Financial Econometrics: Basic assignments for course module "Empirical Asset Pricing"

First batch of assignments

1. Read Cochrane Ch. 1

2. Show that the two period model where

$$\max_{\{\xi\}} \quad u(c_t, c_{t+1}) = u(c_t) + \beta\mathbb{E}[u(c_{t+1})]$$

$$\text{s.t.} \quad c_t = e_t - \xi p_t$$
$$c_{t+1} = e_{t+1} + \xi x_{t+1} \qquad (x_{t+1} = p_{t+1} + d_{t+1})$$

and the multiperiod model where the investor maximises

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j})$$

$$\text{s.t.} \quad c_t = e_t - \xi p_t$$
$$c_{t+1} = e_{t+1} + \xi d_{t+1}$$
$$c_{t+2} = e_{t+2} + \xi d_{t+2}$$
$$\vdots$$

i.e. the investor can buy a dividend stream $\{d_{t+j}\}$ at price p_t
yield the same basic pricing equation

$$p_t = \mathbb{E}_t \left(\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right)$$

Cochrane (2005) p. 24 only sketches the derivation you need to fill the gaps!

3. Solve problem 1b in Cochrane (2005) p. 31!

4. Take the two period problem under uncertainty

$$u(c_t, c_{t+1}) = u(c_t) + \beta\mathbb{E}(u(c_{t+1}))$$
$$\text{assume} \quad u(c_t) = \frac{1}{1-\gamma} \ c_t^{1-\gamma}$$

In $t+1$ the economy can take only three states. The "recession" state occurs with probability p_1, the "normal" state with probability p_2 and the "boom" state with

1

probability p_3.

In the recession state $c_{t+1} = c_1$ and the payoff of an asset is $x_{t+1} = x_1$. In the normal state we have $c_{t+1} = c_2$ and $x_{t+1} = x_2$ and in the boom state we have $c_{t+1} = c_3$ and $x_{t+1} = x_3$.

Derive the fundamental pricing equation in this special case:

$$p_t = \sum_{i=1}^{3} \beta \left(\frac{c_i}{c_t} \right)^{-\gamma} x_i \cdot p_i$$

5.

State	Probability	Payoff x_{t+1}	$\frac{c_{t+1}}{c_t}$
1	0.1	100	1.02
2	0.3	200	0.97
3	0.2	300	1.03
4	0.3	10	0.92
5	0.1	600	1.05

Assume the basic two period model and

$$u(c) = \frac{1}{1 - \gamma} c^{1-\gamma}$$

and $\beta = 0.95$ and $\gamma = 0.8$

5a) Compute the value of the (shadow) risk free rate R^f_{t+1}

5b) Compute the expected payoff of the asset $\mathbb{E}(x_{t+1})$

5c) Compute the price of the asset p_t

5d) Compute the expected return of the asset $\mathbb{E}(R_{t+1})$

Solutions to the *first set* of assignments:

5a) $R^f_{t+1} = 1.034$

5b) $\mathbb{E}(x_{t+1}) = 193$

5c) $p_t = 181.29$

5d) $\mathbb{E}(R_{t+1}) = 1.065$

Second batch of assignments

1. The Law of Total Expectations (also referred to as Law of Iterated Expectations) states that

   a) $\mathbb{E}[\mathbb{E}(X|Y)] \quad = \mathbb{E}(X)$
      and
   b) $\mathbb{E}[\mathbb{E}(X|Y,Z)] \quad = \mathbb{E}(X)$ $\left.\right\}$ Law of Total Expectations
      and
   c) $\mathbb{E}[\mathbb{E}(X|Y,Z)|Z] \quad = \mathbb{E}(X|Z)$ $\left.\right]$ Law of Iterated Expectations

   Show (derive) these results for X, Y, Z continuous random variables with joint density $f_{XYZ}(x,y,z)$.
   Hints:

   $$f_{X|Y,Z}(X|Y,Z) \quad = \quad \frac{f_{XYZ}(x,y,z)}{f_{YZ}} \quad \text{(conditional density of } X|Y,Z)$$

   $$\mathbb{E}(X|Y,Z) \quad = \quad \int_{-\infty}^{\infty} x f_{X|Y,Z}(x|y,z) dx \quad \text{(conditional expectation)}$$

   $$\mathbb{E}(g(X,Y)|X) \quad = \quad \int_{-\infty}^{\infty} g(x,y) \cdot \frac{f_{XY}(x,y)}{f_X(x)} dy$$

2. What does the ergodic theorem state ?

3. What does the multivariate central limit theorem (CLT) for i.i.d. (independent, identically distributed) samples state ?

