Using Stories to Create Qualitative Representations of Motion

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Abstract

Qualitative representations of motion transform kinematic floating point data into a finite set of concepts. Their main advantage is that they usually reflect a human understanding of the moving system, so we can more straightforwardly implement human-like navigation rules; in addition, they reduce the overhead of floating point computations. Therefore, they are an asset for mobile robots or unmanned vehicles—both terrestrial and aerial—especially those that interact with humans. In this paper we provide a method to create new qualitative representations of motion from any qualitative spatial representation by using a story-based approach.

1 Introduction

Description and interpretation of moving entities (humans, animals, robots, or inert objects) are at the core of many disciplines such as mobile robotics, human-robot interaction, geographic information systems, animal behaviour, high-level computer vision, and knowledge representation, among others. Qualitative representations transform the mass of quantitative data (positions and velocities) into a reduced group of concepts. Therefore, they simplify data so that these are easier to understand and to process (e.g. in modelling, planning, learning, or control).

Nonetheless, the work in qualitative representations of *motion* is still reduced in number, when compared to *spatial* representations[1] [5, p. 16][6, p. 5187], and mostly restricted to point-like entities moving in one or two dimensions [21]. Moreover, spatial representations deal with regions [19] and three or more dimensions [10, 1], but this is unusual in representations of motion.

To fill the gap, in this paper we profit from the available spatial representations to systematically increase the number of representations of motion: we introduce a method that creates qualitative representations of motion given any qualitative spatial representation.

This has direct applications, for example, we may create a representation of motion using Hall's spatial categorisation, proxemics [13], which is based on the social distances. Such a representation of motion would describe trajectories according to the personal space and, thus, it could be used to make robot navigation in human environments more friendly.

Our method centres on the concept of 'stories' which, we believe, opens a new perspective in dealing with representations. A spatial representation can classify two static entities, or equivalently, each snapshot of two

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[1]Wherever we mention the term 'representation' throughout this paper, it is understood that we are talking about '*qualitative* representations' — We drop the term 'qualitative' for sake of readability.

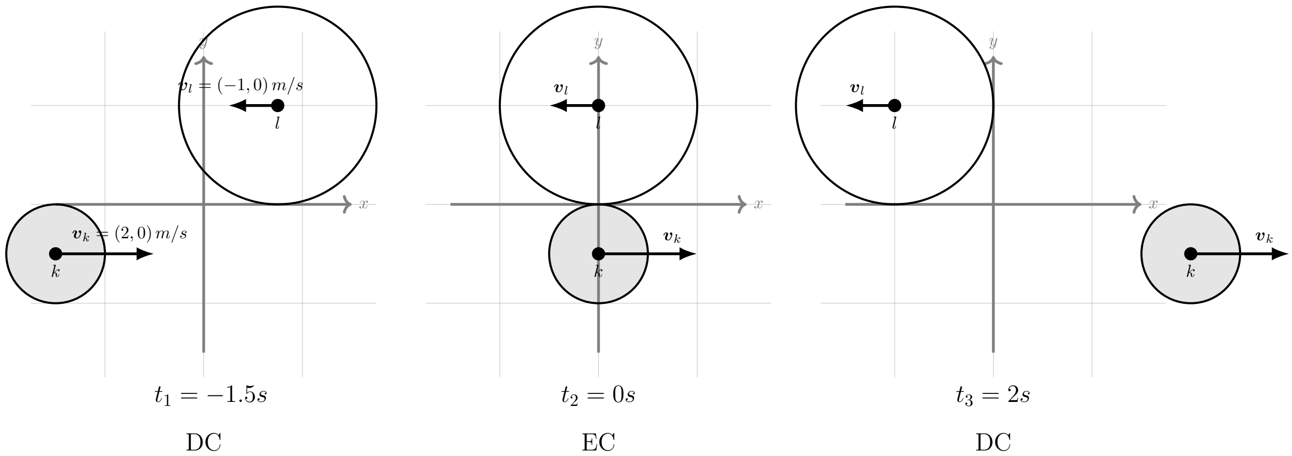

Figure 1: **An RCC story with three qualitative relations.** Two circular entities k (radius = $1m$) and l (radius 2m) moving with velocities $\vec{v}_k = (2, 0)\, m/s$ and $\vec{v}_l = (-1, 0)\, m/s$ in the interval $[t_1, t_3]$. The snapshots depict the *temporal sequence of relations* (DC, EC, DC) in the qualitative representation RCC (Fig. 3).

moving entities. If we therefore consider the complete sequence of snapshots—what we call the 'story' (Def. 2)—, we have a qualitative description of the motion.

