Asset Pricing and Generalized Method of Moments

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Prices equal expected discounted payoff

$$p_t = E_t(\underbrace{m_{t+1}}_{\Downarrow} x_{t+1})$$

$$m_{t+1} = f(data, \underbrace{parameters}_{b})$$

Moment condition: $E(m_{t+1}x_{t+1} - p_t) = 0$

use WLLN: $\frac{1}{n}\sum \underset{p}{\rightarrow} E()$

and GMM to estimate parameters

Different models imply different stochastic discount factors m_{t+1}

Different utility functions: $m_{t+1} = \text{fct(consumption data)}$

Consumption based model, Lettau Ludvigson's scaled asset prices, Garcia Renault Semenov, Yogo, Epstein Zin

Factor pricing models: $m_{t+1} = \text{fct(factors)}$

factors: returns (capm), three portfolio returns (Fama French), macro data (icapm), termstructure data

Generalized method of moments (GMM Hansen 1982)

moment condition: $E\ g(b) = 0$

sample moments: $g_T(b) \equiv \frac{1}{T}\sum_{t=1}^{T} g(b) = E_T[g(b)]$

GMM Idea: make the sample moment as close as possible to the population moment of zero

objective function is minimized: (numerical methods)

$$\underset{\{b\}}{\text{argmin}}\ \ g'_T(b)\ W\ g_T(b) \qquad \rightarrow \qquad \hat{b}$$

W symmetric positive definite weighting matrix

Example: Estimating the parameter of the t-distribution

- ν unknown parameter
- What happens if ν gets large?

Estimating the parameter of the t-distribution by MM

unconditional second moment:

$\mu_2 \equiv E(Y^2) = \nu/(\nu-2)$

The sample second moment:

$\hat{\mu}_2 = (1/N)\sum_{i=1}^{N} y_i^2$

For large N: $\hat{\mu}_2 \underset{p}{\rightarrow} \mu_2$

$$\begin{aligned} \nu(\nu-2) &= \hat{\mu}_2 \\ \hat{\nu} &= \frac{2\cdot\hat{\mu}_2}{\hat{\mu}_2 - 1} \end{aligned}$$

Estimating the parameter of the t-distribution by GMM

unconditional second and fourth moments:

$$
\begin{aligned}
\mu_2 \equiv E(Y^2) &= \nu/(\nu-2) \\
\mu_4 \equiv E(Y^4) &= \frac{3\nu^2}{(\nu-2)(\nu-4)}
\end{aligned}
$$

The sample second and fourth moments are:

$$
\begin{aligned}
\hat{\mu}_2 &= (1/N)\sum_{i=1}^{N} y_i^2 \\
\hat{\mu}_4 &= (1/N)\sum_{i=1}^{N} y_i^4
\end{aligned}
$$

Estimating the parameter of the t-distribution

choose ν so as to be as close as possible to both moments

minimize the objective function:

$$\underset{(1\times1)}{Q(\nu)} \equiv \underset{(1\times2)}{g}' \underset{(2\times2)}{W} \underset{(2\times1)}{g}$$

where

$$g = \begin{bmatrix} \left\{\hat{\mu}_2 - \frac{\nu}{\nu-2}\right\} \\ \left\{\hat{\mu}_4 - \frac{3v^2}{(v-2)(v-4)}\right\} \end{bmatrix}$$

Overview of today's exercise

- Programming rules in GAUSS
- Create a t-distributed random variable
- Create an objective function
- Evaluate the objective function

Programming rules in GAUSS

- ALWAYS start simple!!! Start with the simple calculations! Next enrich your program step by step!

- Start in a procedure ALWAYS from the inside and then go outside!

- CHECK frequently the results of your programming! (check them in the output window.)

- ALWAYS start with small n!

Create a t-distributed random variable procedure

input: ν and number of observations n; output: created t-distributed r.v.

