} m_t \frac{1}{T}\sum_{t=1}^{T} R_t}{\frac{1}{T}\sum_{t=1}^{T} m_t}$$

Performance comparison (2)

Plot $\widehat{\mathbb{E}(R)}$ vs. $\frac{1}{T}\sum_{t=1}^{T} R_t = \bar{R}$

Similarly using excess returns as test assets

$$\text{From} \qquad 0 = \mathbb{E}(mR^e)$$

$$0 = \mathbb{E}(m)\mathbb{E}(R^e) + cov(m, R^e)$$

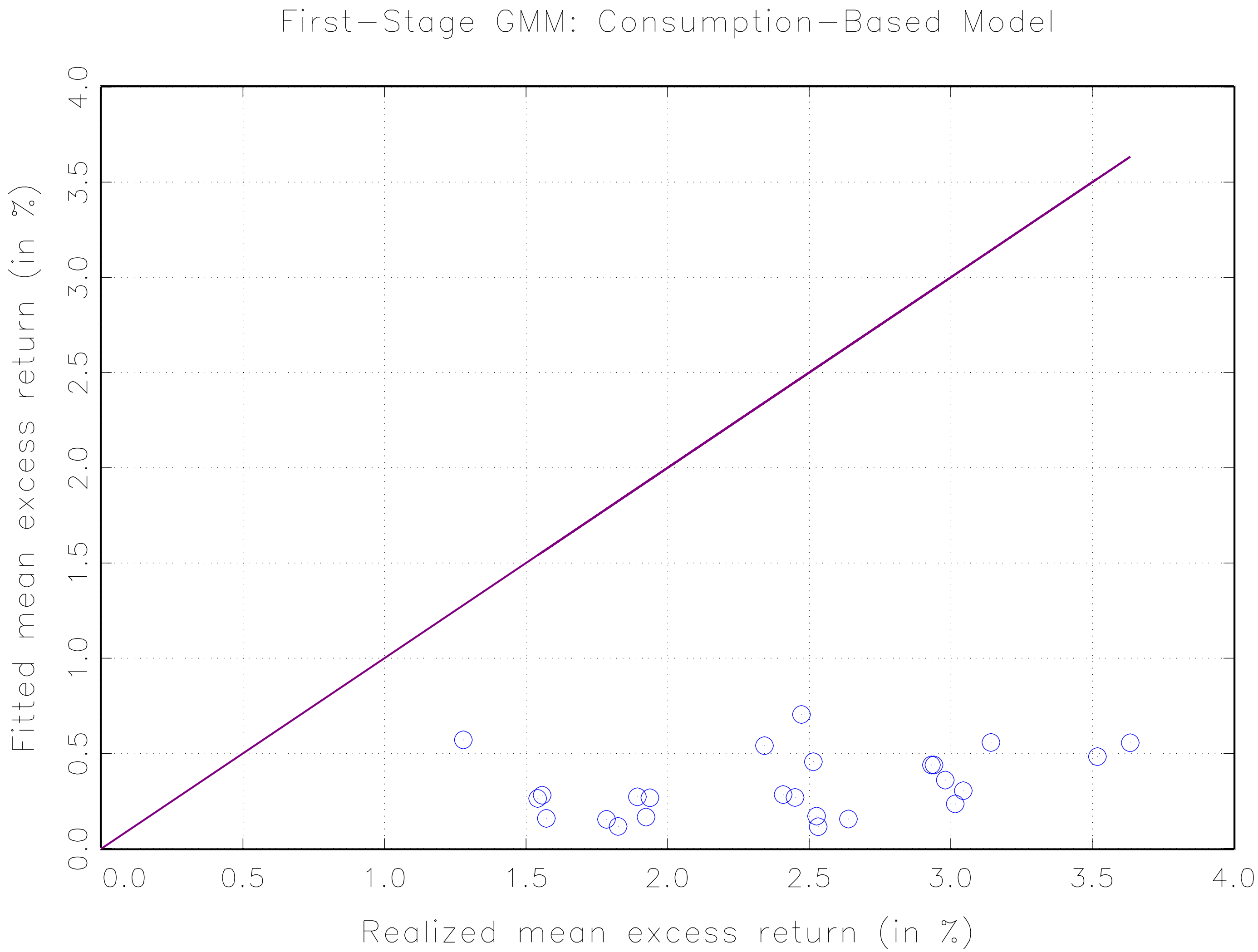
$$\mathbb{E}(R^e) = -\frac{cov(m,R^e)}{\mathbb{E}(m)}$$

Again: replace $\mathbb{E}(\cdot)$ by $\frac{1}{T}\sum(\cdot)$ to obtain $\widehat{\mathbb{E}(R^e)}$

