

Advanced Time Series

Gauss Exercises

1st GAUSS assignment ATS

1. Create a standard normally-distributed random variable

1. Generate a sequence of 1000 standard normal random variables (write the realizations in one column). Produce a graph of the resulting "time series". GAUSS hint: `rndn(r,c)`; plot the time series against a `seqa(1,1,T)`.
2. Compute sample means and sample variance of the time series generated in step 1. Interpret the results.
3. Generate a second series that contains the cumulated standard normal random variables generated in step 1. (i.e. the second row of the new column of this series contains the sum of the first and the second values of the series with standard normal variables et cetera). Plot the resulting sequence.
4. Compute sample means and sample variance of the second time series generated in step 2. Interpret the results.
5. Repeat the steps, i.e. let the program run another time. Compare your results. What happens to your graph?

2. Simulate an AR(1) process

1. Simulate a first order AR (AR(1)) process that is defined as follows:

$$y_t = \phi y_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0,1)$$

$T = 100$. Simulate the process for different parameter values: $\phi = 0$ (white noise) $\phi = -0.5$ $\phi = 0.5$ $\phi = 0.9$ $\phi = 1$ (random walk). Hint: You either can use the Gauss function `recserar` or use a loop statement.

2. Plot the realizations of the process.
3. Write a procedure that delivers one realization ($T = 100$) of an AR(1) process. Hint: Use as input variables ϕ and the number T , as output the resulting AR(1) realization without the starting value.
4. Analyze the stationarity properties of the AR(1) process by using simulations. Evaluate if the requirements for weak stationarity for the AR(1) process are fulfilled. Compute ensemble means and variances for 30 realizations of your process (for every of the four parameter alternatives). Visualize the means and variances in a plot and label the axis.
5. Compute empirical autocorrelations (first order) by using the 30 realizations (Ensembles). Compute for this the correlation of y_1 and y_2 ($\text{Corr}(y_1, y_2)$) and $\text{Corr}(y_2, y_3), \dots, \text{Corr}(y_{99}, y_{100})$). Plot the resulting sequence of autocorrelations in a graph and title it.
6. Compute for one arbitrary realization of the AR(1) process (for every of the four parameter alternatives) the empirical autocorrelation of first order and compare this with the correlations of the ensembles of the previous question.

7. Given your results of the previous questions: Evaluate if the AR(1) process is (weakly) stationary. Does the stationarity depend on the choice of the parameter ϕ ? For which parameter value is the AR(1) process stationary?

3. Simulate a MA(1) process

1. Simulate a first order moving average (MA(1)) process that is defined as follows:

$$y_t = \theta \varepsilon_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0, 1)$$

$T = 100$. Simulate the process for different parameter values: $\theta = 0.5$, $\theta = -0.5$, $\theta = 2$, $\theta = -10$ Plot the realizations of the process in a graph.

2. Write a procedure that delivers one realization of a MA(1) process. Hint: Use as input variables θ and the number T , as output the resulting MA(1) realization.
3. Evaluate whether the requirements for weak stationarity are fulfilled. Compute ensemble means and variances for 30 realizations of your process (for every of the four parameter alternatives). Visualize the means and variances in a plot and label the axis.
4. Compute empirical autocorrelations (first order) by using the 30 realizations (Ensembles). Compute for this the correlation of y_1 and y_2 ($\text{Corr}(y_1, y_2)$ and $\text{Corr}(y_2, y_3), \dots, \text{Corr}(y_{99}, y_{100})$). Plot the resulting sequence of autocorrelations in a graph and title it.
5. Compute for one arbitrary realization of the MA(1) process the empirical autocorrelation of first order and compare this with the correlations of the ensembles. Hint: Use Gauss function `acf`.
6. Given your results of the previous questions: Evaluate if the MA(1) process is (weakly) stationary. Does the stationarity depend on the choice of the parameter θ ?

2nd GAUSS assignment ATS

1. Estimate an AR(1) process

1. Use the AR simulation procedure of the 1st set of assignment. Simulate an AR(1) process with $\phi = 0.5$, $c = 0$, $\sigma^2 = 1$ and $T = 100$.

$$y_t = c + \phi y_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma^2)$$

2. Write a procedure that estimates the parameters of an AR(1) process.

First build the $\varepsilon_t = y_t - c - \phi y_{t-1}$ of an AR sequence. Hint: You need a starting value for y and you need starting values for c , σ and ϕ .

Second compute the value of the likelihood function by plugging in ε_t .

