Solution to the *4th* set of assignment Financial Econometrics

1. GMM inference

$$\frac{\partial g_T(b)}{\partial b'} = \begin{bmatrix} \frac{1}{T}\sum\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} R^a_{t+1} & \frac{1}{T}\sum -\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\ln\left(\frac{c_{t+1}}{c_t}\right) R^a_{t+1} \\ \frac{1}{T}\sum\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} R^b_{t+1} & \frac{1}{T}\sum -\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\ln\left(\frac{c_{t+1}}{c_t}\right) R^b_{t+1} \end{bmatrix}$$

$$\begin{aligned} Var(\hat{\beta}_{GMM}) &= 0.05 \\ Var(\hat{\gamma}_{GMM}) &= 0.1 \end{aligned}$$

critical value : $z = 1.96 \quad \text{with} \quad z \sim N(0,1)$

$$P(-1.96 \leq z \leq 1.96) = 0.975$$

$t_1 : -0.8944$

$\rightarrow$ we can not reject the null hypothesis: $\beta = 1$

$t_2 : 0.3162$

$\rightarrow$ we can not reject the null hypothesis: $\gamma = 0$

2. Application of the δ-method

$$\begin{aligned} Var(\hat{\theta}) &= 0.02 \\ Var(\hat{\phi}) &= 0.03 \\ \hat{r} &= 0.4 \\ \frac{\partial a(b)}{\partial \phi} &= \frac{\theta}{(\phi+\theta)^2} \\ \frac{\partial a(b)}{\partial \theta} &= -\frac{\phi}{(\phi+\theta)^2} \\ A(\hat{b}) &= \left(\frac{\hat{\theta}}{(\hat{\phi}+\hat{\theta})^2}, -\frac{\hat{\phi}}{(\hat{\phi}+\hat{\theta})^2}\right) = (0.6, -0.4) \\ A(\hat{b})\left(\frac{1}{100}\widehat{\Sigma}\right)A(\hat{b})' &= 0.0110 \\ t &= -0.9517 \end{aligned}$$

$\rightarrow$ we can not reject the null hypothesis: $r = 0.5$