

Advanced Time Series

Gauss Exercises

1st GAUSS assignment ATS

1. Create a standard normally-distributed random variable

1. Generate a sequence of 1000 standard normal random variables (write the realizations in one column). Produce a graph of the resulting "time series". GAUSS hint: `rndn(r,c)`; plot the time series against a `seqa(1,1,T)`.
2. Compute sample means and sample variance of the time series generated in step 1. Interpret the results.
3. Generate a second series that contains the cumulated standard normal random variables generated in step 1. (i.e. the second row of the new column of this series contains the sum of the first and the second values of the series with standard normal variables et cetera). Plot the resulting sequence.
4. Compute sample means and sample variance of the second time series generated in step 2. Interpret the results.
5. Repeat the steps, i.e. let the program run another time. Compare your results. What happens to your graph?

2. Simulate a MA(1) process

1. Simulate a first order moving average (MA(1)) process that is defined as follows:
$$y_t = \theta \varepsilon_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0, 1)$$
$$T = 100.$$
 Simulate the process for different parameter values: $\theta = 0.5$, $\theta = -0.5$, $\theta = 2$, $\theta = -10$ Plot the realizations of the process in a graph.
2. Write a procedure that delivers one realization of a MA(1) process. Hint: Use as input variables θ and the number T , as output the resulting MA(1) realization.
3. Evaluate whether the requirements for weak stationarity are fulfilled. Compute ensemble means and variances for 30 realizations of your process (for every of the four parameter alternatives). Visualize the means and variances in a plot and label the axis.
4. Compute empirical autocorrelations (first order) by using the 30 realizations (Ensembles). Compute for this the correlation of y_1 and y_2 ($\text{Corr}(y_1, y_2)$ and $\text{Corr}(y_2, y_3), \dots, \text{Corr}(y_{99}, y_{100})$). Plot the resulting sequence of autocorrelations in a graph and title it.
5. Compute for one arbitrary realization of the MA(1) process the empirical autocorrelation of first order and compare this with the correlations of the ensembles. Hint: Use Gauss function `acf`.
6. Given your results of the previous questions: Evaluate if the MA(1) process is (weakly) stationary. Does the stationarity depend on the choice of the parameter θ ?

3. Simulate an AR(1) process

1. Simulate a first order AR (AR(1)) process that is defined as follows:

$$y_t = \phi y_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0, 1)$$

$T = 100$. Simulate the process for different parameter values: $\phi = 0$ (white noise) $\phi = -0.5$ $\phi = 0.5$ $\phi = 0.9$ $\phi = 1$ (random walk). Hint: You either can use the Gauss function `recserrar` or use a loop statement.

2. Plot the realizations of the process.
3. Write a procedure that delivers one realization of a AR(1) process. Hint: Use as input variables ϕ and the number T , as output the resulting AR(1) realization.
4. Analyze the stationarity properties of the AR(1) process by using simulations. Evaluate if the requirements for weak stationarity for the AR(1) process are fulfilled. Compute ensemble means and variances for 30 realizations of your process (for every of the four parameter alternatives). Visualize the means and variances in a plot and label the axis.
5. Compute empirical autocorrelations (first order) by using the 30 realizations (Ensembles). Compute for this the correlation of y_1 and y_2 ($\text{Corr}(y_1, y_2)$ and $\text{Corr}(y_2, y_3), \dots, \text{Corr}(y_{99}, y_{100})$). Plot the resulting sequence of autocorrelations in a graph and title it.
6. Compute for one arbitrary realization of the AR(1) process (for every of the four parameter alternatives) the empirical autocorrelation of first order and compare this with the correlations of the ensembles of the previous question.
7. Given your results of the previous questions: Evaluate if the AR(1) process is (weakly) stationary. Does the stationarity depend on the choice of the parameter ϕ ? For which parameter value is the AR(1) process stationary?

2nd GAUSS assignment ATS

1. Estimate a MA(1) process

1. Use the MA simulation procedure of the 1st set of assignment. Simulate an MA(1) process with $\theta = 0.5$ and $T = 200$.

$$y_t = \theta \varepsilon_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0, 1)$$

2. Write a procedure that estimates the parameters of a MA(1) process.

First build the ε_t of a MA sequence recursively. Hint: You need starting values for c , σ and θ . You need a starting value for ε . Use a loop statement and initialize a vector for the ε 's e.g. `eps = ones(rows(y), 1)`;

Second compute the value of the likelihood function by plugging in ε_t .