Now write the rest of the procedure around the procedure-body. Hint: Do not forget to define locals.

For estimation do not use the starting values of y and you lose one observation by taking a lag of the series (y_{t-1}). GAUSS hint: `lagn`.

Input variables of the procedure are the data y and a parameter vector that contains parameters to be estimated. (c , σ and ϕ . The parameters enter the procedure as one (3×1 column vector!).

Output variable is a column vector ($(T-1) \times 1$) that contains the likelihood contribution of each observation.

3. Estimate ϕ , c , and σ^2 and discuss your results. (PDF-REPORT)
4. Compute standard errors of the estimators! Test the null hypothesis $H_0 : \text{parameter} = 0$. Compute p -values of the test-statistic. Gauss hint: `cdfnc` (PDF-REPORT)
5. Simulate the process for different parameter values of $\phi = -0.5$, $\phi = 0.5$, $\phi = 0.9$, estimate ϕ and discuss your results. (Note: $|\phi| < 1$!!!) (PDF-REPORT)
6. Simulate the process for different values of $T = 30$, $T = 50$, $T = 100$, $T = 200$ and $\phi = 0.7$. Estimate ϕ and discuss your results. How do your results change? (PDF-REPORT)
7. Write a likelihood function for only two parameters σ and ϕ . Plot the likelihood as a function of σ and ϕ in a three dimensional plot. Write the Gauss-Graph format into a postscript file. GAUSS Graph Hint: `xyz` (PDF-REPORT)

3rd GAUSS assignment ATS

1. Estimate a MA(1) process

1. Use the MA simulation procedure of the 1st set of assignment. Simulate an MA(1) process with $\theta = 0.5$, $c = 0$, $\sigma^2 = 1$, and $T = 100$.

$$y_t = c + \theta\varepsilon_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma^2)$$

2. Write a procedure that estimates the parameters of a MA(1) process.

First build the $\varepsilon_t = y_t - c - \theta\varepsilon_{t-1}$ of a MA sequence recursively. Hint: You need starting values for c , σ and θ . You need a starting value for ε . Use a loop statement and initialize a vector for the ε 's e.g. `eps = ones(rows(y), 1)`;

Second compute the value of the likelihood function by plugging in ε_t .

Now write the rest of the procedure around the procedure-body. Hint: Do not forget to define locals.

Input variables of the procedure are the data y and a parameter vector that contains parameters to be estimated. (c , σ and θ . The parameters enter the procedure as a column vector!).

Output variable is a column vector $((T-1) \times 1)$ that contains the likelihood contribution of each observation.

3. Estimate θ , c , and σ^2 and discuss your results. (PDF-REPORT)
4. Compute standard errors of the estimators! Test the null hypothesis $H_0 : \text{parameter} = 0$. Computer p -values of the test-statistic. Gauss hint: `cdfnc` (PDF-REPORT)
5. Simulate the process for different parameter values of $\theta = -0.5$, $\theta = 1$, $\theta = 0.3$ and $T = 100$. Estimate θ . Discuss the maximum likelihood estimator $\hat{\theta}$. (PDF-REPORT)
6. Simulate the process for different values of $T = 30, 50, 100, 200$ and $\theta = 0.7$. Estimate θ . Discuss the maximum likelihood estimator $\hat{\theta}$ and its estimated standard error. How do your results change? (PDF-REPORT)
7. Write the likelihood function for only two parameters σ and θ . Plot the likelihood as a function of σ and θ in a three dimensional plot. Write the Gauss-Graph format into a postscript file. GAUSS Graph Hint: `xyz` (PDF-REPORT)

Including Source Files

To render your programs less confusing procedures can be written into source files. The source file is then included into the program using:

```
#include mysourcefile.src;
```

A source file consists only of procedure code, no *hard* code should be written into a source file

4th GAUSS assignment ATS

Testing for Stationarity - Simulating the Dickey-Fuller Test Statistics:

- Stationarity test with null hypothesis: unit root (non stationarity), i.e. that $\rho = 1$
- Non-standard asymptotic distribution of unit root processes
- Inference requires simulation of asymptotic distribution

Procedure for the Dickey-Fuller test statistic simulation:

1. Simulate a random walk
2. Conduct an OLS regression and calculate the t-statistic for the null hypothesis that the true value of ρ equals 1
3. Simulate the test statistic: Run Step 1 and 2 $n=10000$ times and sort the t-values into quantiles