1. Create $\nu + 1$ normal random variables: $rnumb = rndn(n, vt + 1)$;

2. Square ν of them and sum them up! $zw = rnumb[., 2 : vt + 1]^2$; $zw = sumc(zw')$;

3. Create a t-distributed random variable: $rant = rnumb[., 1]./sqrt(zw/vt)$;

4. Build a procedure around the creation of the random variable

Create an objective function procedure

input: ν and $data$; output: objective function Q

1. Create $\hat{\mu}_2$ $(mu2)$ and $\hat{\mu}_4$ $(mu4)$.
2. Create the moment conditions: $u1 = mu2 - \nu/(\nu - 2)$;
   $u2 = mu4 - 3 * \nu^2/((\nu - 2) * (\nu - 4))$;
3. Collect the moment conditions in a vector g: $g = u1|u2$;
4. Define the objective function with the identity matrix I as weighting matrix W $W = eye(rows(g))$; $Q = g' * W * g$;

5. Build a procedure around the creation of the objective function and call it *moments*

Evaluating the objective function

input: $startv$,$endv$,$step$ and $data$;

output: values of objective function Q and corresponding estimated $\hat{\nu}$

1. Create a loop that starts with $startv$ ends with $endv$ and has step length of $step$

2. Let evaluate the moments function for all i $(= \hat{\nu})$ and store the objective function values with the corresponding $i = \hat{\nu}$ values in a vector e.g.: Q=$moments$($data$,i);

3. Build a procedure: proc gridsearch($data$,$startv$,$endv$,$step$);

Overview of today's exercise

- Theory of consumption based model (CBM)
- Programming rules in GAUSS 2
- Loading Data
- Create an objective function for the CBM
- Evaluate the objective function for different β and γ
- Plot the objective function for different β and γ in a xyz and surface plot

Basic pricing equation of the consumption-based model

The basic pricing equation is

$$\mathbb{E}_t[m_{t+1}R^i_{t+1}] = 1$$

where m_{t+1} is the stochastic discount factor (SDF)

For the consumption-based model, m_{t+1} becomes

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$$

Thus, m_{t+1} is the marginal rate of substitution between current and future consumption.

Deriving the moment conditions...

...using a power utility function

$$
\begin{aligned}
u(c_t) &= \frac{1}{1-\gamma} c_t^{1-\gamma} \\
u'(c_t) &= c_t^{-\gamma}
\end{aligned}
$$

where γ is the degree of risk aversion of the investor.

We get for the SDF

$$m_{t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma}$$

and derive the moment conditions

$$\mathbb{E}[m_{t+1}R^i_{t+1} - 1] = \mathbb{E}\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} R^i_{t+1} - 1\right] = 0$$

Objective function for GMM

$$\underset{(1\times1)}{Q}(\beta,\gamma) = \underset{(1\times10)}{g'}\underset{(10\times10)}{W}\underset{(10\times1)}{g}$$

where W is a weighting matrix (in the simplest/our case the identity matrix)

and g the sample moment conditions

$$g = \begin{bmatrix} \frac{1}{T}\sum_{t=1}^{T}\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}R_{t+1}^{1}-1\right] \\ \vdots \\ \frac{1}{T}\sum_{t=1}^{T}\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}R_{t+1}^{10}-1\right] \end{bmatrix}$$

Programming rules in GAUSS 2

- Comment your program!!!
- Use useful and sensible names for your variables and programs!
- Create your own program collection!

Loading Data

- Check if your fmt files are in your working path
- Load the data with the `load` command

  `load` consgr="consgr_1947Q2_1993Q4";

  `load` retdec="ret_dec10_1947Q2_1993Q4";
- Time period: 2nd quarter 1947 to 4th quarter 1993
- Have a look at the return deciles and the consumption growth data!
- Plot single time series if you want!