Now write the rest of the procedure around the procedure-body. Hint: Do not forget to define locals.

Input variables of the procedure are the data y and the parameters that are estimated. (c , σ and θ . The parameters enter the procedure as one column vector!).

Output variable is a column vector $((T-1) \times 1)$ that contains the likelihood contribution of each observation.

3. Simulate the process for different parameter values of θ , estimate θ and discuss your results.
4. Compute standard errors of the estimators! Test the null hypothesis $H_0 : \text{parameter} = 0$. Computer p-values of the test-statistic. Gauss hint: `cdfnc`
5. Simulate the process for different values of T . What do you observe? How do your results change?
6. Write the likelihood function for only two parameters σ and θ . Plot the likelihood as a function of σ and θ in a three dimensional plot. GAUSS Graph Hint: `xyz`

2. Estimate an AR(1) process

1. Use the AR simulation procedure of the 1st set of assignment. Simulate an AR(1) process with $\phi = 0.5$ and $T = 200$.

$$y_t = \phi y_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0, 1)$$

2. Write a procedure that estimates the parameters of an AR(1) process.

First build the ε_t of a AR sequence. Hint: You need a starting value for y . Hint: You need starting values for c , σ and ϕ .

Second compute the value of the likelihood function by plugging in ε_t .

Now write the rest of the procedure around the procedure-body. Hint: Do not forget to define locals.

For estimation do not use the starting values of y and you use one observation by taking one lag of the series. GAUSS hint: `lagn`

Input variables of the procedure are the data y and the parameters that are estimated. (c , σ and ϕ . The parameters enter the procedure as one column vector!).

Output variable is a column vector $((T-1) \times 1)$ that contains the likelihood contribution of each observation.

3. Simulate the process for different parameter values of ϕ , estimate ϕ and discuss your results. (Note: $|\phi| < 1!!!$)
4. Compute standard errors of the estimators! Test the null hypothesis $H_0 : parameter = 0$. Computer p-values of the test-statistic. Gauss hint: `cdfnc`
5. Simulate the process for different values of T . What do you observe? How do your results change?
6. Write the likelihood function for only two parameters σ and ϕ . Plot the likelihood as a function of σ and ϕ in a three dimensional plot. GAUSS Graph Hint: `xyz`

3rd GAUSS assignment ATS

Testing for Stationarity - Simulating the Dickey-Fuller Test Statistics

1. Simulate a Random Walk

1. Simulate a random walk that is defined as follows:

$$y_t = \alpha + \phi y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2)$$

$T = 100$. Simulate the process for the following parameter values: $\phi = 1$, $\alpha = 0$, and $\phi = 1$, $\alpha = 2$. Plot the realizations of the process.

2. Write a procedure that delivers one realization of the random walk process. Hint: Use as input variables ϕ , α and the number T , as output the resulting random walk realization.

2. Conduct an OLS regression

1. Estimate the equation that is defined as follows:

$$y_t = \alpha + \rho y_{t-1} + \delta t + \varepsilon_t,$$

$T = 100$. Estimate the unknown parameters. Use as y_t the random walk realizations and as y_{t-1} the lagged realizations of the random walk. First, set the constant $\alpha = 0$ and the time trend $t = 0$. Second, set just the time trend equal to zero ($t=0$). Third, estimate the whole regression equation with constant and time trend. Hint: Order the matrix that contains your regressors as follows: $x = y_{t-1} \sim \text{vector of constants} \sim \text{time trend}$.

2. Estimate the standard error of your parameter $\hat{\rho}$ and calculate the t-statistic for the null hypothesis that the true value of ρ equals 1. Collect them in one row vector.
3. Write a procedure around your OLS estimation. Hint: Use as input variables the matrix of independent variables and the vector of dependent variables, as output specify a row vector that contains your estimate for $\hat{\rho}$, the corresponding standard error and the t-statistic associated with the null hypothesis that the true value $\rho = 1$.

3. Simulation of a test statistic

1. Simulate the two procedures you calculated above $n=10,000$ times. In order to do so, write a loop around the two procedures. Collect the parameter estimates for ρ and the corresponding standard errors and t-values in a matrix. Hint: Initialize a matrix of dimension $n \times 3$. Fill in the results from the simulation.
2. Now, just concentrate on the t-values you estimated. Sort them into quantiles. Hint: Use the command `quantiled`. Compare the quantile values you calculated with those in Hamilton p. 763. Why do they differ?

