## LOGIC COLLOQUIM '10

[2] \_\_\_\_\_, Systems of second order arithmetic with restricted induction I & II (Abstracts), The Journal of Symbolic Logic, vol. 41 (1976), pp. 557–559.

[3] H. JEROME KEISLER, Nonstandard arithmetic and reverse mathematics, The Bulletin of Symbolic Logic, vol. 12 (2006), no. 1, pp. 100–125.

[4] STEPHEN G. SIMPSON, *Subsystems of second order arithmetic*, Perspectives in Mathematical Logic, Springer–Verlag, Berlin, 1999.

[5] KAZUYUKI TANAKA, *The self-embedding theorem of WKL0 and a non-standard method*, *Annals of Pure and Applied Logic*, vol. 84 (1997), no. 1, pp. 41–49.

## ► DENIS I. SAVELIEV, On a question of Mycielski.

Department for Mathematical Logic, Moscow State University, 119991, GSP-1, Vorobievy Gory, Glavnoe zdanie (Main Bldg), Moscow, Russia.

*E-mail*: denissaveliev@mail.ru.

Mycielski asked [1] whether all subsets of the space  $\kappa^{cf\kappa}$  with its lexicographic order topology that are constructible from  $\kappa^{cf\kappa}$  may have the properties that are immediate analogs of the Baire and the perfect set properties, for all  $\kappa$ . He asked also [2] more generally about appropriate descriptive set theories for these spaces. We derive some consequences of Mycielski's proposal, e.g., by showing that then the universe is not constructible from any set and contains a certain portion of large cardinals. We show also that, under some assumptions on  $\kappa$ , most of concepts and results of classical descriptive set theory have immediate analogs for the space  $\kappa^{cf\kappa}$ .

[1] JAN MYCIELSKI, Axioms which imply GCH, Fundamenta Mathematiae, vol. 176 (2003), pp. 193–207.

[2] —, A personal communication, 2004.

► PETER SCHROEDER-HEISTER, An alternative implication-left schema for the sequent calculus.

Wilhelm-Schickard-Institut für Informatik, Universität Tübingen, Sand 13, 72076 Tübingen, Germany.

*E-mail*: psh@informatik.uni-tuebingen.de.

As an alternative to Gentzen's schema  $(\rightarrow L)$  for the introduction of implication on the left side of the sequent sign in the intuitionistic sequent calculus LJ we propose the schema  $(\rightarrow L)^{\circ}$ :

$$(\rightarrow \mathbf{L}) \ \frac{\Gamma \vdash A \ \Delta, B \vdash C}{\Gamma, \Delta, A \rightarrow B \vdash C} \qquad (\rightarrow \mathbf{L})^{\circ} \ \frac{\Gamma \vdash A}{\Gamma, A \rightarrow B \vdash B}$$

In the absence of cut,  $(\rightarrow L)^{\circ}$  is weaker than  $(\rightarrow L)$ . In the system based on  $(\rightarrow L)^{\circ}$ , cut is admissible except for cuts whose left premiss is the conclusion of  $(\rightarrow L)^{\circ}$ , i.e., cuts of the following restricted form:

$$(\rightarrow L)^{\circ} \frac{\vdots}{\Gamma \vdash A} \begin{array}{c} \vdots \\ A, \Delta \vdash C \end{array}$$

$$(\operatorname{cut}) \frac{\Gamma \vdash A}{\Gamma, \Delta \vdash C}$$

Using cut in this restricted form,  $(\rightarrow L)$  and  $(\rightarrow L)^{\circ}$  can be shown to be equivalent. Unlike full cut, applications of restricted cut do not compromise the subformula property and are harmless in this sense. Philosophically,  $(\rightarrow L)^{\circ}$  is motivated by the interpretation of implications as rules [1, 2] and can be viewed as a direct translation of *modus ponens* into the sequent calculus.

[1] P. SCHROEDER-HEISTER, Generalized elimination inferences, higher-level rules, and the implicationsas-rules interpretation of the sequent calculus, Advances in natural deduction (E. H. Haeusler, L. C. Pereira and V. de Paiva, editors), 2010.

[2] \_\_\_\_\_, Implications-as-rules vs. implications-as-links: An alternative implication-left schema for the sequent calculus, Journal of Philosophical Logic, in print.

316