Create an objective function for the CBM

input: β, γ and $data$; output: objective function Q

1. Create a $(T \times 1)$ matrix object that contains for each t the stochastic discount factor $m = \beta * consgr^{-\gamma}$;

2. Create the pricing errors $u = m. * retdec - 1$;

3. Compute sample moment conditions by taking the sample mean of u

4. Define the objective function with the identity matrix I as weighting matrix W: $W = eye(rows(u))$; $Q = g' * W * g$;

5. Build a procedure around the creation of the objective function and call it $Q = cbm\_moments(beta, gamma)$

Evaluate the objective function for different β and γ

input: $startb$,$endb$,$stepb$,$startg$,$endg$,$stepg$ and $data$;

output: values of Q with corresponding estimated $\hat{\beta}$ and $\hat{\gamma}$

1. Create a first loop (with counting index i) that starts with $startb$ ends with $endb$ and has step length of $stepb$

2. Create a second loop (with counting index j) that is nested within the first loop starts with $startj$ ends with $endj$ and has step length of $stepj$

3. Initialize in the top of your program a matrix that collects the output: $outmat = \{\}$

4. Let evaluate the moments function for all i $(=\hat{\beta})$ and j $(=\hat{\gamma})$ and store the objective function values with the corresponding $\hat{\beta}$ and $\hat{\gamma}$ values in the matrix $outmat$ (The first column of $outmat$ contains the Q-values)

5. Build a procedure around the loops. Call it cbm_outmat;

6. Determine the estimators for β and γ:

   $selif(outmat$,minc($outmat$[.,1]) .== $outmat$[.,1]);

XYZ plot of the objective function for different β and γ

1. Choose for $startb = 0.97$,$endb$=1,$stepb$=0.01

   $startg$=239,$endg$=242,$stepg$= 1.

   How many objective function values do you get if you evaluate the objective function on a xyz plot and in a $surface$ plot?

2. Look at the matrix $outmat$. Plot the values of $outmat$ in a xyz plot.

3. choose $stepb$=0.002, $stepg$= 0.01

$Surface$ plot the objective function for different β and γ

1. Rearrange the objective function values (in the first column) in the matrix $outmat$ for a surface plot.

2. For the surface grid you need the coordinate numbers for the β- and γ-axis.

3. Your GAUSS code for the surface plot should look like this:

   $nb = (endb - startb)/stepb + 1;$

   $ng = (endg - startg)/stepg + 1;$

   $bet = seqa(startb, stepb, nb)';$

$$gam = seqa(startg, stepg, ng);$$

$$surface\_outmat = reshape(outmat[., 1], nb, ng)';$$

$$surface(bet, gam, surface\_outmat);$$

4. choose $stepb$=0.002, $stepg$= 0.01

Overview of today's exercise

- Optmum procedure in GAUSS
- GMM Toolbox
- Writing the moments procedure of the
  a) t_ν-distributed random variable **b)** CBM
- Create a GMM Toolbox estimation procedure
- Estimation of **a)** ν and **b)** β and γ using the GMM Toolbox

Optmum procedure

- Numerical optimization of a function using an algorithm
- Input: function to be minimized and starting values for parameters
- Output: vector of parameters and function value at minimum
- Convergence criteria: - number of iterations - convergence tolerance for objective function - convergence tolerance for gradient (1^{st} derivative of the obj. fct. w.r.t the parameter vector)

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Optmum Global variables I

Optmum global:`_opalgr`= scalar indicator for optimization method

`_opalgr`

= 1 SD (steepest descent - default)

= 2 BFGS (Broyden, Fletcher, Goldfarb, Shanno)

= 3 Scaled BFGS

= 4 Self-Scaling DFP (Davidon, Fletcher, Powell)

= 5 NEWTON (Newton-Raphson)

= 6 Polak-Ribiere Conjugate Gradient

Note:

5 should always be a good choice, try 2 if the algorithm breaks down

Optmum Global variables II

Optmum global: `_opgtol=0.000000001;`

`_opgtol` = scalar is a convergence tolerance for gradient of estimated coefficients.

Default = 1e-5.

When this criterion has been satisifed OPTMUM will exit the iterations.

Important!!

Some applications in asset pricing demand a small value in order to prevent convergence on a local minimum!!!!!

GMM Toolbox

- The toolbox was originally written for MATLAB and translated into GAUSS. See Mick Cliff's document for details of the GMM Toolbox.

- Note: The naming convention is a bit different. In the MATLAB version the settings are given with gmmopt.parameter while in the GAUSS version we use the convention `_gmmopt_parameter`.