4. Do we require data to be generated by i.i.d. processes to apply a CLT ?

   Hints for 2.-4.: See Hayashi, Econometrics (2000), Princeton: pp. 88-107

5. void

6. Apply the law of total expectations to

   $$\begin{aligned} p_t &= \mathbb{E}(m_{t+1}x_{t+1}|F_t) &\text{payoffs} \\ 1 &= \mathbb{E}(m_{t+1}R_{t+1}|F_t) &\text{returns} \\ 0 &= \mathbb{E}(m_{t+1}R^e_{t+1}|F_t) &\text{excess returns} \quad . \end{aligned}$$

7. Why is it necessary to perform an "unconditioning" of the pricing equation $p_t = \mathbb{E}_t(m_{t+1}x_{t+1})$ when we want to estimate the unknown parameters by GMM ?

8. Why do we prefer to base the GMM estimation of the basic asset pricing equation on $1 = \mathbb{E}_t(m_{t+1}R_{t+1})$ or $0 = \mathbb{E}_t(m_{t+1}R^e_{t+1})$ instead of $p_t = \mathbb{E}_t(m_{t+1}x_{t+1})$?

Third batch of assignments

1. In the linear regression model

$$Y_t = \alpha + \beta X_t + \varepsilon_t$$

with X_t a scalar random variable we assume

$$\mathbb{E}(\varepsilon_t) = 0$$
$$\mathbb{E}(X_t \varepsilon_t) = 0$$

Show that the moment estimator that results from the unconditional moment restrictions is identical to the least squares estimator obtained by

$$\underset{\{\hat{\alpha}, \hat{\beta}\}}{argmin} \; \Sigma_{t=1}^{T} (Y_t - \alpha - \beta X_t)^2$$

2. In the linear regression model

$$Y_t = \beta_1 X_{t1} + \beta_2 X_{t2} + \cdots \beta X_{tK} + \varepsilon_t = \underline{\beta}' \underline{X}_t + \varepsilon_t$$
$$\underline{\beta} = (\beta_1, \beta_2, \cdots, \beta_K)'$$
$$\underline{X}_t = (X_{t1}, X_{t2}, \cdots, X_{tK})'$$

with <u>endogenous</u> regressors
i.e
$$\mathbb{E}(\varepsilon_t X_{t1}) \neq 0$$
$$\mathbb{E}(\varepsilon_t X_{t2}) \neq 0$$
$$.$$
$$.$$
$$\mathbb{E}(\varepsilon_t X_{tK}) \neq 0$$

we have found K instruments $\underline{Z}_t = (Z_{t1}, \cdots, Z_{tK})'$ for which

$$\mathbb{E}(\varepsilon_t Z_{t1}) = 0$$
$$\mathbb{E}(\varepsilon_t Z_{t2}) = 0$$
$$.$$
$$.$$
$$\mathbb{E}(\varepsilon_t Z_{tK}) = 0$$
$$\text{or} \quad \mathbb{E}(\varepsilon_t \underline{Z}_t) = \underline{0}$$

Show that in this case the moment estimator (here: IV-estimator) is given by

4

$$\hat{\beta}_{IV} = \left[\frac{1}{T}\Sigma_{t=1}^{T}\underline{Z}_t\underline{X}_t'\right]^{-1}\frac{1}{T}\Sigma_{t=1}^{T}\underline{Z}_t Y_t$$

Hint: Proceed as in the OLS case with orthogonal regressors (see lecture) by defining matrices and vectors $\underline{Y}, \underline{e}$ and $\underline{X}$ (and $\underline{Z}$)

so that you can write $g_T(\hat{b}) = \underline{0}$ as $\underline{Z}'\underline{e} = 0$ or $\underline{Z}'(\underline{Y} - \underline{X}\hat{\beta}) = \underline{0}$

Recall: In the lecture we used

$$\underline{X}'\underline{e} = 0$$
$$\underline{X}'(\underline{Y} - \underline{X}\hat{\beta}) = 0$$

3. The CAPM assumes $m_{t+1} = a + \tilde{b}\,R_{t+1}^m$

Write for this case $\mathbb{E}\left(u_t(b, \underline{X}_t)\right) = 0$

What is b? What is $\underline{X}_t$? What is $u_t(b, \underline{X}_t)$?