True Process:

$$y_t = y_{t-1} + \varepsilon_t$$

Case 1

Estimated Regression:

$$y_t = \rho y_{t-1} + \varepsilon_t \quad (1)$$

Case 2

Estimated Regression:

$$y_t = \alpha + \rho y_{t-1} + \varepsilon_t \quad (2)$$

Case 4

Estimated Regression:

$$y_t = \alpha + \rho y_{t-1} + \delta t + \varepsilon_t \quad (3)$$

Programming - Case 1:

1. Simulate a random walk

1. Simulate a random walk that is defined as follows:

$$y_t = \alpha + \rho y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2) \quad (4)$$

$T = 100$. Simulate the process for the following parameter values: $\rho = 1$, $\alpha = 0$, $\sigma^2 = 1$. Plot the realizations of the process.

2. Write a procedure that delivers one realization of the random walk process. Hint: Use as input variables ρ , α and T , as output the resulting random walk realization.

2. Estimate the unknown parameters with OLS

1. Compute the estimator $\hat{\rho}$ in Equation (1). First, implement an OLS estimator. $\beta = (X'X)^{-1}(X'Y)$; $X = y_{t-1}$ and Y contains the dependent variable y_t . For estimation do not use the starting values of y_t and you lose one observation by taking a lag of the series y_t . Hence, the sample size is actually $T + 1$.
2. Estimate the standard error of your parameter $\hat{\rho}$ and calculate the t-statistic for the null hypothesis that the true value of ρ equals 1.
3. Write a procedure around the OLS estimation. The OLS procedure has two input variables. A $(T \times K)$ matrix containing K exogenous regressors (here $X = y_{t-1}$) and a $(T \times 1)$ matrix containing the dependent variable, i.e. $Y = y_t$. As output specify a row **vector** that contains the estimate for $\hat{\rho}$, the corresponding standard error and the t-statistic associated with the null hypothesis that the true value $\rho = 1$.

3. Simulation of the test statistic

1. Simulate the two procedures of Step 1 and 2 $n=10000$ times. In order to do so, write a loop around the two procedures. Collect the parameter estimates for ρ and the corresponding standard errors and t-values in a matrix. Hint: Initialize a matrix of dimension $n \times 3$. Fill in the results from the simulation.
2. Sort the estimated t-values into quantiles (0.01, 0.025, 0.05, 0.1, 0.9, 0.95, 0.975, and 0.99). Hint: Use the GAUSS command `quantiled`. Compare the quantile values you calculated with those in Hamilton p. 763. Why do they differ? (PDF-REPORT)
3. Create a critical values table for the Dickey-Fuller Test for Case 1 for sample size $T = 25$ and $T = 100$ and for the 0.01, 0.025, 0.05, 0.1, 0.9, 0.95, 0.975, and 0.99 Quantile. (PDF-REPORT)

Programming - Case 2:

1. Use the procedure from Case 1 and simulate the random walk (see Equation (4)) with $T = 100$, $\rho = 1$, $\alpha = 0$, $\sigma^2 = 1$.
2. Estimate the unknown ρ and α parameters in Equation (2) with OLS. Hint: Order the matrix that contains your regressors as follows: $X = y_{t-1} \sim$ vector of constants.
3. Simulate the previous two steps $n=10000$ times. Store the parameter estimates for ρ and the corresponding standard errors and t-values in a matrix.
4. Sort the estimated t-values into quantiles.
5. Create a critical values table for the Dickey-Fuller Test for Case 2 for sample size $T = 25$ and $T = 100$ and for the 0.01, 0.025, 0.05, 0.1, 0.9, 0.95, 0.975, and 0.99 Quantile. (PDF-REPORT)

Programming - Case 4:

1. Use the procedure from Case 1 and simulate a random walk (see Equation (4)) with $T = 100$, $\rho = 1$, $\alpha = 0$, $\sigma^2 = 1$.
2. Estimate the unknown ρ , δ and α parameters in Equation (3) with OLS. Hint: Order the matrix that contains your regressors as follows: $X = y_{t-1} \sim$ vector of constants \sim time trend.
3. Simulate the two steps $n=10000$ times. Store the parameter estimates for ρ and the corresponding standard errors and t-values in a matrix.
4. Sort the estimated t-values into quantiles.
5. Create a critical values table for the Dickey-Fuller Test for Case 4 for sample size $T = 25$ and $T = 100$ and for the 0.01, 0.025, 0.05, 0.1, 0.9, 0.95, 0.975, and 0.99 Quantile. (PDF-REPORT)

Interpretation:

A researcher wants to conduct a Dickey-Fuller test to test an economic time series for a unit root. He estimates a regression model of the form:

$$y_t = \alpha + \rho y_{t-1} + \delta t + \varepsilon_t$$

The researcher works under the null hypothesis that the true data generating process is given by:

$$y_t = y_{t-1} + \varepsilon_t$$

Running the regression the researcher computed the estimate $\hat{\rho} = 0.78$. The estimated OLS standard error $s.e.(\hat{\rho}) = 0.021$. The sample size is 100. Use your simulated critical values and interpret the result. (PDF-REPORT)

5th GAUSS assignment ATS

1. Loading Data and Data Description

1. First, load the data into a GAUSS matrix. The data_var.txt file contains Swiss seasonal adjusted macroeconomic data: quarterly Interest Rate in % (first column), quarterly Consumer Price Index (second column), quarterly Gross Domestic Product in Swiss (BIP) in Mio CHF (third column) and quarterly Money Stock M1 in Mio CHF (fourth column). The first observation is from the first quarter of 1974 and the last observation is from the first quarter of 2002. $T = 113$. Gauss hint: `loadm all[] = data_var.txt; all2 = reshape(all,113,4);`
2. Plot the four time series and describe the plot. Decide whether the time series includes
 - a) a constant
 - b) a constant and a time trend
 - c) no constant and no time trend. (PDF-REPORT)

2. Conducting Dickey Fuller Tests

1. Conduct a Dickey Fuller test of non stationarity for all four time series. Estimate a regression model.

Generally, the regression model is written as:

$$\Delta y_t = \beta_1 + \beta_2 y_{t-1} + \beta_3 t + \varepsilon_t$$

with $\Delta y_t = y_t - y_{t-1}$, i.e. first differences of y_t , t as time trend and β_1 as constant.

Based on your observations, if you decided that the series contains a constant, include β_1 in the regression. Else $\beta_1 = 0$. Based on your observations, if you decided that the series contains a time trend, include $\beta_3 \times t$ in the regression. Else $\beta_3 = 0$. For the first parameter estimate $\hat{\beta}_2$ use the standard error to compute the t-statistic for $H_0 : \beta_2 = 0$.

Compare your t -value with the corresponding Dickey-Fuller distribution and decide whether you can reject or not reject the null hypothesis of a unit root. Interpret your result. (PDF-REPORT)

2. Compute log differences of the series, i.e. $\tilde{y}_t = \Delta \log(y_t) = \log(y_t) - \log(y_{t-1})$.
3. Plot the series \tilde{y}_t and describe the plot. Decide whether the time series includes
 - a) a constant
 - b) a constant and a time trend
 - c) no constant and no time trend. (PDF-REPORT)
4. Conduct a Dickey Fuller test on \tilde{y}_t (for all four time series) and discuss your result. Does \tilde{y}_t contain a unit root? (PDF-REPORT)

6th GAUSS assignment ATS

1. Estimate a GARCH(1,1) for a return series

1. First, load the return data into a GAUSS matrix `garch.fmt`. You can load this files with the `load` command (look it up in the GAUSS Help).
2. Plot the return time series. Describe the stylized facts you observe.

A simple model to account for these stylized facts ist the GARCH model.
Mean equation (y_t is a log return time series):

$$y_t = c + \varepsilon_t$$

In the following we assume $c = 0$. Argue why this is a sensible assumption. Therefore,

$$\varepsilon_t = y_t$$

Variance equation:

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \quad (5)$$

Writing the conditional likelihood function GARCH(1,1):

$$f(y_t|y_{t-1}, \dots, y_0; \boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi h_t}} \exp \left[\frac{-(y_t - c)^2}{2h_t} \right]$$

Conditional log likelihood function:

$$\begin{aligned} \mathcal{L}(\boldsymbol{\theta}) &= \sum_{t=1}^T \log f(y_t|y_{t-1}, \dots, y_0; \boldsymbol{\theta}) \\ &= -\frac{T}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^T \log(h_t) - \frac{1}{2} \sum_{t=1}^T \frac{(y_t - c)^2}{h_t} \end{aligned}$$

3. Write a procedure that estimates the parameters of an GARCH(1,1) process. Hint: First read all following remarks. Then work through them step by step.

First build the h_t of GARCH(1,1) model. Hint: You need starting values for h , ω and α , β . The starting value of the h sequence is an appropriate guess of ω and α , β for computing the unconditional expectation of the GARCH(1,1) process in Equation (5), i.e. $h = \omega / (1 - \alpha - \beta)$ with $\omega = 0.1$, $\alpha = 0.1$, and $\beta = 0.6$.