- Generally, the GMM procedure in the GMM toolbox needs as input the raw matrix of moment conditions without taking sample means of the respective moment conditions.

GMM Toolbox- Global Variables -Call

- `_gmmopt_gmmit=1`; Number of GMM iterations: 1 reports only first stage estimates
- `_gmmopt_w0=''I''`;
- `_gmmopt_S=''W''`; denotes the method to estimate the S-matrix (covariance of GMM residuals)
- Call: `gmm(initial,model,matrix1,matrix2,matrix3);`

Load data

a) Use the procedure of the 1st assignment which produces a t-distributed random variable to create a data vector.

b) First, load the consumption growth data and the return data into a GAUSS matrix. Consumption growth data from 2nd quarter 1947 to 4th quarter 1993 are provided in the file `consgr_1947Q2_1993Q4.fmt`. Return data for ten portfolios (1st size decile to 10th size decile) from 2nd quarter 1947 to 4th quarter 1993 are provided in the file `ret_dec10_1947Q2_1993Q4.fmt`. You can load those files with the `load` command (look it up in the GAUSS Help).

Reminder:

Generally, the GMM procedure in the GMM toolbox needs as input the raw matrix of moment conditions without taking sample means of the respective moment conditions. (i.e. a matrix of moment conditions with dimension ($T\times$number of moment conditions) with T the number of observations)

Writing the moments procedure of the t_ν-distributed random variable

Write a procedure (modify the procedure of assignment 1) which returns the matrix

$$u = \begin{bmatrix} y_1^2 - \frac{\nu}{\nu-2} & y_1^4 - \frac{3v^2}{(v-2)(v-4)} \\ y_2^2 - \frac{\nu}{\nu-2} & y_2^4 - \frac{3v^2}{(v-2)(v-4)} \\ \vdots & \vdots \\ y_n^2 - \frac{\nu}{\nu-2} & y_n^4 - \frac{3v^2}{(v-2)(v-4)} \end{bmatrix}$$

Writing the moments procedure of the CBM

Write a procedure (modify the procedure of assignment 2) which returns the matrix

$$u = \begin{bmatrix} \beta\left(\frac{c_1}{c_0}\right)^{-\gamma} R_1^1 - 1 & \beta\left(\frac{c_1}{c_0}\right)^{-\gamma} R_1^2 - 1 & \cdots & \beta\left(\frac{c_1}{c_0}\right)^{-\gamma} R_1^{10} - 1 \\ \beta\left(\frac{c_2}{c_1}\right)^{-\gamma} R_2^1 - 1 & \beta\left(\frac{c_2}{c_1}\right)^{-\gamma} R_2^2 - 1 & \cdots & \beta\left(\frac{c_2}{c_1}\right)^{-\gamma} R_2^{10} - 1 \\ \vdots & \ddots & & \vdots \\ \beta\left(\frac{c_T}{c_{T-1}}\right)^{-\gamma} R_T^1 - 1 & \beta\left(\frac{c_T}{c_{T-1}}\right)^{-\gamma} R_T^2 - 1 & \cdots & \beta\left(\frac{c_T}{c_{T-1}}\right)^{-\gamma} R_T^{10} - 1 \end{bmatrix}$$

Estimation of unknown parameters using the GMM Toolbox

Write a GAUSS procedure containing all the global settings and the estimation procedure. The estimation procedure is called in the following way: `gmm(initial,model,matrix1,matrix2,matrix3);`
where `initial` is a column vector of initial values for your parameters, `model` is a reference to the procedure written in step 1 (e.g. if your procedure creating the moment matrix is called `cbm_moments`, then `model` would be `&cbm_moments`). For the last three arguments `matrix1` to `matrix3` assign an empty matrix and plug it in.

Call estimation procedure

a) Use the simulated data of the t-distribution and estimate with the help of the GMM Toolbox the unknown ν.

Call estimation procedure!

b) Use the consumption growth data to estimate the CBM.

Call estimation procedure!