Derive a moment estimator for a and $\tilde{b}$. Use two asset returns R_{t+1}^a and R_{t+1}^b for which we have

$$\mathbb{E}_t\left((a + \tilde{b}\,R_{t+1}^m)\,R_{t+1}^a\right) = 1$$

$$\mathbb{E}_t\left((a + \tilde{b}\,R_{t+1}^m)\,R_{t+1}^b\right) = 1$$

and proceed as described in the lecture to derive the moment estimator

Solutions to the *third set* of assignments:

3.

$$\text{for convenience}: R_{t+1}^a = R^a, R_{t+1}^b = R^b, R_{t+1}^m = R^m$$

$$a = \frac{\mathbb{E}_T(R^m R^b) - \mathbb{E}_T(R^m R^a)}{\mathbb{E}_T(R^m R^b)\mathbb{E}_T(R^a) -_T (R^m R^a)\mathbb{E}_T(R^b)}$$

$$\tilde{b} = \frac{\mathbb{E}_T(R^a) - \mathbb{E}_T(R^b)}{\mathbb{E}_T(R^m R^b)\mathbb{E}_T(R^a) - \mathbb{E}_T(R^m R^a)\mathbb{E}_T(R^b)}$$

$$\left[\text{NOTE:} \quad \mathbb{E}_T = \frac{1}{T}\sum_{t=1}^{T}\right]$$

Fourth batch of assignments

1. GMM inference

For the GMM estimator $\hat{b}_{GMM}$ resulting from

$$\underset{\{\hat{b}\}}{argmin} \; g_T(\hat{b})'Wg_T(\hat{b})$$

We have $\hat{b}_{GMM} \xrightarrow[p]{} b$ and

$$\sqrt{T}(\hat{b}_{GMM} - b) \xrightarrow[d]{} N(0, Avar(\hat{b}_{GMM}))$$

Where $Avar(\hat{b}_{GMM})$ denotes the asymptotic variance covariance matrix.
In a finite sample we use the approximation

$$\hat{b}_{GMM} \overset{a}{\sim} N(b, \frac{Avar(\hat{b}_{GMM})}{T})$$

to test hypotheses about b.

We have $Avar(\hat{b}_{GMM}) = (d'wd)^{-1}d'wSwd(d'wd)^{-1}$.

To compute $Avar(\hat{b}_{GMM})$ you need to write

$$d = \frac{\partial g_T(b)}{\partial b'} \qquad .$$

$g_T(b)$ is a vector valued function, i.e. it returns, for a given parameter vector $b = (b_1, b_2, ..., b_k)'$, the vector of sample moments:

$$\begin{pmatrix} \mathbb{E}_T(u_t^1(b)) \\ \vdots \\ \mathbb{E}_T(u_t^N(b)) \end{pmatrix} = \begin{pmatrix} \frac{1}{T}\sum_{t=1}^{T} u_t^1(b) \\ \vdots \\ \frac{1}{T}\sum_{t=1}^{T} u_t^N(b) \end{pmatrix}$$

$d = \frac{\partial g_T(b)}{\partial b'}$ is then

$$\begin{pmatrix} \frac{\partial \mathbb{E}_t(u_t^1(b))}{\partial b_1} & \cdots & \frac{\partial \mathbb{E}_t(u_t^1(b))}{\partial b_k} \\ \vdots & \ddots & \vdots \\ \frac{\partial \mathbb{E}_t(u_t^N(b))}{\partial b_1} & \cdots & \frac{\partial \mathbb{E}_t(u_t^N(b))}{\partial b_k} \end{pmatrix} N \times K$$

Write d <u>in detail</u> for the GMM estimation framework of the consumption based model where

$$m_{t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma}.$$

Use two moment restrictions for two asset returns R_{t+1}^a and R_{t+1}^b:

$$\mathbb{E}(m_{t+1} R_{t+1}^a - 1) = 0$$

$$\mathbb{E}(m_{t+1} R_{t+1}^b - 1) = 0$$

What is b?
What is $u_t(b)$?
What is $\mathbb{E}_t(u_t(b))$ and $g_T(b)$?
What is $\frac{\partial g_T(b)}{\partial b'}$?