Our method can use any spatial representation (e.g., $\mathrm{OPRA_m}$ [16], Rectangle Algebra [3]); however, it can be hampered by the generation of the *stories set* (Def. 3), because this is often an arduous manual task. For that reason we used as a example (Ex. 5) the simple and well-known spatial representation RCC [19] (See Fig. 3). As RCC relates regions, our method will generate, in this particular case, a novel representation of motion—we call it 'Motion-RCC' (Eq. (1) on page 6)—that deals with regions, and an extended variant, 'Augmented-Motion-RCC' (Eq. (2) on page 6).

2 Related Work

2.1 Qualitative Representations of Motion

An overview of representations is found in a survey by Dylla et al. [8]: in a total of 40 representations surveyed they classify three as representations of motion: QRPC [12], RfDL-3-12 [15], and, the most used, QTC [21]. The survey of spatial representations of Chen et al. [4] also mentions three motion representations: Dipole Calculus [17], DIA [20], and QTC.

Representations of orientation and relative direction, such as OPRA [16] or Dipole Calculus [17], are sometimes used to represent moving entities; nevertheless, they are not primarily intended for such a task.

All the aforementioned representations are limited to point-like entities moving in one or two dimensions. There is, however, a particular qualitative relation of motion for regions [22] that is built combining RCC and distances.

2.2 Sequences of Qualitative Relations

Continuous sequences of qualitative relations, such as the *temporal* sequences of Def. 1 (p. 4), are based on Freska's foundational concept *conceptual neighbourhood* [11]. Connecting the qualitative relations of a certain representation that are conceptual neighbours we obtain the *conceptual neighbourhood graph*[9] (See example in Fig. 3). So paths in the conceptual neighbourhood graph and continuous sequences of qualitative relations are equivalent.

Sequences of relations are used to analyse real data by Delafontaine et al. [6], and specifically in human-robot interaction by Hanheide et al. [14] from which we borrow the term *'temporal sequence of qualitative relations'* (Def. 1).

