

Advanced Econometrics - 1st assignment sheet

Task 1

Show that these functions are Kernels:

$$K(\psi) = (2\pi)^{-1/2} \exp(-0.5\psi^2) \quad (1)$$

$$K(\psi) = \mathbf{1}\{|\psi| \leq 1\} 3/4 \cdot \{1 - \psi^2\} \quad (2)$$

$$K(\psi) = \mathbf{1}\{|\psi| \leq 1/2\} \quad (3)$$

Task 2

Show that the relation

$$\int_{-\infty}^{+\infty} \psi^k K(\psi) d\psi = 0 \quad (4)$$

holds, for $k = 1, 3, 5, \dots$

Task 3

Proof the following Collary:

Corollary 1 *Let assumptions DE I-IV hold. Then*

$$\hat{f}_x(x_0) \xrightarrow{p} f_x(x_0) \quad (5)$$

$$plim \hat{f}_x(x_0) \rightarrow f_x(x_0) \quad (6)$$

Task 4

Show that the following theorem holds:

Theorem 1 *Under ADE I', ADE II, ADE III' and ADE IV we obtain*

$$E[\hat{f}_x(x_0)] = f_x(x_0) + \frac{h^r}{r} \mu_r \partial_x^r f_x(x_0) + o(h^r) \quad (7)$$

$$\text{Var}[\hat{f}_x(x_0)] = (nh)^{-1} \kappa_0 f_x(x_0) + o((nh)^{-1}) \quad (8)$$