Overview of today's exercise

- Introduction to factor models
- CAPM Fama/French Model
- Writing the moments procedure of the CAPM Model
- Estimation of the CAPM parameters
- Writing the moments procedure of the Fama/French Model
- Estimation of the Fama/French parameters

Factor Models

Motivation: CBM does not work well in practice (measurement of consumption, utility function, etc ...)

Tie the discount factor m_{t+1} to other data

Linear factor pricing models are most popular in finance

$$m_{t+1} = a + b' f_{t+1}$$

What should one use for f_{t+1}?

$$m_{t+1} = a + b' f_{t+1}$$

- Variables that indicate/measure the state of the economy
- Factor pricing models look for variables that are good proxies for aggregate marginal utility growth (Consumption is related to returns on broad based portfolios, IR, GDP growth, investment, other macroeconomic variables, return on real production processes)

- Payoffs in the economy add up to aggregate wealth and drives marginal utility
- All factor models are derived as a specialization of the CBM

Proxy Consumption Growth

- Consumption growth is the least predictable macroeconomic time series
- Proxy consumption growth means to choose variables that should not be highly predictable
- Practice: choose right units, e.g. growth rather than level, returns rather than prices, excess returns rather than returns.

CAPM

$$m_{t+1} = a + bR^m_{t+1}$$

Proxy R^m_{t+1} ("return on total wealth") by broad based stock portfolios, e.g. NYSE, S&P500

In excess returns:

$$m_{t+1} = a + b(R^m_{t+1} - R^f_{t+1})$$

Estimate the model, i.e. using $E(m_{t+1}R_{t+1}) = 1$ with the GMM toolbox in GAUSS.

Fama/French Model

Estimate the Fama/French model, i.e. using $E(mR) = 1$ with the GMM toolbox in GAUSS. The stochastic discount factor m is formulated as a linear function of three factors:

$$m_{t+1} = b_0 + b_m R^{em}_{t+1} + b_{SMB} SMB_{t+1} + b_{HML} HML_{t+1}$$

where

$$\begin{aligned} R^{em}_{t+1} &= (R^m_{t+1} - R^f_{t+1}) \\ SMB_{t+1} &= (R^H_{t+1} - R^L_{t+1}) \\ HML_{t+1} &= (R^S_{t+1} - R^B_{t+1}) \end{aligned}$$

- R^S denotes the return of a portfolio of *small* firms (in terms of market capitalization)

- R^B denotes the return of a portfolio of *big* firms

- R^H denotes the return of a portfolio of firms with a *high* ratio of book value to market value

- R^L denotes the return of a portfolio of firms with a *low* ratio of book value to market value

All factors in the Fama/French model are excess returns.

Construction of SMB and HML

Assets are sorted according to the two variables:

- Book to market value (HML "high minus low")
- Market capitalization (SMB "small minus big")

Compute deciles of variables' empirical distribution

Portfolios are constructed according to those deciles

Use the upper decile and the lower decile for calculating the return difference

Load data

First, load the factor data and the return data into a GAUSS matrix. **Factor data** from 2nd quarter 1947 to 4th quarter 1993 are provided in the file `ff_factors.fmt`. The first column contains the market excess return over the T-bill rate. The second column contains the series `SMB` and the third column contains the series `HML`. **Return data** for ten portfolios (1st size decile to 10th size decile) from 2nd quarter 1947 to 4th quarter 1993 are provided in the file `ret_dec10_1947Q2_1993Q4.fmt`. You can load those files with the `load` command (look it up in the GAUSS Help).

Writing the moments procedure of the CAPM

Moment conditions:

$E(mR - 1) = 0$:

Write a procedure which returns the pricing error u matrix with dimension $10 \times T$ of the CAPM:

$u = mR - 1$

with $m = a + bR^m$, R^m the time series of excess market return and R the time series matrix of the ten portfolio deciles.

Estimation of unknown parameters using the GMM Toolbox

Use the estimation GAUSS procedure from the last exercise.

`estimate_gmm(initial,model)`

The procedure contains all global settings

Use this procedure to estimate a and b of the CAPM model.