4th GAUSS assignment ATS

1. Estimate a GARCH(1,1) for a FDAX return series

1. First, load the return data into a GAUSS matrix. The first observation is on July 1st, 2002 and the last observation is December 31th, 2005 `r_fdax_is_no_30min.fmt`. The log return data is from an equity index future of the DAX traded on the Eurex. A equity future is a standardized contract between two parties agreeing to purchase/sell the financial underlying (here the DAX equity index) of the contract based on specific predetermined conditions (e.g. date, price, size). The maximum lifetime of a future contract is limited to nine months. For constructing a continuous time series the last (most frequently traded) 3 months are taken. You can load this files with the `load` command (look it up in the GAUSS Help).

2. Plot the return time series. Describe the stylized facts you observe.

A simple model to account for these stylized facts ist the GARCH model.

Mean equation (y_t is a log return time series):

$$y_t = c + \varepsilon_t$$

In the following we assume $c = 0$. Argue why this is a sensible assumption. Therefore,

$$\varepsilon_t = y_t$$

Variance equation:

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \quad (1)$$

3. Write a procedure that estimates the parameters of an GARCH(1,1) process. Hint: First read all following remarks. Then work through them step by step.

First build the h_t of GARCH(1,1) model. Hint: You need starting values for h , ω and α , β . The starting value of the h sequence is the unconditional variance of the GARCH(1,1) process, i.e. $h = \omega + (\alpha + \beta) * var_0$ with $var_0 = vcx(y - meanc(y))$; $\omega = 0.2$, $\alpha = 0.2$, and $\beta = 0.7$. $\varepsilon_t = y_t$.

Second compute the value of the log likelihood function by plugging in ε_t and the conditional variance sequence of h_t .

Now write the rest of the procedure around the procedure-body. Hint: Do not forget to define locals. Input variables of the procedure are the data y and the parameters that are estimated. (ω , α , β . The parameters enter the procedure as one column vector!)

Output variable is a column vector that contains the likelihood contribution of each observation.

4. Call the procedure and check your output.

5. In a GARCH estimation you have to ensure that the left hand side variable of Equation (1) is always positive. You have to impose restrictions on your parameters. Modify your procedure the following way: $parameter = \exp(parameter^*)$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$$

The estimated equation is then

$$h_t = \exp(\omega^*) + \exp(\alpha^*) \varepsilon_{t-1}^2 + \exp(\beta^*) h_{t-1}$$

6. Use CML to estimate the unknown parameters; ω^* , α^* , β^* . Take as starting values the log of the starting values given above. Choose as global variables:

```
.cml_Algorithm =1;
.cml_LineSearch=2;
.cml_DirTol = 1e-5;
.cml_CovPar = 2;
```

7. After Estimation. Transfer the estimated parameter back by: taking $parameter = \exp(parameter^*)$.
8. To compute standard errors of the estimators ω , α , β we need the delta method. Compute standard errors of the estimators using the delta method. Gauss hint: gradp.

First, write a procedure that returns you the nonrestricted parameters. Input variable is one column vector θ^* containing ω^* , α^* , β^* . Output variable is one column vector θ containing ω , α , β .

Second, write a procedure that returns the estimated variance covariance matrix of $\theta = \omega$, α , β . Input variables are estimated θ^* and the estimated variance covariance matrix of θ^* . You can download estimated θ^* and estimated $\text{cov}(\theta^*)$ from the course homepage.

9. Test the null hypothesis $H_0 : \beta = 0.9$. $H_A : \beta \neq 0.9$. Compute p-values of the test-statistic. Gauss hint: cdfnc

5th GAUSS assignment ATS

1. Loading Data and Data Description

1. First, load the data into a GAUSS matrix. The .txt file contains Swiss seasonal adjusted macroeconomic data: quarterly Interest Rate in % (first column), quarterly Consumer Price Index (second column), quarterly Gross Domestic Product in Swiss (BIP) in Mio CHF (third column) and quarterly Money Stock M1 in Mio CHF (fourth column). The first observation is from the first quarter of 1974 and the last observation is from the first quarter of 2002. $T = 113$. Gauss hint: `loadm all[] = data_var.txt; all2 = reshape(all,113,4);`
2. Plot the series and describe the plot. Decide whether the time series includes
 - a) a constant
 - b) a constant and a time trend
 - c) no constant and no time trend.

2. Conducting Dickey Fuller Tests

Start with the money stock series. After you finished all the following tasks repeat the procedure for the other three time series.