Write all in greatest detail!

You have succeeded in computing a consistent estimate of $Avar(\hat{b}_{GMM})$ for your GMM application.

$$\widehat{Avar(\hat{b}_{GMM})} = \begin{pmatrix} 5 & 0.3 \\ 0.3 & 10 \end{pmatrix}$$

You have used $T = 100$ observations. Your GMM estimates are given by

$$\hat{\beta}_{GMM} = 0.8 \quad \hat{\gamma}_{GMM} = 0.1$$

Compute an estimate of $Var(\hat{\beta}_{GMM})$ and $Var(\hat{\gamma}_{GMM})$.
Test the hypotheses

$$
\begin{array}{cc}
H_0: \quad \beta = 1 & H_0: \quad \gamma = 0 \\
\text{versus} & \text{versus} \\
H_A: \quad \beta \neq 1 & H_A: \quad \gamma \neq 0
\end{array}
$$

using the t-statistics

$$t_1 : \frac{\hat{\beta}_{GMM} - 1}{\sqrt{\widehat{Var}(\hat{\beta}_{GMM})}} \qquad t_2 : \frac{\hat{\gamma}_{GMM}}{\sqrt{\widehat{Var}(\hat{\gamma}_{GMM})}}$$

t_1 and t_2 are approximately N(0,1) under the respective Null-Hypothesis.

2. Application of the δ-method

Suppose you have obtained a GMM estimator for $b = \begin{bmatrix} \theta \\ \phi \end{bmatrix}$ $i.e.$ $\hat{b} = \begin{bmatrix} \hat{\theta} \\ \hat{\phi} \end{bmatrix}$.

We have

$$\sqrt{T}(\hat{b} - b) \xrightarrow{d} N(0, \Sigma)$$

where Σ is the asymptotic variance covariance matrix.

A consistent estimate of Σ, denoted $\widehat{\Sigma}$, is given by

$$\widehat{\Sigma} = \begin{pmatrix} 2 & 0.2 \\ 0.2 & 3 \end{pmatrix}$$

The sample has $T = 100$ observations.
Provide estimates of $Var(\hat{\theta})$ and $Var(\hat{\phi})$ using this information. The GMM estimates are $\hat{\theta} = 0.6$ and $\hat{\phi} = 0.4$

You are interested in testing whether

$$r = \frac{\phi}{\phi + \theta} = 0.5$$

Construct a suitable test statistic (again, a t-statistic). For this purpose compute an estimate of the variance of $\hat{r} = \frac{\hat{\phi}}{\hat{\phi} + \hat{\theta}}$, $Var(\hat{r})$, by using the δ-method.

Hints:

$$a(b) = \frac{\phi}{\phi + \theta} = r$$

$$\hat{r} = a(\hat{b}) \xrightarrow{p} a(b)$$

$$\sqrt{T}(a(\hat{b}) - a(b)) \xrightarrow{d} N(0, A(b)\Sigma A(b)')$$

where $A(b) = \frac{\partial a(b)}{\partial b'} = \left(\frac{\partial a(b)}{\partial \phi}, \frac{\partial a(b)}{\partial \theta} \right)$

The test statistic is

$$t = \frac{\hat{r} - 0.5}{\sqrt{\widehat{Var}(\hat{r})}}$$

t is approximately N(0,1) under the Null Hypothesis that $r = 0.5$.

Solutions to the *fourth set* of assignments:

1. **GMM inference**

$$\frac{\partial g_T(b)}{\partial b'} = \begin{bmatrix} \frac{1}{T}\sum \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} R_{t+1}^a & \frac{1}{T}\sum -\beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} \ln\left(\frac{c_{t+1}}{c_t}\right) R_{t+1}^a \\ \frac{1}{T}\sum \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} R_{t+1}^b & \frac{1}{T}\sum -\beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} \ln\left(\frac{c_{t+1}}{c_t}\right) R_{t+1}^b \end{bmatrix}$$

$$
\begin{aligned}
Var(\hat{\beta}_{GMM}) &= 0.05 \\
Var(\hat{\gamma}_{GMM}) &= 0.1
\end{aligned}
$$

critical value : $\qquad z = 1.96 \qquad$ with $\qquad z \sim N(0,1)$
$$P(-1.96 \le z \le 1.96) = 0.975$$