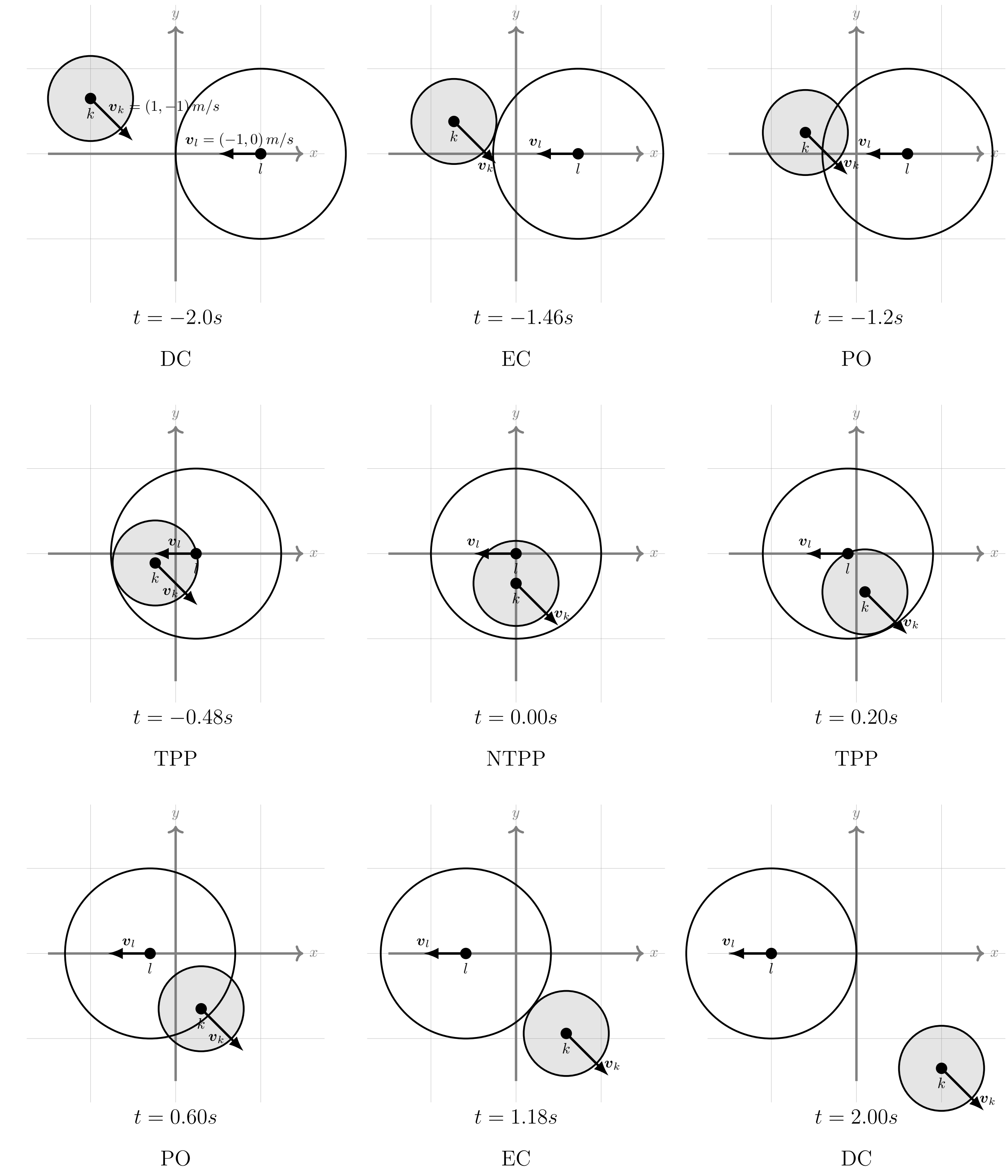

Figure 2: **A RCC story with nine qualitative relations.** Two circular entities k (radius $= 1m$) and l (radius 2m) moving in uniform motion with velocities $\vec{v}_k = (1,-1)\,m/s$ and $\vec{v}_l = (-1,0)\,m/s$. They depict the Temporal Sequence of Relations (DC, EC, PO, TPP, NTPP, TPP, PO, EC, DC) in the qualitative representation RCC (Fig. 3). The snapshots correspond to different increasing times.
This sequence is a *story*, because it remains the same, even if we extend the interval to $(-\infty, +\infty)$. It corresponds to the story S_5 of the created representation of motion 'Motion-RCC' (Sect. 5).

3 Temporal Sequences of Relations and Stories

In this section, we define and illustrate the key concepts—*stories* and *stories set*—that we use to create qualitative representations of motion (Sect. 5). But first of all we define the underlying concept: *temporal sequence of relations*.

Definition 1. A **Temporal Sequence of Relations** [14] is a chronologically ordered sequence of qualitative relations of any kind, e.g., space or motion, generated by the motion of two entities in a *time interval* (t_a, t_b).

The time interval (t_a, t_b) can be freely chosen, e.g., it can be totally unbounded, i.e., extend to the whole time $(-\infty, \infty)$, be half-bounded $(-\infty, t_b)$, or bounded (t_a, t_b).

We obtain the temporal sequence of relations of two entities in a certain time interval by mapping their trajectories $\vec{x}_k(t)$ and $\vec{x}_l(t)$ into the qualitative relations of the representation we are using. We describe a sequence of relations as a list in parenthesis: $(R_1, R_2, \dots, R_i, \dots)$. We say a temporal sequence of relations is *finite*, if it has a finite number of relations, or *infinite*, if it has an infinite number. Notice that even though the entities' motion occurs in a continuous space throughout a continuous time interval, the temporal sequences are finite, when the trajectories have a finite number of transitions between qualitative relations; this happens in Fig. 1, the sequence is finite, (DC, EC, DC), because we have only two transitions: DC $\rightarrow$ EC and EC $\rightarrow$ DC.

Now, based on the temporal sequences, we define the *stories*.

Definition 2. A **Story** is a temporal sequence of relations of two entities that is defined over the whole unbounded time interval $(-\infty, \infty)$.

A story describes the qualitative relation of two moving entities at any instant of time. Thus, any temporal sequence of relations is a substring of a certain story. We can see each story as a complete qualitative description of the motion of a two-entities system. We characterise stories with the letter S and, if necessary, an appropriate subscript.

