I. Principles of Financial Economics

Reference:
Cochrane (2001), Ch. 1 (without 1.5)

Empirical asset pricing - Introduction (1)

Asset pricing (Valuation of financial assets)

delay of payoff } account for { risk of payoff

$\Rightarrow$ risk correction

50 years US stocks: 9% average return (real) p.a.
1% real interest rate p.a. (treasury bills)

8% premium earned for holding risk
What is the risk that is priced?

Asset pricing

normative

how should the world work?
are the prices "wrong"?
- trading opportunities?
- cost of capital
- non traded assets: "fair" price

positive

how does the world work?

Empirical asset pricing - Introduction (2)

Basic : Prices equal discounted $\underbrace{\text{expected}}_{\text{What probability measure?}}$ payoff

Absolute Asset Pricing
↓
exposure to "fundamental" macroeconomic risk

Asset priced given other asset prices (e.g. option pricing)
↑
Relative Asset Pricing

e.g. CAPM:

$$\mathbb{E}\left(R^i\right) = R^f + \beta_i \left(\underbrace{\mathbb{E}(R^m) - R^f}\right)$$

$$\beta_i = \frac{cov\left(R^i, R^m\right)}{var(R^m)}$$

Market price of risk (factor)risk premium not explained

Empirical asset pricing - Introduction (3)

Basic pricing equation $p_t = \mathbb{E}_t(m_{t+1} x_{t+1})$

- p_t → asset price at t
- m_{t+1} → stochastic discount factor (r.v.)
- x_{t+1} → payoff (r.v.)

$$m_{t+1} = f(\underbrace{\text{data , parameters}}_{\text{the model}})$$

Moment condition: $\mathbb{E}_t(m_{t+1} x_{t+1}) - p_t \;\; = 0$

use $\frac{1}{n}\sum \;\; \rightarrow \mathbb{E}()$ WLLN

Generalized Method of Moments (GMM) to estimate parameters

Empirical asset pricing - Introduction (4)

time line of discovery traditional

Portfolio theory

Mean-Variance frontier

CAPM

APT

Option pricing

contingent claims state preference

consumption-based modell

stochastic discount factor

Cochrane's approach

From an utility maximising investor's first order conditions we obtain the basic asset pricing formula (1)

Basic objective: find p_t, the present value of stream of uncertain payoff x_{t+1}

$$x_{t+1} = p_{t+1} + d_{t+1}$$

dividend

price of asset in t+1

Utility function

period utility function

$$U(c_t, c_{t+1}) = u(c_t) + \beta \mathbb{E}_t [u(c_{t+1})]$$

consumption

expected utility

subjective discount factor

consumption level without asset purchase (other income)

$$c_t = e_t - p_t \xi$$

$$c_{t+1} = e_{t+1} + x_{t+1} \xi$$

quantity of asset bought/sold

Random variables: $p_{t+1},\ d_{t+1},\ x_{t+1},\ e_{t+1},\ c_{t+1},\ u(c_{t+1})$ $\quad \mathbb{E}_t[\cdot] \triangleq \mathbb{E}[\cdot \mid \mathcal{F}_t]$

From an utility maximising investor's first order conditions we obtain the basic asset pricing formula (2)

$$\max_{(\xi)} \left[U\left(c_t, c_{t+1}\right)\right] \text{ s.t.}$$

$$c_t = e_t - p_t\xi;\; c_{t+1} = e_{t+1} + x_{t+1}\xi$$

$$\max_{(\xi)} \left\{u\left(e_t - p_t\xi\right) + \beta\mathbb{E}_t\left[u\left(e_{t+1} + x_{t+1}\xi\right)\right]\right\}$$

$$-p_t \cdot u'\left(c_t\right) + \beta \cdot \mathbb{E}_t\left[u'\left(c_{t+1}\right) \cdot x_{t+1}\right] = 0$$

utility loss if investor buys another unit of the asset

discounted expected utility increase from extra payoff

$$\begin{aligned} p_t u'\left(c_t\right) &= \mathbb{E}_t\left[\beta u'\left(c_{t+1}\right) x_{t+1}\right] \\ p_t &= \mathbb{E}_t\left[\beta \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} x_{t+1}\right] \end{aligned}$$

Investor continues to buy or sell the asset until marginal loss equals marginal gain.

