7th set of assignments Financial Econometrics

Choose one of the following alternatives to estimate an asset pricing model where the stochastic discount factor is a linear function of consumption growth:

$$m_{t+1} = b_0 + b_1 \cdot \Delta c_{t+1}$$

- as a dependent variable, use the excess return of our ten test assets (substract `avustret` from each of the ten asset returns `decile1` to `decile10`)
- for each of the alternatives, use the variable `cnsqdifferenz` as a factor

1. Alternative 1: Use standard GMM techniques in an EViews `System` environment to estimate the model. Write down the classical moment conditions according to the basic pricing equation
$$E(mR^{ei}) = 0$$
Proceed as in the 5th assignment sheet!

2. Alternative 2: Use the two stage regression approach discussed in Cochrane, chapter 12.2. Therefore, you have to conduct time series regression first to estimate the β_i (see assignment sheet 6 for details). Then, compute the average excess return of your test assets $E_T(R^{ei})$ and regress them on the estimated β_i in order to get an OLS estimate for λ. Compute the standard error for $\hat{\lambda}$ as follows:
$$Var(\hat{\lambda}) = \frac{1}{T}\left[(\hat{\beta}'\hat{\beta})^{-1}\hat{\beta}'\hat{\Sigma}\hat{\beta}(\hat{\beta}'\hat{\beta})^{-1}(1+\hat{\lambda}'\hat{\Sigma}_f^{-1}\hat{\lambda}) + \hat{\Sigma}_f\right]$$
where
$$\begin{aligned}\hat{\beta} &= (\hat{\beta}_1, \cdots, \hat{\beta}_N)' \\ \hat{\lambda} &= (\hat{\lambda}_1, \cdots, \hat{\lambda}_K)' \\ \hat{\Sigma} &= \text{VC-matrix of the first stage regression residuals} \\ &\quad\ \text{(Note: differs slightly from the lecture)} \\ \hat{\Sigma}_f &= \text{VC-matrix of the factors}\end{aligned}$$
Hints, how to proceed: First, conduct a time series regression in a `Pool` object. Save your $\hat{\beta}_i$ coefficients in a vector. Collect the average excess return of each asset i in a vector. Estimate λ by computing the OLS estimator in matrix notation:
$$\hat{\lambda} = (\hat{\beta}'\hat{\beta})^{-1}\hat{\beta}'E_T(R^e)$$
Having saved the residuals of the first stage time series regression and computed their VC-matrix as well as the VC-matrix of the factors (here, in the one factor case this is just a variance) you have all the ingredients to calculate the variance of $\hat{\lambda}$. In order to test if all the pricing errors $\hat{\alpha}$ are zero, compute the test statistic
$$\hat{\alpha}'cov(\hat{\alpha})^{-1}\hat{\alpha} \sim \chi^2_{N-1}$$
where
$$cov(\hat{\alpha}) = \frac{1}{T}(I_N - \hat{\beta}(\hat{\beta}'\hat{\beta})^{-1}\hat{\beta}')\hat{\Sigma}(I_N - \hat{\beta}(\hat{\beta}'\hat{\beta})^{-1}\hat{\beta}') \times (1+\hat{\lambda}'\hat{\Sigma}_f^{-1}\hat{\lambda})$$
and
$$\hat{\alpha} = (\hat{\alpha}_1, \cdots, \hat{\alpha}_N) \ \text{ with } \ \hat{\alpha}_i = R^{ei} - \hat{\beta}_i\hat{\lambda}$$

3. Alternative 3: Estimate simultaneously all the β_i and λ in a GMM framework using the `System` object and formulating the moment conditions as follows:

$$g_T(\beta, \lambda) = \begin{bmatrix} E_T[R^{e1} - a - \beta_1 f_t] \\ E_T[(R^{e1} - a - \beta_1 f_t) f_t] \\ \vdots \\ E_T[R^{eN} - a - \beta_N f_t] \\ E_T[(R^{eN} - a - \beta_N f_t) f_t] \\ E_T[R^{e1} - \lambda \beta_1] \\ \vdots \\ E_T[R^{eN} - \lambda \beta_N] \end{bmatrix}$$

Now, in order to test if λ is equal to zero you can refer to the usual GMM test statistics delivered by EViews. The same is true for testing if the model is correctly specified. A usual J-test is applicable here.