Example 1. The temporal sequence $S =$(DC, EC, DC) in Fig. 1 is a story. Any proper substring is not a story, but just a temporal sequence of relations, because it does not happen in the whole unbounded interval $(-\infty, \infty)$. For instance, the substring (EC,DC) is not a story, because it happens on $[0, +\infty)$.

Example 2. The temporal sequence (DC, EC, PO, TPP, NTPP, TPP, PO, EC, DC) in Fig. 2 is a story. Substrings, such as (PO, TPP, NTPP, TPP) or (DC, EC, PO), are not stories, but just temporal sequence of relations.

Definition 3. The **Stories Set** is the set of all possible stories of two entities.

If there is no constraint on the stories, the stories set contains an infinite number of stories. We refer to the *stories set* with the letter Σ (see Sect. 5); we add a subscript, e.g., Σ_0, when we deal with a set of stories that is not the stories set, but a subset thereof.

4 Restricting the Stories: Uniform Motion

The central idea of this paper is to classify motions through *stories*: we assign the same category to the motions that belong to the same story. (Sect. 5). Thus, the total number of categories in our novel motion representation is the cardinality of the *stories set*, i.e., its number of elements. However, an awkward situation arises: the cardinality of the stories set is infinite—some stories are also infinite—, if we do not restrict the motions that create the stories.

Consequently, we suggest restricting the type of motions considered in order to obtain a tractable motion representation. We choose to restrict the stories by considering, from now on, only *uniform motion*, i.e., the velocity vectors are *constant*. This has two desirable properties:

i. Each story in uniform motion is *finite*, i.e., has a finite number of relations (See Prop. 1 in Appendix A).

ii. The set of all possible stories in uniform motion, i.e., the *stories set* (Def. 3), is *finite* (See Prop. 2 in in Appendix A). Consequently it partitions the whole *phase space*, i.e., the coordinates space of the positions and velocities of the two entities $(\vec{x}_k, \vec{v}_k; \vec{x}_l, \vec{v}_l)$.

The restriction to uniform motion *stories* is a standard assumption, if we classify motion situations that are specified only by the current position and velocity of two entities, i.e., $(\vec{x}_k, \vec{v}_k; \vec{x}_l, \vec{v}_l)$—the acceleration is disregarded, as in QTC [21]. Though we note that our method may remain valid with other kind of restrictions.

Definition 4. A **Rigid Story** is the story of two entities that move with the same velocity, i.e., $\vec{v}_k = \vec{v}_l$.

Rigid stories play a special role in uniform motion: each of them is a *singleton*—it has a single element, a constant spatial relation. But not all singleton stories are rigid, e.g., the story S_{11} = (DC) is not rigid but is a singleton (Fig. 4).

5 Creating New Qualitative Representations of Motions

We describe the method to create a representation of motion from any given spatial representation. In practice, our method yields always two representations of motion: the simple one, which is just formed by the stories, and the *augmented* variant, which is refined by adding the spatial relations to each story—we combine the power of 'story' and 'snapshots'. We illustrate the method in the example below using the spatial representation RCC (Fig. 3), and thus, the two new generated representations of motion are *Motion-RCC* (Eq. (1) on page 6, and Fig. 4), and its augmented variant *Augmented-Motion-RCC* (Eq. (2) on page 6).

The method is as follows:

1. We have a spatial representation.

2. We calculate the *stories set*, Σ, for the given spatial representation. In case it is a finite set, e.g., when restricted to uniform motion, we can work out a method to calculate it.

3. The obtained *stories set* is a novel representation of motion, where each story is a qualitative relation—every motion state is classified according to the story it belongs to.

4. (optional) We can create the *augmented representation* of motion from the first one by specifying the spatial relations in each story.

Example: Creating a representation of motion from RCC

We illustrate the method above using the spatial representation RCC. (Fig. 3). RCC relates two regions according to their overlapping. So it yields 8 possible relations: DC, regions do not overlap; EC, regions are tangent non-overlapping; PO, regions overlap in the interior but none is contained in the other; TPP, region x is contained in y and is tangent to the border; TPPI, region y is contained in x and is tangent to the border; EQ, both regions overlap completely; NTPP, x is contained in y and does not overlap the border of y; NTPPI, y is contained in x and does not overlap the border of x.