$t_1 : -0.8944$

$\rightarrow$ we can not reject the null hypothesis: $\qquad \beta = 1$

$t_2 : 0.3162$

$\rightarrow$ we can not reject the null hypothesis: $\qquad \gamma = 0$

2. **Application of the δ-method**

$$
\begin{aligned}
Var(\hat{\theta}) &= 0.02 \\
Var(\hat{\phi}) &= 0.03 \\
\hat{r} &= 0.4 \\
\frac{\partial a(b)}{\partial \phi} &= \frac{\theta}{(\phi+\theta)^2} \\
\frac{\partial a(b)}{\partial \theta} &= -\frac{\phi}{(\phi+\theta)^2} \\
A(\hat{b}) &= \left(\frac{\hat{\theta}}{(\hat{\phi}+\hat{\theta})^2}, -\frac{\hat{\phi}}{(\hat{\phi}+\hat{\theta})^2}\right) = (0.6, -0.4) \\
A(\hat{b})\left(\frac{1}{100}\hat{\Sigma}\right)A(\hat{b})' &= 0.0110 \\
t &= -0.9517 \\
\rightarrow \qquad & \text{we can not reject the null hypothesis:} \qquad r = 0.5
\end{aligned}
$$

Fifth batch of assignments

1. Estimate a CAPM with help of the system environment in EViews. The stochastic discount factor m is specified as:

$$m = a + b(R^m - R^f)$$

Moment conditions result from:

$$E(mR - 1) = 0$$

Test for the significance of b and compute and interpret the J-statistic as well as its p-value (see hint below).

- Use the data set provided on the home page of the course
- For R^m, optionally, the series `avvwret` or `avewret` can be chosen
- R^f is the series `avustret`
- The returns R are collected in the series `decile1` to `decile10`

- <u>Hint for the J-statistic</u>: The J-statistic, EViews returns and displays has to be multiplied with the number of observations used for estimation. Computation of the J-statistic and the p-value can be conducted in the following way: Save your system and name it (e.g. as "CAPM"). Then use the commands (see User Guide, page 293):
  ```
  scalar jval=CAPM.@regobs*CAPM.@jstat
  scalar pval=1-@cchisq(jval,8)
  ```

2. Estimate a CAPM for *excess returns* with help of the system environment in EViews. The stochastic discount factor m is still specified as:

$$m = a + b(R^m - R^f)$$

Moment conditions do now result from:

$$E(mR^e) = 0$$

<u>Note: Mind the trap!!!</u>

Test for the significance of b and compute and interpret the J-statistic as well as its p-value (see hint above).

- Use the data set provided on the home page of the course

- For R^m, optionally, the series `avvwret` or `avewret` can be chosen
- R^f is the series `avustret`
- The returns R are collected in the series `decile1` to `decile10`. To compute excess returns, subtract `avustret` from the return of each decile.

3. Estimate the famous Fama/French model based on *excess returns*, i.e. using $E(mR^e) = 0$ with help of the system environment in EViews. Here, the stochastic discount factor m is formulated as a linear function of three factors:

$$m = a + b_1 f_1 + b_2 f_2 + b_3 f_3$$

where

$$
\begin{aligned}
f_1 &= (R^m - R^f) \\
f_2 &= (R^H - R^L) \\
f_3 &= (R^S - R^B)
\end{aligned}
$$

R^S denotes the return of a portfolio of *small* firms (in terms of market capitalization). R^B denotes the return of a portfolio of *big* firms. R^H denotes the return of a portfolio of firms with a *high* ratio of book value to market value. R^L denotes the return of a portfolio of firms with a *low* book to market ratio. In order to construct those portfolios, distribution deciles of the respective variable (e.g. book value/market value) are created for a set of assets. Then, portfolios are constructed according to those deciles. Typically, one uses the upper decile and the lower decile for calculating the return difference. Note, that all factors in the Fama/French model are excess returns.

Test for the significance of b and compute and interpret the J-statistic as well as its p-value (see hint above). Additionally, conduct an F-test for joint significance of b, where the null hypothesis is H_0: $b_1 = b_2 = b_3 = 0$ (see hint below)

- Use the data set provided on the home page of the course
- f_1 can be constructed from R^m (optionally, the series `avvwret` or `avewret` can be chosen) and R^f (`avustret`) as above for the CAPM
- f_2 is provided directly in `HML_r`
- f_3 is provided directly in `SMB_r`
- The returns R are collected in the series `decile1` to `decile10`. To compute excess returns, subtract `avustret` from the return of each decile as above.

