3rd set of assignments Financial Econometrics

1. In the linear regression model

$$Y_t = \alpha + \beta X_t + \varepsilon_t$$

with X_t a scalar random variable we assume

$$E(\varepsilon_t) = 0$$
$$E(X_t \varepsilon_t) = 0$$

Show that the moment estimator that results from the unconditional moment restrictions is identical to the least squares estimator obtained by

$$\underset{\{\hat{\alpha},\hat{\beta}\}}{argmin}\ \Sigma_{t=1}^{T}(Y_t - \alpha - \beta X_t)^2$$

2. In the linear regression model

$$Y_t = \beta_1 X_{t1} + \beta_2 X_{t2} + \cdot\cdot\ \beta X_{tK} + \varepsilon_t = \underline{\beta}' \underline{X}_t + \varepsilon_t$$
$$\underline{\beta} = (\beta_1, \beta_2, \cdot\cdot, \beta_K)'$$
$$\underline{X}_t = (X_{t1}, X_{t2}, \cdot\cdot, X_{tK})'$$

with endogenous regressors
i.e
$E(\varepsilon_t X_{t1}) \neq 0$
$E(\varepsilon_t X_{t2}) \neq 0$
.
.
$E(\varepsilon_t X_{tK}) \neq 0$
we have found K instruments $\underline{Z}_t = (Z_{t1}, \cdot\cdot, Z_{tK})'$ for which

$$E(\varepsilon_t Z_{t1}) = 0$$
$$E(\varepsilon_t Z_{t2}) = 0$$
$$.$$
$$.$$
$$E(\varepsilon_t Z_{tK}) = 0$$
$$\text{or} \quad E(\varepsilon_t \underline{Z}_t) = \underline{0}$$

Show that in this case the moment estimator (here: IV-estimator) is given by

$$\hat{\beta}_{IV} = \left[\frac{1}{T}\Sigma_{t=1}^{T}\underline{Z}_t \underline{X}_t'\right]^{-1} \frac{1}{T}\Sigma_{t=1}^{T}\underline{Z}_t Y_t$$

Hint: Proceed as in the OLS case with orthogonal regressors (see lecture) by defining matrices and vectors $\underline{Y}, \underline{e}$ and $\underline{X}$ (and $\underline{Z}$)

so that you can write $g_T(\hat{b}) = \underline{0}$ as $\underline{Z}'\underline{e} = 0$ or $\underline{Z}'(\underline{Y} - \underline{X}\hat{\beta}) = \underline{0}$

Recall: In the lecture we used

$$\underline{X}'\underline{e} = 0$$
$$\underline{X}'(\underline{Y} - \underline{X}\,\hat{\beta}) = 0$$

3. The CAPM assumes $m_{t+1} = a + \tilde{b}\, R^m_{t+1}$
Write for this case $E\Big(u_t(b, \underline{X}_t)\Big) = 0$
What is b? What is $\underline{X}_t$? What is $u_t(b, \underline{X}_t)$?
Derive a moment estimator for a and $\tilde{b}$. Use two asset returns R^a_{t+1} and R^b_{t+1} for which we have

$$E_t\Big((a + \tilde{b}\, R^m_{t+1})\, R^a_{t+1}\Big) = 1$$
$$E_t\Big((a + \tilde{b}\, R^m_{t+1})\, R^b_{t+1}\Big) = 1$$

and proceed as described in the lecture to derive the moment estimator