1. Conduct a Dickey Fuller test of non stationarity. Estimate a regression model. Generally, the regression model is written as:

$$\Delta y_t = \beta_1 + \beta_2 y_{t-1} + \beta_3 t + \varepsilon_t$$

with $\Delta y_t = y_t - y_{t-1}$, i.e. first differences of y_t , t as time trend and β_1 as constant.

Based on your observations, if you decided that the series contains a constant, include β_1 in the regression. Else $\beta_1 = 0$. Based on your observations, if you decided that the series contains a time trend, include $\beta_3 \times t$ in the regression. Else $\beta_3 = 0$. For the first parameter estimate $\hat{\beta}_2$ use the standard error to compute the t-statistic for $H_0 : \beta_2 = 0$.

2. Compare your t -value with the corresponding Dickey-Fuller distribution (either use your own simulation results (large n !) from assignment sheet 3 or use tables from Hamilton) and decide whether you can reject or not reject the null hypothesis of a unit root.
3. Compute log differences of the series, i.e. $\tilde{y}_t = \Delta \log(y_t) = \log(y_t) - \log(y_{t-1})$.
4. Plot the series \tilde{y}_t and describe the plot. Decide whether the time series includes
 - a) a constant
 - b) a constant and a time trend
 - c) no constant and no time trend.
5. Conduct a Dickey Fuller test on \tilde{y}_t . Does \tilde{y}_t contain a unit root?

6th GAUSS assignment ATS

1. Loading Data and Data Description

1. First, load the data into a GAUSS matrix. The .txt file contains Swiss seasonal adjusted macroeconomic data: quarterly Interest Rate in % (first column), quarterly Consumer Price Index (second column), quarterly Gross Domestic Product in Swiss (BIP) in Mio CHF (third column) and quarterly Money Stock M1 in Mio CHF (fourth column). The first observation is from the first quarter of 1974 and the last observation is from the first quarter of 2002. $T = 113$. Gauss hint: `loadm all[] = data_var.txt; all2 = reshape(all,113,4);`

- m = money stock M1
- r = quarterly average of three month Swiss franc LIBOR rate of interest
- p = Consumer price index
- y = GDP in 1990 Swiss francs

Create the following series: \tilde{m}_t , \tilde{r}_t , \tilde{p}_t , \tilde{y}_t

Denote: $\tilde{m}_t = \Delta \log m_t$; $\tilde{r}_t = \Delta r_t$; $\tilde{p}_t = \Delta \log p_t$; $\tilde{y}_t = \Delta \log y_t$;

($\Delta \log m_t$, Δr_t , $\Delta \log p_t$, $\Delta \log y_t$) with $\Delta \log p_t = \log p_t - \log p_{t-1}$

2. Conducting Dickey Fuller Tests

1. Proceed as described in the 5th assignment. Conduct Dickey Fuller Tests for: p_t , y_t , m_t , r_t , \tilde{p}_t , \tilde{y}_t , \tilde{m}_t , \tilde{r}_t
2. Estimating a SVAR, why would you be interested in testing the time series for non-stationarity?

3. Estimation of a structural VAR

You are interested to model the effects of Swiss Monetary policy. You consider the following Swiss variables for your analysis: the consumer price index, the GDP in 1990 Swiss francs, the money stock M1 variable, and the quarterly average of three month Swiss franc LIBOR rate of interest.

Structural Vector Autoregression (SVAR) in primitive form

Note: Ordering of the variables is important!!

$$\begin{aligned}\tilde{m}_t &= \gamma_{10} && + \gamma_{11}\tilde{m}_{t-1} + \gamma_{12}\tilde{r}_{t-1} + \gamma_{13}\tilde{p}_{t-1} + \gamma_{14}\tilde{y}_{t-1} + \varepsilon_{1t} \\ \tilde{r}_t &= \gamma_{20} - b_{21}\tilde{m}_t && + \gamma_{21}\tilde{m}_{t-1} + \gamma_{22}\tilde{r}_{t-1} + \gamma_{23}\tilde{p}_{t-1} + \gamma_{24}\tilde{y}_{t-1} + \varepsilon_{2t} \\ \tilde{p}_t &= \gamma_{30} - b_{31}\tilde{m}_t - b_{32}\tilde{r}_t && + \gamma_{31}\tilde{m}_{t-1} + \gamma_{32}\tilde{r}_{t-1} + \gamma_{33}\tilde{p}_{t-1} + \gamma_{34}\tilde{y}_{t-1} + \varepsilon_{3t} \\ \tilde{y}_t &= \gamma_{40} - b_{41}\tilde{m}_t - b_{42}\tilde{r}_t - b_{43}\tilde{p}_t && + \gamma_{41}\tilde{m}_{t-1} + \gamma_{42}\tilde{r}_{t-1} + \gamma_{43}\tilde{p}_{t-1} + \gamma_{44}\tilde{y}_{t-1} + \varepsilon_{4t}\end{aligned}$$