No complete solution: endogenous variables

Turning off uncertainty we are in the standard two-goods case (1)

$$\max\left[u\left(c_t\right)+\beta u\left(c_{t+1}\right)\right] \text{ s.t. } c_t=e_t-p_t\cdot\xi,\ c_{t+1}=e_{t+1}+x_{t+1}\cdot\xi$$

$$\frac{\partial U\left(c_t,c_{t+1}\right)}{\partial\xi}=-p_t\cdot\frac{\partial u\left(c_t\right)}{\partial c_t}+\beta\cdot x_{t+1}\cdot\frac{\partial u\left(c_{t+1}\right)}{\partial c_{t+1}}=0$$

$$p_t\cdot u'\left(c_t\right) = x_{t+1}\cdot\beta u'\left(c_{t+1}\right)$$

$$p_t = x_{t+1}\cdot\frac{\beta u'\left(c_{t+1}\right)}{u'\left(c_t\right)}$$

marginal valuation of consumption in t+1 in terms of consumption in t →

$$-\frac{dc_t}{dc_{t+1}}=\frac{\beta\cdot u'\left(c_{t+1}\right)}{u'\left(c_t\right)}=\frac{p_t}{x_{t+1}}$$

← opportunity cost to transfer consumption from t to t+1

$$p_t u'\left(c_t\right) = \mathbb{E}_t\left[\beta u'\left(c_{t+1}\right)x_{t+1}\right]$$

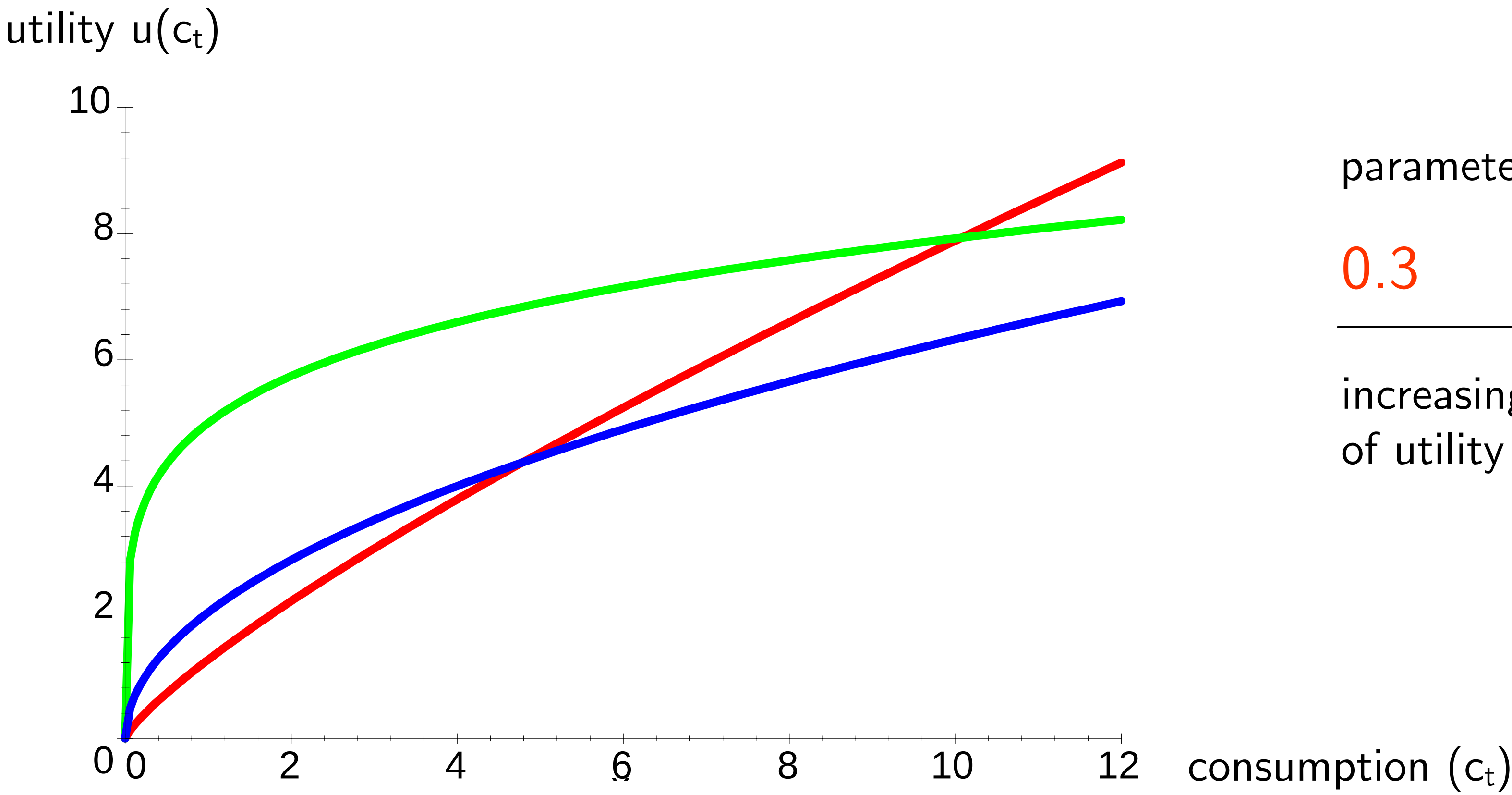
$$p_t = \mathbb{E}_t\left[\beta\frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)}x_{t+1}\right]$$