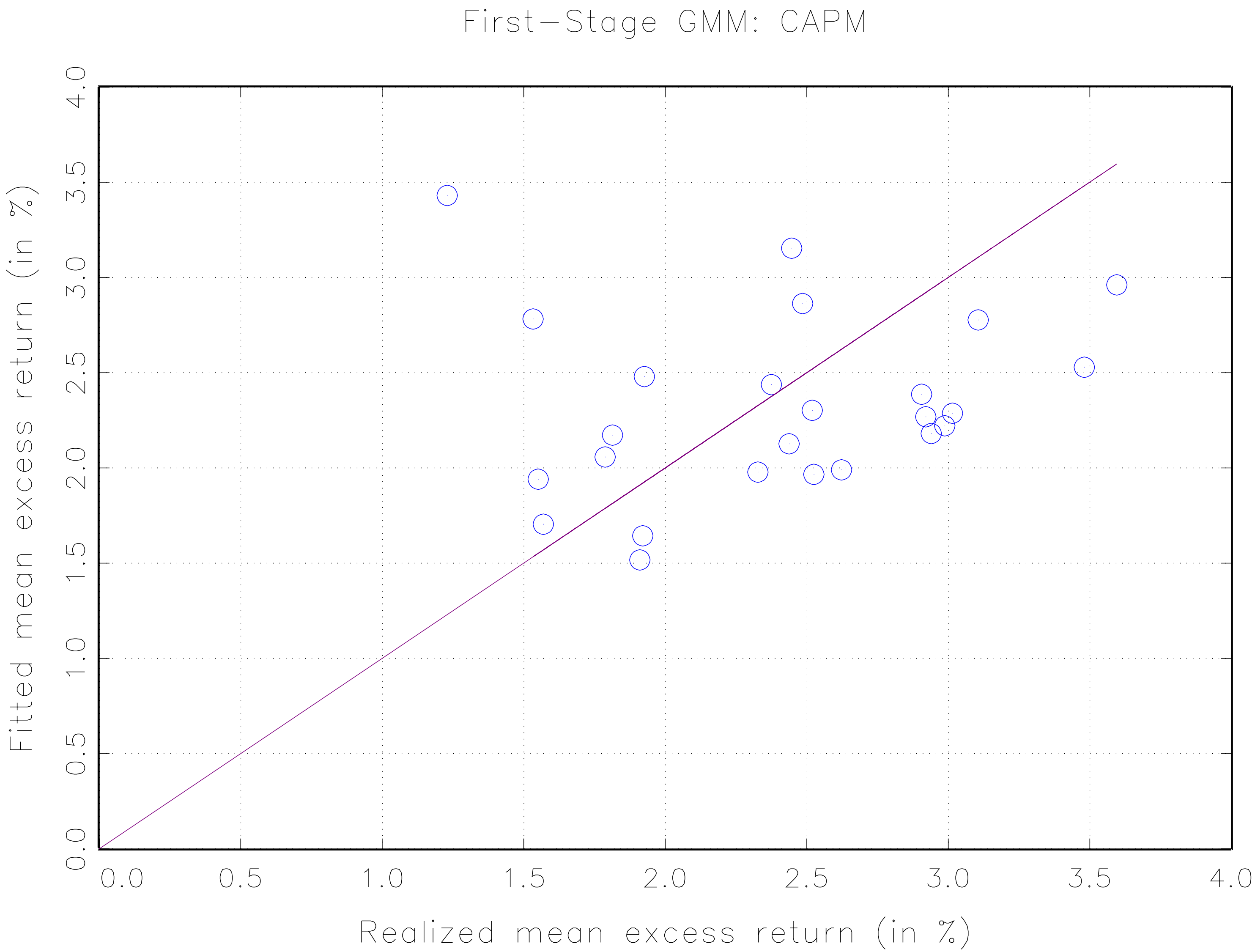
Plot $\widehat{\mathbb{E}(R^e)}$ against $\bar{R}^e$

RMSE $= \sqrt{\sum_{j=1}^{N}\left[\widehat{\mathbb{E}(R^j)} - \bar{R}^j\right]^2}$ or $= \sqrt{\sum_{j=1}^{N}\left[\widehat{\mathbb{E}(R^{ej})} - \bar{R}^{ej}\right]^2}$ used to rank and compare alternative models