$$Bx_t = \Gamma_0 + \Gamma_1 x_{t-1} + \varepsilon_t$$

$$E(\varepsilon_t \varepsilon_t') = \begin{cases} D & \text{for } t = \tau \\ 0 & \text{otherwise.} \end{cases}$$

No distinction between exogenous and endogenous variables.

Writing the VAR in standard form "solves" the system

$$x_t = B^{-1}\Gamma_0 + B^{-1}\Gamma_1 x_{t-1} + B^{-1}\varepsilon_t$$

$$x_t = A_0 + A_1 x_{t-1} + e_t$$

$$\tilde{m}_t = a_{10} + a_{11}\tilde{m}_{t-1} + a_{12}\tilde{r}_{t-1} + a_{13}\tilde{p}_{t-1} + a_{14}\tilde{y}_{t-1} + e_{1t}$$

$$\tilde{r}_t = a_{20} + a_{21}\tilde{m}_{t-1} + a_{22}\tilde{r}_{t-1} + a_{23}\tilde{p}_{t-1} + a_{24}\tilde{y}_{t-1} + e_{2t}$$

$$\tilde{p}_t = a_{30} + a_{31}\tilde{m}_{t-1} + a_{32}\tilde{r}_{t-1} + a_{33}\tilde{p}_{t-1} + a_{34}\tilde{y}_{t-1} + e_{3t}$$

$$\tilde{y}_t = a_{40} + a_{41}\tilde{m}_{t-1} + a_{42}\tilde{r}_{t-1} + a_{43}\tilde{p}_{t-1} + a_{44}\tilde{y}_{t-1} + e_{4t}$$

GAUSS Hint: use var_code.src; ar = **var**(x,lags,z,nameout);

The innovations of a VAR in standard form are, by construction, contemporaneously correlated (composite innovations/shocks)

To obtain the idiosyncratic shocks (ε_t) from the composite shocks we need the structural parameters, the matrix B

covariance matrix of e_t :

$$\mathbb{E}(e_t e_t') = \Omega$$

relation between shocks in VAR and SVAR: $e_t = B^{-1}\varepsilon_t$

$$\begin{aligned} \mathbb{E}(e_t e_t') &= B^{-1} \mathbb{E}(\varepsilon_t \varepsilon_t') [B^{-1}]' \\ &= B^{-1} D [B^{-1}]' \end{aligned}$$

To identify the structural parameters B , we decompose the variance covariance matrix of composite innovations (Cholesky-Decomposition)

some maths:

for any real symmetric positive definite matrix Ω exists a lower triangular matrix A with ones along the principal diagonal and a unique diagonal matrix D with positive elements on the principal diagonal

$$\Omega = ADA'$$

i.e. $A = B^{-1}$

Ω : real symmetric positive definite matrix

A : lower triangular matrix with ones along the principal diagonal

D : diagonal matrix with positive elements

GAUSS hint: `chol` (look up in the reference!) computes $\Omega = Y'Y$; $Y = chol(\Omega)$; with $A \cdot C = Y'$

$$\Omega = ACC'A' \text{ with } C = \begin{bmatrix} \sqrt{d_1} & 0 & 0 & \dots & 0 \\ 0 & \sqrt{d_2} & 0 & \dots & 0 \\ 0 & 0 & \sqrt{d_3} & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & \sqrt{d_k} \end{bmatrix}$$

$$D = CC' = \begin{bmatrix} d_1 & 0 & 0 & \dots & 0 \\ 0 & d_2 & 0 & \dots & 0 \\ 0 & 0 & d_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & d_k \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ a_{21} & 1 & 0 & \dots & 0 \\ a_{31} & a_{32} & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ a_{k1} & a_{k2} & a_{k3} & \dots & 1 \end{bmatrix}$$

GAUSS code:

`load vcv = varcv;`

either do it by hand:

```
y = chol(vcv); /* note: y'*y = vcv*/  
y = y';
```

or use the program code: `y_inv,y = reca0(vcv,ind);`

Note: `ind` gives the ordering, check the descriptions in the code.

compute A and D from Y

```
c = (diag(y)'*ones(k,k).*eye(k));
```

```
a = y*inv(c);
```

```
d = eye(k).*diag(y)*eye(k).*diag(y);
```

7th GAUSS assignment ATS

Cointegration: Engle and Granger Method

1. Loading Data and Data Description

Load the fmt-File data_abx.fmt. The file contains data on a Canadian Stock (ABX- Barrick Gold Corporation), which is traded on the Toronto Stock Exchange and the New York Stock Exchange simultaneously. The data includes the first two hours of trading, ranging from 1st January 2004 to 31st March 2004, sampled at 5 minute intervals.