We often use a convenient power utility function (1)

$$u(c_t) = \frac{1}{1-\gamma} c_t^{1-\gamma} \qquad \lim_{\gamma \to 1} \left(\frac{1}{1-\gamma} c_t^{1-\gamma} \right) = \ln(c_t)$$

$$u'(c_t) = c_t^{-\gamma} \qquad \frac{dc_t}{dc_{t+1}} = \frac{\beta u'(c_{t+1})}{u'(c_t)} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma}$$

← negative of marginal rate of substitution



parameter γ:

0.3 0.5 0.8

increasing concavity of utility function

Prices, payoffs, excess returns

	Price p_t	Payoff x_{t+1}
stock	p_t	$p_{t+1} + d_{t+1}$
return	1	R_{t+1}
excess return	0	$R^e_{t+1} = R^a_{t+1} - R^b_{t+1}$
one \$ one period discount bond	p_t	1
risk-free rate	1	R^f

Payoff x_{t+1} divided by price $p_t \Rightarrow$ gross return $R_{t+1} = \frac{x_{t+1}}{p_t}$
Return: payoff with price one

$$1 = \mathbb{E}_t \left(m_{t+1} \cdot R_{t+1} \right)$$

Zero-cost portfolio:
Short selling one stock, investing proceeds in another stock
$\Rightarrow$excess return R^e

Example: Borrow $1\$$ at R^f, invest it in risky asset with return R.
Pay no money out of the pocket today $\longrightarrow$ get payoff $R^e = R - R^f$.

Zero price does not imply zero payoff.

The *covariance* of the payoff with the discount factor rather than its *variance* determines the risk-adjustment

$$
\begin{aligned}
cov\left(m_{t+1}, x_{t+1}\right) &= \mathbb{E}\left(m_{t+1} \cdot x_{t+1}\right) - \mathbb{E}\left(m_{t+1}\right)\mathbb{E}\left(x_{t+1}\right) \\
p_t &= \mathbb{E}\left(m_{t+1} \cdot x_{t+1}\right) \\
&= \mathbb{E}\left(m_{t+1}\right)\mathbb{E}\left(x_{t+1}\right) + cov\left(m_{t+1}, x_{t+1}\right)
\end{aligned}
$$

$$
\begin{aligned}
R^f &= \frac{1}{\mathbb{E}\left(m_{t+1}\right)} \\
p_t &= \frac{\mathbb{E}\left(x_{t+1}\right)}{R^f} + cov\left(m_{t+1}, x_{t+1}\right)
\end{aligned}
$$

$$
\begin{aligned}
p_t &= \frac{\mathbb{E}\left(x_{t+1}\right)}{R^f} + cov\left(\beta\frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)}, x_{t+1}\right) \\
p_t &= \underbrace{\frac{\mathbb{E}\left(x_{t+1}\right)}{R^f}}_{\text{price in risk-neutral world}} + \underbrace{\beta\frac{cov\left(u'\left(c_{t+1}\right), x_{t+1}\right)}{u'\left(c_t\right)}}_{\text{risk adjustment}}
\end{aligned}
$$

Marginal utility declines as consumption rises.

Price is lowered if payoff covaries positively with consumption. (makes consumption stream more volatile)

Price is increased if payoff covaries negatively with consumption. (smoothens consumption) Insurance !

Investor does not care about volatility of an individual asset, if he can keep a steady consumption.