Second compute the value of the log likelihood function by plugging in ε_t and the conditional variance sequence of h_t .

Now write the rest of the procedure around the procedure-body. Hint: Do not forget to define locals. Input variables of the procedure are the data y and the parameters that are estimated. (ω, α, β . The parameters enter the procedure as one column vector!)

Output variable is a column vector that contains the likelihood contribution of each observation.

4. Call the procedure and check your output.
5. In a GARCH estimation you have to ensure that the left hand side variable of Equation (5) is always positive. You have to impose restrictions on your parameters. Modify your procedure the following way: $parameter = \exp(parameter^*)$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$$

The estimated equation is then

$$h_t = \exp(\omega^*) + \exp(\alpha^*) \varepsilon_{t-1}^2 + \exp(\beta^*) h_{t-1}$$

6. Use CML to estimate the unknown parameters; $\omega^*, \alpha^*, \beta^*$. Take as starting values the log of the starting values given above. Choose as global variables:


```

_cml_Algorithm = 1;
_cml_LineSearch = 2;
_cml_DirTol = 1e-5;
_cml_CovPar = 2;
      
```
7. After Estimation. Transfer the estimated parameter back by: taking $parameter = \exp(parameter^*)$.

2. Compute standard errors using the delta method

1. To compute standard errors of the estimators ω, α, β the delta method is required. Compute standard errors of the estimators using the delta method. Gauss hint: gradp.

First, write a procedure that returns you the nonrestricted parameters. Input variable is one column vector θ^* containing $\omega^*, \alpha^*, \beta^*$. Output variable is one column vector θ containing ω, α, β .

Second, write a procedure that returns the estimated variance covariance matrix of $\theta = \omega, \alpha, \beta$. Input variables are estimated θ^* and the estimated variance covariance matrix of θ^* .

2. Test the null hypothesis $H_0 : \beta = 0.7$. $H_A : \beta \neq 0.7$. Compute p-values of the test-statistic. Interpret. Gauss hint: cdfnc

Delta Method

Suppose that $\{\mathbf{x}_n\}$ is a sequence of K -dimensional random vectors such that $\mathbf{x}_n \xrightarrow{p} \boldsymbol{\beta}$ and

$$\sqrt{n}(\mathbf{x}_n - \boldsymbol{\beta}) \xrightarrow{d} N(\mathbf{0}, \boldsymbol{\Sigma})$$

then

$$\sqrt{n}(\mathbf{a}(\mathbf{x}_n) - \mathbf{a}(\boldsymbol{\beta})) \xrightarrow{d} N(\mathbf{0}, \mathbf{A}(\boldsymbol{\beta})\boldsymbol{\Sigma}\mathbf{A}(\boldsymbol{\beta})')$$

where $\mathbf{A}(\boldsymbol{\beta})$ is the matrix of continuous first derivatives of $\mathbf{a}(\boldsymbol{\beta})$ evaluated at $\boldsymbol{\beta}$:

$$\mathbf{A}(\boldsymbol{\beta}) = \frac{\partial \mathbf{a}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}'}$$

Hint: Look in the GAUSS help how the `gradp` function works.

7th GAUSS assignment ATS

1. Loading Data and Data Description

1. First, load the data into a GAUSS matrix. The .txt file contains Swiss seasonal adjusted macroeconomic data: quarterly Interest Rate in % (first column), quarterly Consumer Price Index (second column), quarterly Gross Domestic Product in Swiss (BIP) in Mio CHF (third column) and quarterly Money Stock M1 in Mio CHF (fourth column). The first observation is from the first quarter of 1974 and the last observation is from the first quarter of 2002. $T = 113$. Gauss hint: `loadm all[] = data_var.txt; all12 =reshape(all,113,4);`

- m = money stock M1
- r = quarterly average of three month Swiss franc LIBOR rate of interest
- p = Consumer price index
- y = GDP in 1990 Swiss francs

Create the following series: $\tilde{m}_t, \tilde{r}_t, \tilde{p}_t, \tilde{y}_t$

Denote: $\tilde{m}_t = \Delta \log m_t; \tilde{r}_t = \Delta r_t; \tilde{p}_t = \Delta \log p_t; \tilde{y}_t = \Delta \log y_t;$

($\Delta \log m_t, \Delta r_t, \Delta \log p_t, \Delta \log y_t$) with $\Delta \log p_t = \log p_t - \log p_{t-1}$