Interpret the estimation result

Writing the moments procedure of the Fama/French model

Moment conditions:

$E(mR-1)=0$:

Write a procedure which returns the pricing error u matrix with dimension $10 \times T$ of the Fama/French:

$u = mR - 1$

with $m = b_0 + b_m R^{em} + b_{SMB} SMB + b_{HML} HML$, R^{em} the excess market return and R the time series matrix of the ten portfolio deciles.

Estimation of unknown parameters using the GMM Toolbox

Use the estimation GAUSS procedure from the last exercise.

```
estimate_gmm(initial,model)
```

The procedure contains all global settings

Use this procedure to estimate the unknown parameters of the Fama/French model.

Interpret the estimation results.

Overview of today's exercise

- Repetition
- F-Test for joint significance
- Plot the time series of the SDF
- Plot of average return vs. predicted return

Overview of programmed procedures

- cbm_moments
- capm_moments
- ff_moments
- estimate_gmm - GMM estimation procedure

Test: Are the coefficients of the Fama/French factors statistically significant different from zero?

Fama/French model

$$
\begin{aligned}
E(mR-1) &= 0 \\
m &= b_0 + b_m R^{em} + b_{SMB} SMB + b_{HML} HML
\end{aligned}
$$

Test: Are the coefficients of the Fama/French factors statistically significant different from zero?

Use the estimated variance covariance matrix (GMM: toolbox _gmmout_bcov) to compute an F-statistic for joint significance of the coefficients:

$$F \equiv (\mathbf{Rb} - \mathbf{r})'[\mathbf{R}\widehat{Var(\mathbf{b}|\mathbf{X})}\mathbf{R}']^{-1}(\mathbf{Rb} - \mathbf{r})/\#\mathbf{r}$$

where $\#\mathbf{r}$ is the dimension of $\mathbf{r}$ (number of restrictions).

Example:

For the construction of the matrix $\mathbf{R}$ and the vector $\mathbf{r}$, suppose you have estimated the parameter vector $\mathbf{b} = \begin{pmatrix} b_1 & b_2 & b_3 & b_4 \end{pmatrix}'$ and want to test the joint hypotheses whether the true parameter $\beta_2 = \beta_3$ and $\beta_1 = 0$. Then, you can write the null hypotheses as a system of linear equations:

$$H_0 : \mathbf{R}\boldsymbol{\beta} = \mathbf{r}$$

In this example, it follows for $\mathbf{R}$ and $\mathbf{r}$:

$$\mathbf{R} = \begin{bmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{r} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Plot of the stochastic discount factor time series

$\mathbb{E}_t[m_{t+1}R^i_{t+1}] = 1$ where m_{t+1} is the stochastic discount factor (SDF)

- Estimate an asset pricing model of your choice and save the estimated coefficients in a vector.

- Write a GAUSS procedure which returns the time series of the stochastic discount factor for a specific asset pricing model (e.g. Fama/French, CAPM etc.).

- Then, use this procedure in a second procedure which plots the SDF series.

Plot average returns vs. predicted returns

The predicted returns R^i for each return decile can be calculated from

$$E(R^i) = \frac{1 - cov(m, R^i)}{E(m)}$$

- Estimate an asset pricing model of your choice and save the estimated coefficients in a vector. Use the procedure which returns the SDF series together with the matrix of returns to compute the predicted mean returns for each return decile.
- Further, calculate the realized mean returns $\bar{R}^i$ for each return decile and collect them in a vector.

- Plot the realized mean returns on the x-axis versus the predicted mean returns on the y-axis.

- Draw an additional 45° line to provide an illustration how well the model fits the data.

Overview of today's exercise

- Estimation of the CBM in excess returns using conditioning information
- Estimation of the CAPM in excess returns using conditioning information
- Plot average excess return vs. predicted excess returns

Estimation of asset pricing models using conditioning information

- Download the instruments and the Treasury bill data from the course homepage

- In the GAUSS file `instruments.fmt` you find the two instruments used by Cochrane(JPE 1996).