1. We have the spatial representation RCC

2. We calculate the RCC *stories set* restricted to uniform motion as $\Sigma = \Sigma_0 \cup \Sigma_1$. Σ_0 = {(DC), (EC), (PO), (TPP), (NTPP)} are the *rigid stories* and Σ_1 ={(DC), (DC, EC, DC), (DC, EC, PO, EC, DC), (DC, EC, PO, TPP, PO, EC, DC), (DC, EC, PO, TPP, NTPP, TPP, PO, EC, DC)} are the non-rigid stories. We rename the rigid stories into S_{0i}, $\Sigma_0 = \{S_{01}, S_{02}, S_{03}, S_{04}, S_{05}\}$, and the non-rigid which we rename into S_{1i}, $\Sigma_1 = \{S_{11}, S_{12}, S_{13}, S_{14}, S_{15}\}$ according to Fig. 4.

3. The stories set Σ is the qualitative representation of motion—note, though, that that story S_{01} and S_{11} are equal to (DC) therefore S_{01} drops to avoid repetition. We call this representation 'Motion-RCC':

$$\textbf{Motion-RCC} = \{S_{02}, S_{03}, S_{04}, S_{05}, S_{11}, S_{12}, S_{13}, S_{14}, S_{15}\} \tag{1}$$

This representation assigns to every motion state $(\vec{x}_k, \vec{v}_k; \vec{x}_l, \vec{v}_l)$ the corresponding story S_i, i.e., the corresponding relation of motion.

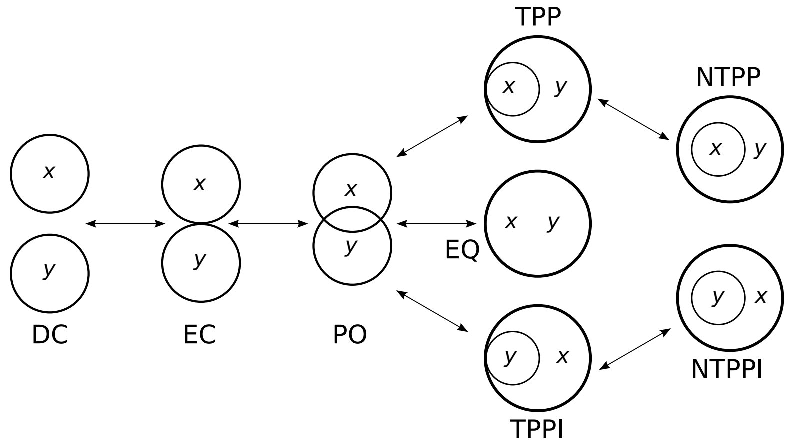

Figure 3: The RCC qualitative relations depend on how two entities overlap. This Figure depicts the 8 RCC relations: DC, EC, PO, TPP, NTPP, EQ, TPPI, and NTPPI as a *conceptual neighbourhood graph* [11, 9]: the arrows connect relations that are *conceptual neighbours* [11] — we switch between conceptual neighbours by a continuous translation without going through any other relation.

4. (optional) We can augment the resolution of the representation of motion *Motion-RCC* by specifying the spatial relations in each story—for the singleton stories this process is redundant, as they have a single spatial relation. So we obtain the representation of motion 'Augmented-Motion-RCC'.

$$\begin{aligned}
&\textbf{Augmented-Motion-RCC} = \{ \\
&S_{02}(\mathrm{EC}), S_{03}(\mathrm{PO}), S_{04}(\mathrm{TPP}), S_{05}(\mathrm{NTPP}), \\
&S_{11}(\mathrm{DC}), S_{12}(\mathrm{DC}_-), S_{12}(\mathrm{EC}), S_{12}(\mathrm{DC}_+), \\
&S_{13}(\mathrm{DC}_-), S_{13}(\mathrm{EC}_-), S_{13}(\mathrm{PO}), S_{13}(\mathrm{EC}_+), S_{13}(\mathrm{DC}_+), \\
&S_{14}(\mathrm{DC}_-), S_{14}(\mathrm{EC}_-), S_{14}(\mathrm{PO}_-), S_{14}(\mathrm{TPP}), \\
&S_{14}(\mathrm{PO}_+), S_{14}(\mathrm{EC}_+), S_{14}(\mathrm{DC}_+), \\
&S_{15}(\mathrm{DC}_-), S_{15}(\mathrm{EC}_-), S_{15}(\mathrm{PO}_-), S_{15}(\mathrm{TPP}_-), S_{15}(\mathrm{NTPP}), \\
&S_{15}(\mathrm{TPP}_+), S_{15}(\mathrm{PO}_+), S_{15}(\mathrm{EC}_+), S_{15}(\mathrm{DC}_+)\}
\end{aligned} \tag{2}$$

For example, the relation $S_{12}(\mathrm{EC})$ indicates that the entities are moving in the story S_{12} at the moment of tangency, i.e., EC. If the spatial relation appears multiple times in the story, such as EC in S_3, we distinguish each appearance, for example, $S_{13}(\mathrm{EC}_-)$ is chronologically the first EC, and $S_{13}(\mathrm{EC}_+)$, the last EC.