The first column of the data matrix consist of the time series of the log home market (TSX) midquotes $((bid+ask)/2)$, the second contains the log foreign market (NYSE) midquotes , the log CAD/USD exchange rate is included in the third column, and in the last column you find a dummy variable indicating the first observation of a day (i.e., indicator = 1, if it is the first observation of the day, otherwise zero)

2. Preliminary Analysis

After loading the data into Gauss, convert the NYSE midquote series into Canadian dollars (note: we are talking about log series here!).

Plot the home market and the converted foreign market midquote series into one graph. Interpret the graph. Which cointegration vector would you suggest?

Conduct a Dickey Fuller unit root test for both series. Include a constant but no time trend into the estimated regression.

(Critical value: 5% level of significance: -2,86)

3. Engle-Granger Method, Step 1: Auxiliary Regression

In order to estimate the normalized cointegration vector, run a regression of the TSX midquotes on the NYSE midquotes, back out the estimated parameter and the residual series. What is the estimated cointegration vector? What does *normalized* mean in this context?

(If you are unsure whether or not to include a constant in the regression, have look again at the graph you've made above).

Test the residual series for stationarity.

4. Engle-Granger Method, Step 2: VECM Estimation

Create return series for the midquotes (i.e. first differences). Since we want to exclude overnight returns from our estimation, the first observation of each day has to be deleted. To do so, make use of the first day indicator and the *delif* comand (Look it up in the command reference). Note: You have to apply the same selection to the residual series to keep both series at the same dimension!

Estimate the VECM using the residual series as the error correction term. (i.e. the matrix of the two return series is the dependent variable matrix, the regressor matrix consists of ones (constant), the residual series and the first lag of the return series):

$$\begin{aligned}\Delta y_{1t} &= a_{10} + \gamma_1 \hat{\varepsilon}_{t-1} + a_{11} \Delta y_{1t-1} + a_{12} \Delta y_{2t-1} + u_{1t} \\ \Delta y_{2t} &= a_{20} + \gamma_2 \hat{\varepsilon}_{t-1} + a_{21} \Delta y_{1t-1} + a_{22} \Delta y_{2t-1} + u_{2t}\end{aligned}$$

Back out the estimated parameters and interpret your results. What could be a sensible interpretation of the coefficients on the correction terms (γ) ?

8th GAUSS assignment ATS

Impulse Response and Variance Decomposition

1. Loading Data and Data Description

You are interested to model the effects of Swiss Monetary policy. The data set `data_var.txt` contains the consumer price index, the GDP in 1990 Swiss francs, the money stock M1 variable, and the quarterly average of three month Swiss franc LIBOR rate of interest. (see 6th assignment for loading data and conducting stationarity tests.)

2. Estimating Vector Autoregression Models

The GAUSS code for this assignment sheet uses a procedure collection written for VAR estimation (`var_code.src`) by Paul Fackler. We estimate a VAR model with GAUSS. In the 6th assignment you find the model written in structural and standard form.

1. First, use the procedure `var()` to estimate the VAR model. (This should be already done in the 6th assignment. Read the documentation of the procedure code to apply the correct global settings.) GAUSS hint: e.g. `ar = var(x,lags,z,nameout)`; Note: The last rows of the `ar` matrix last row contains the constant. The first k rows contain the first lag. The second k rows contain the second lag. Etc...
2. Use the procedure `reca0()` to perform Cholesky decomposition of the innovation covariance matrix. GAUSS hint: e.g. `{y_inv,y} = reca0(vcv,ind)`;