- First column contains the term spread (yield on long term government bonds less yield on 3-month Treasury bills)

- Second column contains the dividend/price ratio of the equally weighted NYSE portfolio. Instead of using the d/p ratio directly, use $1 + 100 \times [(d/p) - 0.04]$ to keep the scale of the moments comparable.

Moment conditions using excess returns

$$g_T(b) = \begin{bmatrix} E[m_{t+1}R^{e,1}_{t+1}] \\ \vdots \\ E[m_{t+1}R^{e,10}_{t+1}] \\ E[(m_{t+1}R^{e,1}_{t+1})z^1_t] \\ \vdots \\ E[(m_{t+1}R^{e,10}_{t+1})z^1_t] \\ E[(m_{t+1}R^{e,1}_{t+1})z^2_t] \\ \vdots \\ E[(m_{t+1}R^{e,10}_{t+1})z^2_t] \end{bmatrix}$$

where z_t^1 is the term spread and z_t^2 is the dividend/price ratio.

Important: Check the time index.

Hint: If excess returns are used you need an additional moment restriction to identify the parameters. This moment restriction follows directly from the fact that $E(mR^F) = 1$.

Working load for CBM and CAPM

- Download data
- Transform data
- Cochrane uses only the 1st, 2nd, 5th and 10th test asset. We reproduce Cochrane's(1996) results.
- Write a procedure for the moment conditions using excess returns
- Write a procedure for the moment conditions using excess returns and the instruments

Plot the average excess return vs. predicted excess return

- Predicted excess returns can be computed as:

$$E(R^{e,i}) = -\frac{cov(m, R^{e,i})}{E(m)}$$

- Calculate the realized mean returns $\bar{R}^{e,i}$ for each return decile and collect them in a vector

- Plot the predicted mean returns on the x-axis versus the realized mean returns on the y-axis

- Draw an additional 45° line to provide an illustration how well the model fits the data

Overview of today's exercise

- Lettau Ludvigson JPE (2001) model
- Load data and write moment conditions procedure for LL
- Estimating LL model with scaled factors
- Estimating the CAPM with scaled factors
- Estimating the CAPM with scaled factors and managed portfolios

Lettau/Ludvigson(JPE 2001)

In the GAUSS file `cay.fmt` you find the instrument used by Lettau/Ludvigson(JPE 2001). To replicate the results of Lettau/Ludvigson, use the 25 Fama/French portfolios provided in `ff_25.fmt`, the market return provided in `mkret_ll.fmt` and the T-bill rate provided in `tbill_ll.fmt`. The stochastic discount factor is specified as:

$$m_{t+1} = a_1 + a_2 cay_t + b_1 R^m + b_2 (R^m_{t+1} \times cay_t)$$

The moment conditions are collected in a vector $g_T(b)$:

$$g_T(b) = \begin{bmatrix} E[m_{t+1}R_{t+1}^{e,1}] \\ \vdots \\ E[m_{t+1}R_{t+1}^{e,10}] \end{bmatrix}$$

Estimation of CAPM with scaled factors

Estimate the CAPM with scaled factors for the Cochrane deciles. Scale the factors with the two instruments term spread and dividend/price ratio provided in `instruments.fmt`. Provide unconditional estimates as in i). The stochastic discount factor is specified as :

$$m_{t+1} = a_1 + b_1 R^m + b_2(R^m_{t+1} \times d/p) + b_3(R^m_{t+1} \times term)$$

Note, that the constant is not scaled by the instruments (hence, the constant is not time varying), since Cochrane does not include the instruments themselves as factors. The moment conditions for unconditional estimates of the scaled factor model are again:

$$g_T(b) = \begin{bmatrix} E[m_{t+1} R^{e,1}_{t+1}] \\ \vdots \\ E[m_{t+1} R^{e,10}_{t+1}] \end{bmatrix}$$