6 Applications of Qualitative Representations of Motion

We outline two possible applications of qualitative representations

- *Recognition of trajectories (i.e., motion patterns)*
  Through the qualitative relations in the new representation of motion, we can characterise and therefore recognise certain types of motion [6, 14], for example an 'avoidance manoeuvre', as in Eq. (3). This motion sequence begins with the collision story, $S_{15}(\mathrm{DC}_-)$, and ends with a collision free story,$S_{11}(\mathrm{DC})$—the augmented indices, DC, show that nowhere a collision takes place.

$$S_{15}(\mathrm{DC}_-) \rightarrow S_{14}(\mathrm{DC}_-) \rightarrow S_{13}(\mathrm{DC}_-) \rightarrow S_{12}(\mathrm{DC}_-) \rightarrow S_{11}(\mathrm{DC}) \tag{3}$$

- *Trajectory control*
  We can use the *conceptual neighbourhood graph* of our new representation of motion to take decisions in order to control trajectories [7]. For example, in the case of Motion-RCC, if we want to avoid a collision we have necessarily to reach the relation $S_{11}(\mathrm{DC})$. Accordingly, the shortest paths in the conceptual neighbourhood graph leading to the relation $S_{11}(\mathrm{DC})$ may provide the needed control operations to avoid the collision.

