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WIRTSCHAFTS- UND
SOZIALWISSENSCHAFTLICHE
FAKULTÄT

Chair of Statistics, Econometrics and Empirical Economics

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S414
Advanced Mathematical Methods

Exercises

WS 2022/23

Integration

Exercise 1 **Indefinite Integrals**

Find the indefinite integrals. Assume that the respective domains are given appropriately.

a) $\int x^{12} dx$ b) $\int x^{-4} dx$ c) $\int \frac{1}{x^3} dx$

d) $\int 5x^9 - 3x^7 dx$ e) $\int x \cdot \sqrt[3]{x} dx$ f) $\int 2e^{3x+1} - 1 dx$

g) $\int \frac{1}{1+x} dx$ h) $\int \frac{1}{1-x} dx$ i) $\int \frac{x+3}{x^2+x-6} dx$

j) $\int \frac{x-1}{\sqrt{x}-1} dx$ k) $\int \frac{(e^x)^2 - 1}{e^x + 1} dx$

Exercise 2 **Definite Integrals**

Compute the values of the definite integrals:

a) $\int_0^2 2 - x \, dx$ b) $\int_{-2}^0 \mid x + 1 \mid \, dx$

Exercise 3 **Integration by Parts**

Use partial integration to solve the indefinite integrals.

a) $\int x^2 \cdot \ln(x) \, dx$ b) $\int x \cdot \ln(x) \, dx$

c) $\int x^\rho \cdot \ln(x) \, dx \quad (\rho \neq -1)$ d) $\int \ln(x)^2 \, dx$

e) $\int x^2 \cdot e^x \, dx$ f) $\int \frac{x}{e^x} \, dx$

Exercise 4 **Integration by Parts**

Determine the values of the definite integrals.

a) $\int_{-1}^{1} x \ln(x+2)\, dx$ b) $\int_{0}^{2} x\, 2^x\, dx$

Exercise 5 **Integration by Substitution**

Solve the following indefinite integrals:

a) $\int \frac{x}{1+x^2}\, dx$ b) $\int \frac{1}{x \cdot \ln(x)}\, dx$

c) $\int \frac{\ln(x)^3}{x}\, dx$ d) $\int \frac{x}{\sqrt{1+x}}\, dx$

e) $\int x^5 \sqrt{(4-x^3)}\, dx$

Exercise 6 **Integration by Substitution**

Determine the values of the following definite integrals:

a) $\int_{1}^{2{,}5} (2x-1)^6\, dx$ b) $\int_{10}^{10.000} \frac{1}{x \cdot \ln(x)}\, dx$ c) $\int_{a}^{b} \frac{f'(x)}{f(x)}\, dx$

Exercise 7 **Improper Integrals**

Determine the values of the following improper integrals:

a) $\int_1^\infty \frac{1}{\sqrt[3]{x^2}}\,dx$ b) $\int_0^\infty xe^{-ax}\,dx \qquad a>0$

c) $\int_{-\infty}^{-1} \frac{3}{x^3}\,dx$ d) $\int_0^1 \frac{1}{\sqrt{x}}\,dx$

e) $\int_0^1 \ln(x)\,dx$

Exercise 8 **Multiple Integrals**

Solve the following multiple integrals:

a) $\int_0^2\int_0^1 (2x+3y+4)\;dx\,dy$ b) $\int_0^a\int_0^b (x-a)(x-b)\,dx\,dy$

c) $\int_1^3\int_1^2 \frac{x-y}{x+y}\,dx\,dy$ d) $\int_0^{1/2}\int_0^{2\pi} y^3\sin(xy^2)\,dx\,dy$

Exercise 9 **Multiple Integrals**

Find

$$I=\int_0^1\int_0^1\cdots\int_0^1 (x_1^2+x_2^2+\cdots+x_n^2)\,dx_1\,dx_2\,...\,dx_n$$

Solution Exercise 1:

a) $\frac{1}{13}x^{13} + C$

b) $-\frac{1}{3}x^{-3} + C$

c) $-\frac{1}{2x^2} + C$

d) $\frac{1}{2}x^{10} - \frac{3}{8}x^8 + C$

e) $\frac{3}{7}x^{\frac{7}{3}} + C$

f) $\frac{2}{3}e^{3x+1} - x + C$

g) $\ln(\mid 1 + x \mid) + C$

h) $-\ln(\mid 1 - x \mid) + C$

i) $\ln(\mid x - 2 \mid) + C$

j) $\frac{2}{3}x^{\frac{3}{2}} + x + C$

k) $\int e^x - 1dx = e^x - x + C$

Solution Exercise 2:

a) 2

b) 1

Solution Exercise 3:

a) $\frac{1}{3}x^3 \ln(x) - \frac{1}{9}x^3 + C$

b) $\frac{1}{2}x^2 \ln(x) - \frac{1}{4}x^2 + C$

c) $\frac{1}{\rho+1}x^{\rho+1} \ln(x) - \frac{1}{(\rho+1)^2}x^{\rho+1} + C$

d) $x(\ln(x)^2 - 2\ln(x) + 2) + C$

e) $e^x(x^2 - 2x + 2) + C$

f) $-e^{-x}(x + 1) + C$

Solution Exercise 4:

a) $2 - \frac{3}{2}\ln 3$

b) $\frac{8}{\ln 2} - \frac{3}{(\ln 2)^2}$

Solution Exercise 5:

a) $\frac{1}{2}\ln(\mid 1+x^2 \mid)+C$

b) $\ln(\mid \ln(x) \mid)+C)$

c) $\frac{1}{4}\ln(x)^4+C$

d) $\frac{2}{3}(1+x)^{\frac{3}{2}}-2\sqrt{1+x}+C$

e) $\frac{-8}{9}(4-x^3)^{\frac{3}{2}}+\frac{2}{15}(4-x^3)^{\frac{5}{2}}+C^*$

Solution Exercise 6:

a) $\frac{1}{14}(4^7-1^7)\approx 1,170.21$

b) $[\ln(\ln(x))]_{10}^{10.000}\approx 1.3863$

c) $\ln(\mid f(b) \mid)-\ln(\mid f(a) \mid)$

Solution Exercise 7:

a) does not converge!

b) $\frac{1}{a^2}$

c) $-\frac{3}{2}$

d) 2

e) -1

Solution Exercise 8:

a) 16

b) $\frac{1}{6}b^2a(3a-b)$

c) $8\ln(4)-5\ln(5)-3\ln(3)=16\ln(2)-5\ln(5)-3\ln(3)$

d) $\frac{1}{8}-\frac{1}{4\pi}$

Solution Exercise 9:

- $I=\frac{n}{3}$