Estimation of CAPM with scaled factors using managed portfolios

To replicate the results in Cochrane(JPE 1996), you have to provide conditional estimates of the scaled model (use only return decile 1, 2, 5 and 10 together with the "managed portfolios" $R^{e,i}z^{i}$). The stochastic discount factor is specified as before in 1. The moment conditions for conditional estimates of the scaled factor model are now (as in the managed portfolio case):

$$g_T(b) = \begin{bmatrix} E[m_{t+1}R^{e,1}_{t+1}] \\ \vdots \\ E[m_{t+1}R^{e,10}_{t+1}] \\ E[(m_{t+1}R^{e,1}_{t+1})z^1_t] \\ \vdots \\ E[(m_{t+1}R^{e,10}_{t+1})z^1_t] \\ E[(m_{t+1}R^{e,1}_{t+1})z^2_t] \\ \vdots \\ E[(m_{t+1}R^{e,10}_{t+1})z^2_t] \end{bmatrix}$$

where z^1_t is the term spread and z^2_t is the dividend/price ratio.

Plot the average excess return vs. predicted excess return

Modify your procedure which returns the series for the stochastic discount factor to account for the different specifications of the SDF and produce the plot fitted return vs. realized return.

Overview of today's exercise

- The MRR model
- Load stock data
- Writing a moment procedure for the MRR model
- Estimating the MRR model with the GMM toolbox
- Computing standard errors of the implied spread using δ method
- Computing standard errors of the asym. info. share using δ method

The MRR model - order flow

- $Q_t = +1$ if purchase at ask price
- $Q_t =$ -1 if sale at bid price
- $P(Q_t = -1) = P(Q_t = +1) = 0.5$ unconditional probabilities
- $E(Q_t) = 0; Var(Q_t) = E(Q_t^2) = 1$
- time series $\{Q_t\}$: order flow

The MRR model - suprise in order flow matters

Evolution of fundamental value

$$\mu_t = \mu_{t-1} + \quad \text{private info} \quad + \text{public info}$$

$$\mu_t = \mu_{t-1} + \quad \theta(Q_t - E[Q_t | Q_{t-1}]) \quad + \varepsilon_t$$

$$p_t = \mu_t + \phi Q_t + \xi_t$$

ξ_t: disturbance term -accounts for rounding errors (price discretion)

θ: measures degree of information asymmetry

ϕ: order processing/inventory component

The MRR model - estimable equation

$$\text{ask price: } P_i^a = \mu_{i-1} + \theta(1 - E[Q_i|Q_{i-1}]) + \phi + \varepsilon_i \quad (1)$$

$$\text{bid price: } P_i^b = \mu_{i-1} - \theta(1 + E[Q_i|Q_{i-1}]) - \phi + \varepsilon_i \quad (2)$$

$E[Q_t|Q_{t-1}]$: conditional expectation $Q_t = \rho \cdot Q_{t-1}$ + error

ρ: measures correlation in order flow

equation to be estimated: Q_t and p_t observed

$$\Delta p_t = \theta(Q_t - \rho Q_{t-1}) + \phi \Delta Q_t + u_t$$

The moment conditions of the MRR model

$$E \begin{bmatrix} u_t \\ u_t Q_t \\ u_t Q_{t-1} \\ Q_t Q_{t-1} - \rho Q_t^2 \end{bmatrix} = 0$$

Load data

In the GAUSS files `ads_tim.fmt`, `bmw_tim.fmt` and `dcx_tim.fmt` you find trade data for the three stocks over a period from 1st February 2004 to 10th February 2004. variables that are necessary for estimation the MRR model. delta_p_t q_t q_tlag

δ-Method

write a procedure that has as a input variable $\beta = (\phi, \theta, \rho)'$ and as output variable the asymmetric information share $r = \theta/(\theta + \phi)$.

write a procedure that has as a input variable $\beta = (\phi, \theta, \rho)'$ and as output variable the implied spread $s_E = 2(\phi + \theta)$

write a procedure that has as input variables beta and its variance covariance matrix and as output variables the implied spread and the asymmetric information share with standard errors

Note: the Gauss command `gradp` is useful for the δ-Method.

Estimate the model for all three stocks

Estimate the model for all three stocks and save your results in a matrix. Note, that if you want to add information having character format you have to convert your numeric values first since mixed matrices (numeric and character) are not allowed in GAUSS.