3. Produce impulse response functions

1. To compute impulse response functions you need the coefficient matrices of the VMA representation. Use the procedure `vma()` to back out the VMA parameters. GAUSS hint: e.g. `ma = vma(ar,lags,malags)`; Note: Denote `malags` as the number of lags used in the VMA. `malags` is 10. The last rows of the `ma` matrix last row contains the constant. The first k rows contain the first lag. The second k rows contain the second lag. Etc...
2. In order to examine the effect of a shock in one variable on all variables in the VAR compute impulse response functions with the procedure `impulse()`. Plot your results in a comprehensive way, i.e. plot the response of one variable to its own shock and the shock in the other three variables. GAUSS hint: e.g. `irf = impulse(ma,y_inv,outcode)`; Note: The `irf` collects in the rows the time and in the columns the IR function values. The first column corresponds to the responds of the first variable to a shock in the first variable. The second column corresponds to the responds in the second variable to a shock in the first variable. etc... The fifth column corresponds to the responds in the first variable to a shock in the second variable. The sixth column corresponds to the responds in the second variable to a shock in the second variable. etc...
3. Describe and compare patterns of impulse response functions by answering following questions: How big/small are the responses to shocks, i.e. the response of a variable to

its 'own' shock and to the shock of other variables? How long do shocks persist in the series?

4. Conduct a variance decomposition

1. Conduct a variance decomposition with the procedure `fedecomp` for each economic variable and plot your results. GAUSS hint: e.g. `varcom = fedecomp(irf,outcode);`
Note: The `varcom` collects in the rows the time and in the columns the Variance Decomposition values. The first column corresponds to percent of the variance in the first variable due to the first variable. The second column corresponds to percent of the variance in the first variable due to the second variable. etc... The fifth column corresponds to percent of the variance in the second variable due to the first variable. The sixth column corresponds to percent of the variance in the second variable due to the second variable.
2. Discuss the results and draw a conclusion from the plots of the variance decomposition. In particular analyze how a shock in the GDP affects the other variables e.g. How is the proportion of the movements in a sequence due to its 'own' shocks versus shocks to the other variables?

9th GAUSS assignment ATS

1. Theoretical Background

Using US data on consumption, investment, and output we want to estimate the parameters of a Vector Error Correction model by Maximum Likelihood. Assuming that the variables follow a VAR of order one, the VECM can be written as:

$$\Delta y_t = \zeta_1 \Delta y_{t-1} + \alpha + \zeta_0 y_{t-1} + \varepsilon_t,$$

where ζ_1 is a (3×3) parameter matrix, α a (3×1) vector of constants and

$$\zeta_0 = -BA' .$$

We know that there exist two cointegration relations between those three variables, and therefore B and A are (3×2) matrices, where B contains the adjustment coefficients and A denotes the cointegration matrix, which if normalized as proposed by Phillips (see Hamilton page 576) is of the form :

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ a_{31} & a_{32} \end{pmatrix}$$

The disturbances ε_t are Gaussian and the conditional log likelihood function can be written as:

$$\log L(\Omega, \zeta_1, \alpha, \zeta_0)$$

$$= \left(-\frac{Tn}{2}\right) \ln(2\pi) - \left(\frac{T}{2}\right) \ln |\Omega|$$

$$- \frac{1}{2} \sum_{t=1}^T [(\Delta y_t - \zeta_1 \Delta y_{t-1} - \alpha - \zeta_0 y_{t-1})' \Omega^{-1} (\Delta y_t - \zeta_1 \Delta y_{t-1} - \alpha - \zeta_0 y_{t-1})]$$

where Ω denotes the covariance matrix of the error terms, n is the number of variables and T the number of observations.

2. Data Processing

1. Load the data contained in `ciy.txt` into Gauss. The file contains quarterly US data for the period 1947Q1 - 1988Q4 for the three variables log consumption (column one), log investment (column two), and log output (column three), the number of observations for each variable is 168.
2. Generate the following four matrices: the first contains the three variables as they have been read in from the data file, the second includes the lag of the three variables, the third the first differences, and the fourth the first lag of the differences. Make sure that there are no missings in any of these matrices and that all have the same dimensions.

3. Computing the Log-Likelihood Function

We will estimate the system by the Gauss procedure `cml` and therefore we need a procedure that returns the log-likelihood contribution of each observation. Before you start, please read all points listed below and work through them step by step.