- Hint for the F-test: To conduct a test for joint significance, click on `VIEW` in the system object and choose `WALD COEFFICIENT TESTS`, then enter your restrictions.

Solutions to the *fifth set* of assignments:

see Financial Econometrics Assignments_5to7_solution.pdf file

Sixth batch of assignments

1. Conduct a (pooled) time series regression: Regress the excess returns for each for all assets on the three Fama/French factors (a brief description of those factors is provided in the fifth batch of assignments): $R^{ei} = \alpha_i + \beta_{i1} f_{t1} + \beta_{i2} f_{t2} + \beta_{i3} f_{t3} + \epsilon_t^i$.

   where

$$\begin{aligned}
f_{t1} &= (R_t^m - R_t^f) \\
f_{t2} &= (R_t^H - R_t^L) \\
f_{t3} &= (R_t^S - R_t^B)
\end{aligned}$$

Test the hypothesis that all parameters α_i are jointly equal to zero ($H_0 : \alpha_1 = \alpha_2 = \ldots = \alpha_N = 0$). The asymptotic test statistic is the following:

$$T \left[1 + \left(E_T(f_t)' \hat{\Omega}^{-1} E_T(f_t) \right) \right]^{-1} \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha} \sim \chi_N^2$$

where

$$\begin{aligned}
f_t &= (f_{t1}, f_{t2}, f_{t3})' \\
\hat{\alpha} &= (\alpha_1, \alpha_2, \cdots, \alpha_N)' \\
\hat{\Omega} &= \frac{1}{T} \sum_{t=1}^{T} [f_t - E_T(f_t)][f_t - E_T(f_t)]' \\
\hat{\Sigma} &= \frac{1}{T} \sum_{t=1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_t' \\
\hat{\varepsilon}_t &= (\varepsilon_t^1, \cdots, \varepsilon_t^N) \quad \text{with} \quad \varepsilon_t^i = R^{ei} - \hat{\alpha}_i - \beta' f_t
\end{aligned}$$

- Use the data set provided on the home page of the course
- For R^m, optionally, the series `avvwret` or `avewret` can be chosen
- R^f is the series `avustret`
- The returns R are collected in the series `decile1` to `decile10`. To compute excess returns, subtract `avustret` from the return of each decile.
- f_1 can be constructed from R^m (optionally, the series `avvwret` or `avewret` can be chosen) and R^f (`avustret`) as above for the CAPM
- f_2 is provided directly in `HML_r` (**IMPORTANT**: subtract 1 from the series to obtain excess returns instead of gross returns)
- f_3 is provided directly in `SMB_r` (**IMPORTANT**: subtract 1 from the series to obtain excess returns instead of gross returns)

- Hints for the application in EViews:
  - Create a new Pool object and name it (e.g. assets).
  - Next, you are asked to provide 'cross section identifiers'. Write 1-10 in that window.
  - Now, you can estimate the regressions by specifying as the dependent variable decile? where ? is used as a wildcard for the cross section identifiers. As explanatory variables, specify the three factors and a c as intercept (deactivate the default intercept). Note, that all β_{ij} are asset specific.
  - Save the residuals via `Proc - Make Residuals` in a group and name the group (e.g. residuals)
  - Create a group with the three factors
  - Calculate the mean vector and the VC-matrix of the factors and the VC-matrix of the residuals and store them in vectors, respectively matrices (use the functions `@mean()` and `@cov()`)
  - Store the number of observations (use `scalar t=assets.@regobs`) and the α_i in a scalar, respectively vector (if the α_i are the first ten elements in the coefficient vector then write
    `vector alpha=@subextract(assets.@coefs,1,1,10,1)`)
  - Now, compute the test statistic given above (though the result is a scalar, assign the test as a matrix since matrix operations have to be conducted (`@transpose` and `@inverse`)). Further, specify the 1 in the test statistic as a 1×1-matrix.
  - Calculate the p-value of the test as in the fifth batch of assignments

Solutions to the *sixth set* of assignments:

see Financial Econometrics Assignments_5to7_solution.pdf file

Seventh batch of assignments

Choose one of the following alternatives to estimate an asset pricing model where the stochastic discount factor is a linear function of consumption growth:

$$m_{t+1} = b_0 + b_1 \cdot \Delta c_{t+1}$$

- as a dependent variable, use the excess return of our ten test assets (substract `avustret` from each of the ten asset returns `decile1` to `decile10`)

- for each of the alternatives, use the variable `cnsqdiff` as a factor

1. Alternative 1: Use standard GMM techniques in an EViews `System` environment to estimate the model. Write down the classical moment conditions according to the basic pricing equation

$$E(mR^{ei}) = 0$$

Proceed as in the 5th assignment sheet!