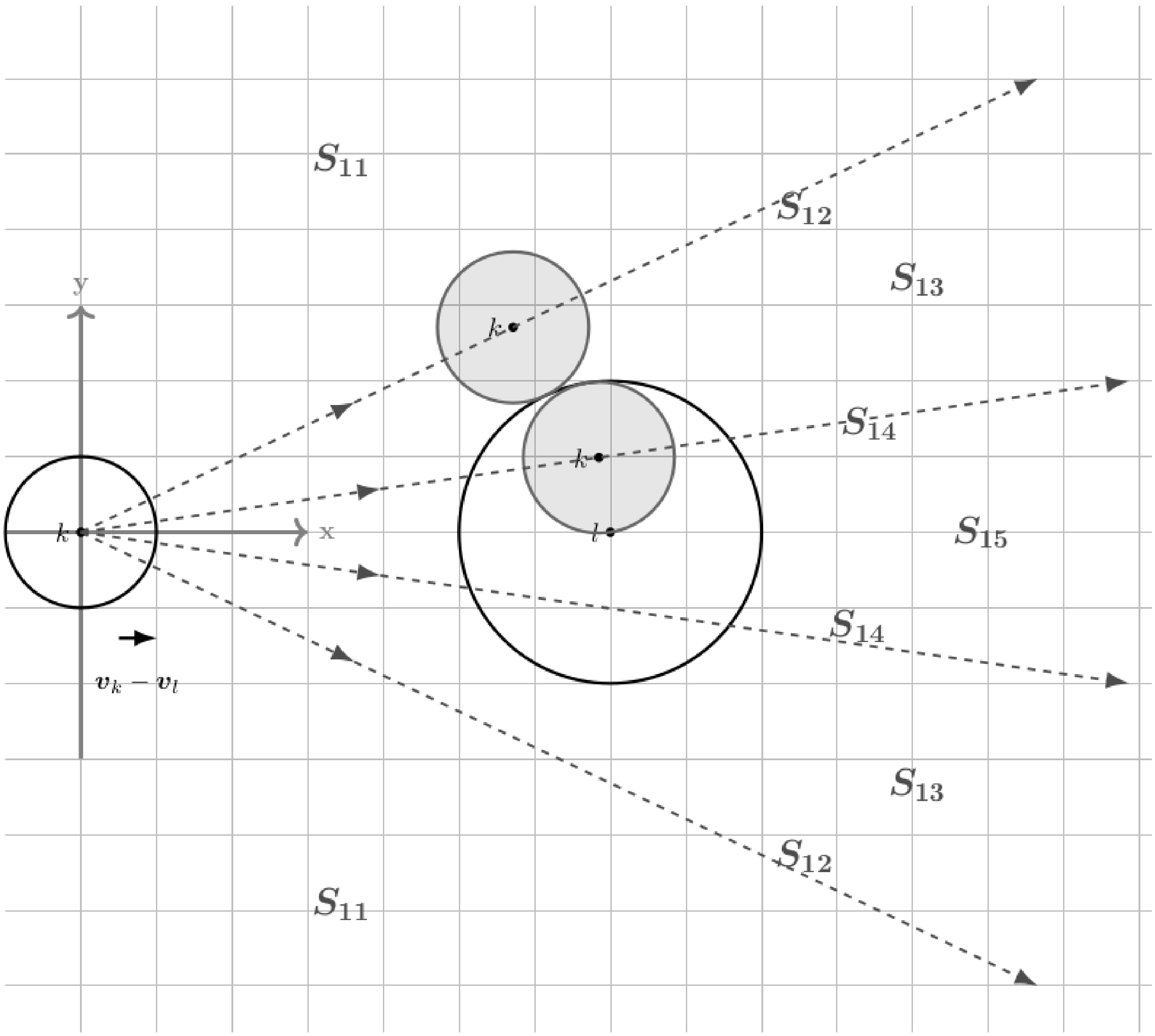

Figure 4: In the representation RCC these are all the possible non-rigid stories, Σ_1, i.e, the stories of two circles k and l moving in uniform motion with velocities $\vec{v}_k$ and $\vec{v}_l$, so that $\vec{v}_k \neq \vec{v}_l$. The total number is five. Two stories are associated with directions: S_{12} = (DC, EC, DC); S_{14} = (DC, EC, PO, TPP, PO, EC, DC). The remaining three stories are associated with the regions between the directions: S_{11} = (DC), S_{13} = (DC, EC, PO, EC, DC), S_{15} = (DC, EC, PO, TPP, NTPP, TPP, PO, EC, DC).
Note: The figure represents an equivalent simplification that considers l being motionless and k moving with the difference of velocities $\vec{v}_{kl} = \vec{v}_k - \vec{v}_l$. The story depends on the direction of $\vec{v}_{kl}$.

7 Discussion

We have presented a a *story-based* method (Sect. 5) that should be able to generate qualitative representations of motion out of any spatial representation. The created representation of motion inherits the properties of the used spatial representation, e.g., dimensions, or type of entities considered. The method has proven to be effective to generate meaningful qualitative representations of motions for the representation RCC (Sect. 5). With our generated motion representation, *Augmented-Motion-RCC*, we have outlined two applications of motion representations: recognition of trajectories, i.e., motion patterns; and control of trajectories.

Our generating method is most effective, when we restrict the trajectories of the entities, e.g., setting velocity constant, so that our stories set is finite. This can be seen as a limitation or as the advantage to tailor the generated representation of motion to the features of our trajectories. We have restricted the trajectories to have uniform motion.

We argue that the use of 'stories' to classify motions borrows from a cognitive idea: we can better recall a series of items, when they are linked by way of a story—Stories seem quite a natural way for humans to relate, connect, or classify items.

The next steps are to test the effectiveness of this method with other spatial representations, for instance, three dimensional [10] or those dealing with orientation [18].

A Appendix

Proposition 1. ***Finitude of the Stories in Uniform Motion***
We can reasonably show that for two regular enough[2] *entities the stories in uniform motion are finite.*

We build the proof on two properties: first, stories in uniform motion have *extreme relations* (Lemma 1); second, temporal sequences of relations in uniform motion are finite over a finite time interval (Lemma 2).

Proof. According to Lemma 1 two regular enough entities in uniform motion have extreme relations. That is, we can find two time instants t_a and t_b, with $t_a < t_b$, so that in the time interval $(-\infty, t_a)$ the entities' relation remains constant—we call it r_a—and in the time interval $(t_b, +\infty)$ the entities' relation remains constant.—we call it r_b.

Now, According to Lemma 2, these regular enough entities moving in uniform motion have a finite temporal sequence of relations in the interval $[t_a, t_b]$, say $(r_1, \ldots, r_n)$.