All assets have an expected return equal to the risk-free rate, plus risk adjustment

$$
\begin{aligned}
1 &= \mathbb{E}\left(m_{t+1} \cdot R^i_{t+1}\right) \\
1 &= \mathbb{E}\left(m_{t+1}\right)\mathbb{E}\left(R^i_{t+1}\right) + cov\left(m_{t+1}, R^i_{t+1}\right) \\
R^f &= \frac{1}{\mathbb{E}\left(m_{t+1}\right)};\ 1 - \frac{1}{R^f}\mathbb{E}\left(R^i_{t+1}\right) = cov\left(m_{t+1}, R^i_{t+1}\right. \\
\mathbb{E}\left(R^i_{t+1}\right) - R^f &= -R^f \cdot cov\left(m_{t+1}, R^i_{t+1}\right) \\
\mathbb{E}\left(R^i_{t+1}\right) - R^f &= -\frac{1}{\mathbb{E}\left(\beta\frac{u'(c_{t+1})}{u'(c_t)}\right)} \cdot cov\left(\beta\frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)}, R^i_{t+1}\right)
\end{aligned}
$$

$$
\overbrace{\mathbb{E}\left(R^i_{t+1}\right) - R^f}^{\text{excess return}} = -\frac{cov\left(u'\left(c_{t+1}\right), R^i_{t+1}\right)}{\mathbb{E}\left(u'\left(c_{t+1}\right)\right)}
$$

Investors demand higher excess returns for assets that covary positively with consumption.
Investors may accept expected returns below the risk-free rate. Insurance !

The basic pricing equation has an expected return-beta representation

$$\mathbb{E}\left(R^i_{t+1}\right) - R^f = -R^f \cdot cov\left(R^i_{t+1}, m_{t+1}\right)$$

$$\mathbb{E}\left(R^i_{t+1}\right) - R^f = -\frac{cov\left(R^i_{t+1}, m_{t+1}\right)}{Var\left(m_{t+1}\right)} \frac{Var\left(m_{t+1}\right)}{\mathbb{E}\left(m_{t+1}\right)}$$

$$\mathbb{E}\left(R^i_{t+1}\right) = R^f - \left(\frac{cov\left(R^i_{t+1}, m_{t+1}\right)}{Var\left(m_{t+1}\right)}\right) \cdot \left(\frac{Var\left(m_{t+1}\right)}{\mathbb{E}\left(m_{t+1}\right)}\right)$$

asset specific quantity of risk

price of risk for all assets

Beta-pricing model:

$$\mathbb{E}\left(R^i\right) = R^f + \beta_{R^i,m} \cdot \lambda_m$$

With $m = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}$ and lognormal consumption growth $\frac{c_{t+1}}{c_t}$

$$\mathbb{E}\left(R^i\right) = R^f + \beta_{R^i,\Delta c} \cdot \lambda_{\Delta c}$$

$$\lambda_{\Delta c} \approx \gamma \cdot Var\left(\Delta \ln c\right)$$

The more risk averse the investors or the riskier the environment, the larger the expected return premium for risky (high-beta) assets.

Marginal utility weighted prices follow martingales (1)

Basic first order condition:

$$p_t u'(c_t) = \mathbb{E}_t \left(\beta \left(u'(c_{t+1}) \right) (\overbrace{p_{t+1} + d_t}^{\mathsf{x_{t+1}}}) \right)$$

Market efficiency $\Leftrightarrow$ Prices follow martingales (random walks)? **NO!**

Required:
- Risk neutral investors u'( )=const. or no variation in consumption
- $\beta = 1$ $\Leftarrow$ OK short time horizon
- no dividends

Then:

$$\begin{aligned} p_t &= \mathbb{E}(p_{t+1}) \\ p_{t+1} &= p_t + \varepsilon_{t+1} \\ \text{if} \quad \sigma^2(\varepsilon_{t+1}) &= \sigma^2 \end{aligned}$$

= Random Walk

$\Rightarrow$ Returns are not predictable $\mathbb{E}\left(\frac{p_{t+1}}{p_t}\right) = 1$

Marginal utility weighted prices follow martingales (2)

With risk aversion (but no dividends) and β=1

$$\tilde{p}_t = \mathbb{E}(\tilde{p}_{t+1})$$

$$\tilde{p}_t = p_t \cdot u'(c_t)$$

Scale prices by marginal utility, correct for dividends and apply risk neutral valuation formulas

Predictability in the short horizon?

consumption
risk aversion } does not change day by day

⇒ Random Walks successful ⇒ Predictability of asset returns (day by day)?

Technical analysis, media reports...

Some popular linear factor models

Factor pricing models

return on wealth portfolio

CAPM : $m_{t+1} = a + bR^w_{t+1}$

Free parameters

Compatible with utility maximisation ?

ICAPM : $m_{t+1} = a + b' f_{t+1}$

parameter vector

factors

factors (macro, term spread, price-earnings ratio help forecast conditional distribution of future asset returns)

APT : similar, but factors determined by principal component analysis of payoff covariance matrix

Practice : just test $m = b' f$ and don't worry about derivations