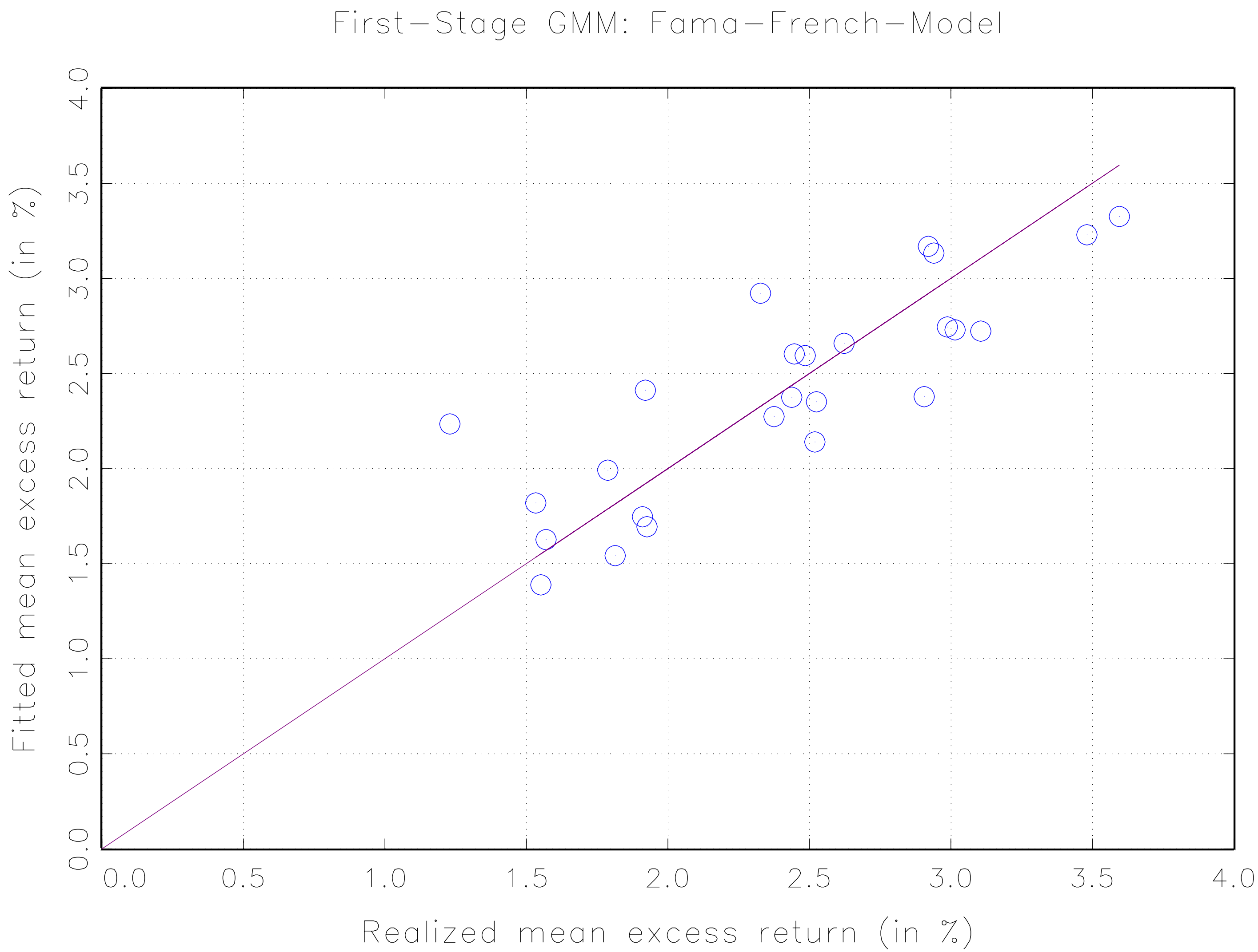
Performance comparison. Example: Consumption-Based Model estimated on 25 Fama-French portfolios



Performance comparison. Example: CAPM estimated on 25 Fama-French portfolios



Performance comparison. Example: Fama-French two factor model estimated on 25 Fama-French portfolios



GMM estimation using the Gauss library: Ingredients and recipe

1. Supply data
2. Provide GMM/optimization settings (number of iterations, weighting matrix)
3. Supply initial parameter values
4. Call GMM minimization procedure

iteratively calls procedure to compute GMM residuals $u_t(b)$

5. Check parameter estimates and test statistics

Parameter values

b

Procedure to compute GMM residuals $u_t(b)$

$u_t(b)$: object with unconditional expectation equal to zero

$$u_t(b) = \begin{pmatrix} u_1^1(b) & \cdots & u_1^N(b) \\ \vdots & \ddots & \vdots \\ u_T^1(b) & \cdots & u_T^N(b) \end{pmatrix}$$

Procedure returns $u_t(b)$:GMM residuals evaluated at b

„Global“ control variables like

model version

specification details

Data:

-Returns

-Factors

-Economic Variables

The canoncical example: Estimate the CBM by GMM

For consumption based model with power utility

$$\mathbb{E}_T(u_t(b)) = \frac{1}{T}\sum_{t=1}^{T} \beta \left(\frac{c_{t+1}}{c_t}\right)^{\gamma} \cdot R_t^i - 1 = 0$$

Exercise: 10 test assets (NYSR decile portfolios)
Perform GMM estimation of γ and β using EXCEL solver.

Input: Time series of returns and consumption growth.

$$\begin{bmatrix} R_1^1 & \cdots & R_1^{10} & R_1^f & dc_1 \\ \vdots & & \vdots & & \vdots \\ R_T^1 & & R_1^{10} & R_1^f & dc_T \end{bmatrix}$$