Consequently the story of the two entities, i.e., the temporal sequence of relations in the interval $(-\infty, t_a) \bigcup [t_a, t_b] \bigcup (t_b, \infty)$, would be finite, as it is obtained by concatenating the two extreme relations and the temporal sequence: $(r_a,\ r_1, \ldots, r_n,\ r_b)$. In case any extreme relation coincides with its border relation, i.e., $r_a = r_1$ or $r_b = r_n$, we exclude the repeated ones. □

Definition 5. Extreme Relations
The *extreme relations* are those relations of a story that remain unchanged when $t \to -\infty$ or $t \to +\infty$. That is, a relation r_a is extreme in $t \to -\infty$, if and only if $\exists t_a$, so that in the time interval $(-\infty, t_a)$ the relation between entities is r_a. Analogously, a relation r_b is extreme in $t \to +\infty$ if and only if $\exists t_b$, so that in the time interval $(t_b, +\infty)$ the relation between entities is r_b.

Lemma 1. ***Existence of extreme relations for two entities in uniform motion.***
Two regular enough[2] *entities that move in uniform motion and are described by a qualitative representation based on overlapping, intersection, or orientation, have a story with extreme relations both for $t \to -\infty$ and $t \to +\infty$.*

Proof. We name the entities k and l and they have constant velocities $\vec{v}_k$ and $\vec{v}_l$.

1. In the case $\vec{v}_k = \vec{v}_l$ the relation between both entities, r_i, remains constant — this relation is the whole story —, therefore, trivially, r_i is the extreme relation for both $t \to -\infty$ and $t \to +\infty$.

2. In the case $\vec{v}_k \neq \vec{v}_l$ we distinguish two subcases regarding what feature the representation bases on: overlapping-intersection of entities, or relative orientation.

[2] *Enough regular* entities are those finite in size with a finite number of features, i.e., a finite number of vertices, edges, concavities, holes, . . .

(a) Representations based on *overlapping-intersection* of finite entities have either one or two qualitative relations for the case of 'no overlapping-intersection', e.g., the relation DC in RCC (Fig. 3); the relation *disjoint* in 9-Int [10]; or the relations '<' and '>' in Allen's Algebra [2]. The mentioned relations must be the extreme relations for each representation, because the distance between two entities that move at different velocities tends to infinity for $t \to \pm\infty$; and consequently the entities do not overlap-intersect any more.

(b) Representations based on *relative orientation* between entities use the *connecting unit vector* between them, i.e., $\hat{\vec{kl}}(t) = \frac{\vec{x}_l(t)-\vec{x}_k(t)}{\|\vec{x}_l(t)-\vec{x}_k(t)\|}$, for which in uniform motion, i.e., $\vec{x}_k(t) = \vec{v}_k t + \vec{x}_{k0}$ and $\vec{x}_l(t) = \vec{v}_l t + \vec{x}_{l0}$, we obtain both limits :

$$\lim_{t\to+\infty} \hat{\vec{kl}}(t) = \frac{\vec{v}_l - \vec{v}_k}{\|\vec{v}_l - \vec{v}_k\|} \ (4a) \qquad \lim_{t\to-\infty} \hat{\vec{kl}}(t) = -\lim_{t\to+\infty} \hat{\vec{kl}}(t) \ (4b)$$

Because both limits for the connecting vector exist, the extreme relations of any story exist; they are the relations neighbouring each limit.

□

Lemma 2. ***Finitude of the Temporal Sequences of Relations in Finite Time Intervals***
In uniform motion, for regular enough[2] entities, a temporal sequence of relations in a finite time interval is also finite.

Proof. A qualitative representation partitions the phase space of two *regular enough* finite entities in a finite number of regions, i.e., the qualitative relations. Therefore by moving in uniform motion in a finite time interval the system goes through a finite number of such regions, i.e., the resultant temporal sequence of relations must be finite. □

Proposition 2. ***Finitude of the Stories Set***
The set of stories in uniform motion, i.e., the stories set, is finite.

Proof. We cannot rigorously prove that the stories set is finite, but Lemma 3 gives an equivalent condition that help us to see that the number of possible stories must be finite in most qualitative representations: if we prove that there is a story with more or an equal number of relations than any other, then the stories set must be finite. This is the case in RCC (Fig. 4), where the longest story is S_5. □

Lemma 3. ***The longest story***
The stories set is finite, if and only if it exists a longest story, i.e., a story that has more or equal relations than any other.

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