

# Financial Econometrics

Second set of assignments:

1. The Law of Total Expectations (also referred to as Law of Iterated Expectations)

states that

$$\left. \begin{array}{l} \text{a) } \mathbb{E}[\mathbb{E}(X|Y)] = \mathbb{E}(X) \\ \text{and} \\ \text{b) } \mathbb{E}[\mathbb{E}(X|Y, Z)] = \mathbb{E}(X) \\ \text{and} \\ \text{c) } \mathbb{E}[\mathbb{E}(X|Y, Z)|Z] = \mathbb{E}(X|Z) \end{array} \right\} \begin{array}{l} \text{Law of Total Expectations} \\ \\ \text{Law of Iterated Expectations} \end{array}$$

Show (derive) these results for  $X, Y, Z$  continuous random variables with joint density  $f_{XYZ}(x, y, z)$ .

Hints:

$$\begin{aligned} f_{X|Y,Z}(X|Y, Z) &= \frac{f_{XYZ}(x, y, z)}{f_{YZ}} \quad (\text{conditional density of } X|Y, Z) \\ \mathbb{E}(X|Y, Z) &= \int_{-\infty}^{\infty} x f_{X|Y,Z}(x|y, z) dx \quad (\text{conditional expectation}) \\ \mathbb{E}(g(X, Y)|X) &= \int_{-\infty}^{\infty} g(x, y) \cdot \frac{f_{XY}(x, y)}{f_X(x)} dy \end{aligned}$$

2. What does the ergodic theorem state ?
3. What does the multivariate central limit theorem (CLT) for i.i.d. (independent, identically distributed) samples state ?
4. Do we require data to be generated by i.i.d. processes to apply a CLT ?

Hints for 2.-4.: See Hayashi, Econometrics (2000), Princeton: pp. 88-107

5. Describe the fundamental differences between the two theories of scientific discovery referred to as "inductivism" and "critical rationalism".

6. Apply the law of total expectations to

$$\begin{aligned} p_t &= \mathbb{E}(m_{t+1}x_{t+1}|F_t) && \text{payoffs} \\ 1 &= \mathbb{E}(m_{t+1}R_{t+1}|F_t) && \text{returns} \\ 0 &= \mathbb{E}(m_{t+1}R_{t+1}^e|F_t) && \text{excess returns} \quad . \end{aligned}$$

7. Why is it necessary to perform an "unconditioning" of the pricing equation

$$p_t = \mathbb{E}_t(m_{t+1}x_{t+1}) \text{ when we want to estimate the unknown parameters by GMM ?}$$

8. Why do we prefer to base the GMM estimation of the basic asset pricing equation on  $1 = \mathbb{E}_t(m_{t+1}R_{t+1})$  or  $0 = \mathbb{E}_t(m_{t+1}R_{t+1}^e)$  instead of  $p_t = \mathbb{E}_t(m_{t+1}x_{t+1})$ ?