1. Compute a column vector that contains the log-likelihood contribution of each observation, i.e. compute the log-likelihood function.
2. All parameters needed for the log-likelihood will be passed into the procedure by a column vector. Consequently, we have to read out the respective elements from this column vector and reshape them into their former dimensions. The vector of parameters is assumed to be structured as follows:

$$x0 = \begin{pmatrix} \zeta_{11} \\ \zeta_{12} \\ \zeta_{13} \\ \zeta_{21} \\ \zeta_{22} \\ \zeta_{23} \\ \zeta_{31} \\ \zeta_{32} \\ \zeta_{33} \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \omega_{11} \\ \omega_{22} \\ \omega_{33} \\ \omega_{21} \\ \omega_{31} \\ \omega_{32} \\ a_{31} \\ a_{32} \\ b_{11} \\ b_{21} \\ b_{31} \\ b_{12} \\ b_{22} \\ b_{32} \end{pmatrix}$$

Read out the elements of ζ_1 and reshape them into a (3×3) matrix.

Next build a column vector that contains the constants $(\alpha_1, \alpha_2, \alpha_3)'$

Then rebuild the normalized cointegration matrix A and the matrix B by addressing the according elements in the vector.

As far as the covariance matrix Ω is concerned, we will use a little trick to make sure that the covariance matrix will always stay positive definite. We define: $\Omega = PP'$ (Cholesky decomposition). So: Build a matrix P as given below :

$$P = \begin{pmatrix} \omega_{11} & 0 & 0 \\ \omega_{21} & \omega_{22} & 0 \\ \omega_{31} & \omega_{32} & \omega_{33} \end{pmatrix}$$

and get Ω .

3. Write a procedure around the reshaping commands and the log-likelihood. Output argument is the value of the log-likelihood function .Input arguments are the parameter vector $x0$ and the data set (in our case we pass the data into the procedure via globals, but `cml` demands those two input arguments). Do not forget to define local variables in the likelihood procedure.

4. CML Estimation

1. Read in the vector contained in `x1.fmt`. It contains the starting values for x_0 .
2. Further use the following options in `cml`:

```
cmlset;  
_cml_GridSearch=1;  
_cml_algorithm=3;  
_cml_linesearch=2;  
_cml_covpar=2;  
_cml_GradMethod=1;  
_cml_DirTol=1e-4;  
_output=1;
```

Since we estimate 26 parameters it might be of help to define labels for the estimated coefficients. This can be done by:

```
_cml_parnames=  
"zeta11"|"zeta12"|"zeta13" |  
"zeta21"|"zeta22"|"zeta23" |  
"zeta31"|"zeta32"|"zeta33" |  
"const1"|"const2"|"const3" |  
"omega11"|"omega22"|"omega33" |  
"omega12"|"omega31"|"omega32" |  
"a31"|"a32" |  
"b11"|"b21"|"b31" |  
"b12"|"b22"|"b32";
```

3. Call `cml` and estimate the parameters. Interpret your results.

5. Likelihood Ratio Test

1. Estimate the unrestricted model, i.e. ζ_0 is no more equal $-BA'$, but simply a (3×3) parameter matrix. The starting values are contained in `x2.fmt` in the following order:

$$x0 = \begin{pmatrix} \zeta_{11}^1 \\ \zeta_{12}^1 \\ \zeta_{13}^1 \\ \zeta_{21}^1 \\ \zeta_{22}^1 \\ \zeta_{23}^1 \\ \zeta_{31}^1 \\ \zeta_{32}^1 \\ \zeta_{33}^1 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \omega_{11} \\ \omega_{22} \\ \omega_{33} \\ \omega_{21} \\ \omega_{31} \\ \omega_{32} \\ \zeta_{11}^0 \\ \zeta_{12}^0 \\ \zeta_{13}^0 \\ \zeta_{21}^0 \\ \zeta_{22}^0 \\ \zeta_{23}^0 \\ \zeta_{31}^0 \\ \zeta_{32}^0 \\ \zeta_{33}^0 \end{pmatrix}$$

Use the following `cml` settings:

```
cmlset;
_cml_GridSearch=1;
_cml_algorithm=1;
_cml_linesearch=2;
_cml_covpar=2;
_cml_GradMethod=1;
_cml_DirTol=1e-4;
__output=1;
```

2. Conduct a Likelihood Ratio Test :

$$LR = 2 * (L(\hat{\theta}) - L(\tilde{\theta}))$$

where $L(\hat{\theta})$ denotes the the value of the log-likelihood function at the unrestricted estimates and $L(\tilde{\theta})$ the value of the log-likelihood function at the restricted estimates (note: `cml` returns the mean log-likelihood, so it has to be multiplied . The test statistic is asymptotically $\chi^2(m)$ distributed, where m is the number of restrictions. You can calculate the critical value in Gauss by using the `cdfchi` command (look it up in the command reference)

Can we reject the restriction? Interpret your results.