2. Alternative 2: Use the two stage regression approach discussed in Cochrane, chapter 12.2. Therefore, you have to conduct time series regression first to estimate the β_i (see assignment sheet 6 for details). Then, compute the average excess return of your test assets $E_T(R^{ei})$ and regress them on the estimated β_i in order to get an OLS estimate for λ. Compute the standard error for $\hat{\lambda}$ as follows:

$$Var(\hat{\lambda}) = \frac{1}{T}\left[(\hat{\beta}'\hat{\beta})^{-1}\hat{\beta}'\hat{\Sigma}\hat{\beta}(\hat{\beta}'\hat{\beta})^{-1}(1 + \hat{\lambda}'\hat{\Sigma}_f^{-1}\hat{\lambda}) + \hat{\Sigma}_f\right]$$

where

$$
\begin{aligned}
\hat{\beta} &= (\hat{\beta}_1, \cdots, \hat{\beta}_N)' \\
\hat{\lambda} &= (\hat{\lambda}_1, \cdots, \hat{\lambda}_K)' \\
\hat{\Sigma} &= \text{VC-matrix of the first stage regression residuals} \\
&\quad \text{(Note: differs slightly from the lecture)} \\
\hat{\Sigma}_f &= \text{VC-matrix of the factors}
\end{aligned}
$$

Hints, how to proceed: First, conduct a time series regression in a `Pool` object. Save your $\hat{\beta}_i$ coefficients in a vector. Collect the average excess return of each asset i in a vector. Estimate λ by computing the OLS estimator in matrix notation:

$$\hat{\lambda} = (\hat{\beta}'\hat{\beta})^{-1}\hat{\beta}'E_T(R^e)$$

Having saved the residuals of the first stage time series regression and computed their VC-matrix as well as the VC-matrix of the factors (here, in the one factor case this is just a variance) you have all the ingredients to calculate the variance of $\hat{\lambda}$. In order to test if all the pricing errors $\hat{\alpha}$ are zero, compute the test statistic

$$\hat{\alpha}' cov(\hat{\alpha})^{-1}\hat{\alpha} \sim \chi^2_{N-1}$$

where

$$cov(\hat{\alpha}) = \frac{1}{T}(I_N - \hat{\beta}(\hat{\beta}'\hat{\beta})^{-1}\hat{\beta}')\hat{\Sigma}(I_N - \hat{\beta}(\hat{\beta}'\hat{\beta})^{-1}\hat{\beta}') \times (1 + \hat{\lambda}'\hat{\Sigma}_f^{-1}\hat{\lambda})$$

and

$$\hat{\alpha} = (\hat{\alpha}_1, \cdots, \hat{\alpha}_N) \text{ with } \hat{\alpha}_i = R^{ei} - \hat{\beta}_i\hat{\lambda}$$

3. <u>Alternative 3</u>: Estimate simultaneously all the β_i and λ in a GMM framework using the `System` object and formulating the moment conditions as follows:

$$g_T(\beta, \lambda) = \begin{bmatrix} E_T[R^{e1} - a - \beta_1 f_t] \\ E_T[(R^{e1} - a - \beta_1 f_t)f_t] \\ \vdots \\ E_T[R^{eN} - a - \beta_N f_t] \\ E_T[(R^{eN} - a - \beta_N f_t)f_t] \\ E_T[R^{e1} - \lambda\beta_1] \\ \vdots \\ E_T[R^{eN} - \lambda\beta_N] \end{bmatrix}$$

Now, in order to test if λ is equal to zero you can refer to the usual GMM test statistics delivered by EViews. The same is true for testing if the model is correctly specified. A usual J-test is applicable here.

Solutions to the *seventh set* of assignments:

see Financial Econometrics Assignments_5to7_solution.pdf file