2. Conducting Dickey Fuller Tests

1. Proceed as described in the 5th assignment. Conduct Dickey Fuller Tests for: $p_t, y_t, m_t, r_t, \tilde{p}_t, \tilde{y}_t, \tilde{m}_t, \tilde{r}_t$ and interpret the results. (PDF-REPORT)
2. Estimating a SVAR, why would you be interested in testing the time series for non-stationary? (PDF-REPORT)

3. Estimation of a structural VAR

You are interested to model the effects of Swiss Monetary policy. You consider the following four ($k = 4$) Swiss variables for your analysis: the consumer price index, the GDP in 1990 Swiss francs, the money stock M1 variable, and the quarterly average of three month Swiss franc LIBOR rate of interest.

The GAUSS code for this assignment sheet uses a procedure collection written for VAR estimation (`var_code.src`) by Paul Fackler.

Structural Vector Autoregression (SVAR) in primitive form

$$Bx_t = \Gamma_0 + \Gamma_1 x_{t-1} + \varepsilon_t$$

$$E(\varepsilon_t \varepsilon_t') = \begin{cases} D & \text{for } t = \tau \\ 0 & \text{otherwise.} \end{cases}$$

$$\begin{aligned} \tilde{m}_t &= \gamma_{10} & + & \gamma_{11}\tilde{m}_{t-1} + \gamma_{12}\tilde{r}_{t-1} + \gamma_{13}\tilde{p}_{t-1} + \gamma_{14}\tilde{y}_{t-1} + \varepsilon_{1t} \\ \tilde{r}_t &= \gamma_{20} - b_{21}\tilde{m}_t & + & \gamma_{21}\tilde{m}_{t-1} + \gamma_{22}\tilde{r}_{t-1} + \gamma_{23}\tilde{p}_{t-1} + \gamma_{24}\tilde{y}_{t-1} + \varepsilon_{2t} \\ \tilde{p}_t &= \gamma_{30} - b_{31}\tilde{m}_t - b_{32}\tilde{r}_t & + & \gamma_{31}\tilde{m}_{t-1} + \gamma_{32}\tilde{r}_{t-1} + \gamma_{33}\tilde{p}_{t-1} + \gamma_{34}\tilde{y}_{t-1} + \varepsilon_{3t} \\ \tilde{y}_t &= \gamma_{40} - b_{41}\tilde{m}_t - b_{42}\tilde{r}_t - b_{43}\tilde{p}_t & + & \gamma_{41}\tilde{m}_{t-1} + \gamma_{42}\tilde{r}_{t-1} + \gamma_{43}\tilde{p}_{t-1} + \gamma_{44}\tilde{y}_{t-1} + \varepsilon_{4t} \end{aligned} \quad (1)$$

Note: Ordering of the variables is important!!

Writing the VAR in standard form "solves" the system

$$x_t = B^{-1}\Gamma_0 + B^{-1}\Gamma_1x_{t-1} + B^{-1}\varepsilon_t$$

$$x_t = A_0 + A_1x_{t-1} + e_t$$

$$\begin{aligned}\tilde{m}_t &= a_{10} + a_{11}\tilde{m}_{t-1} + a_{12}\tilde{r}_{t-1} + a_{13}\tilde{p}_{t-1} + a_{14}\tilde{y}_{t-1} + e_{1t} \\ \tilde{r}_t &= a_{20} + a_{21}\tilde{m}_{t-1} + a_{22}\tilde{r}_{t-1} + a_{23}\tilde{p}_{t-1} + a_{24}\tilde{y}_{t-1} + e_{2t} \\ \tilde{p}_t &= a_{30} + a_{31}\tilde{m}_{t-1} + a_{32}\tilde{r}_{t-1} + a_{33}\tilde{p}_{t-1} + a_{34}\tilde{y}_{t-1} + e_{3t} \\ \tilde{y}_t &= a_{40} + a_{41}\tilde{m}_{t-1} + a_{42}\tilde{r}_{t-1} + a_{43}\tilde{p}_{t-1} + a_{44}\tilde{y}_{t-1} + e_{4t}\end{aligned}\tag{2}$$

The innovations of a VAR in standard form are, by construction, contemporaneously correlated (composite innovations/shocks).

⇒ Estimate the VAR in standard form (see Equation (2)).

GAUSS Hint: `ar = var(x,lags,z,nameout);`

Read the documentation of the procedure code to apply the correct global settings. Note: The last row of the `ar` matrix contains the constant. The first k rows contain the matrix of parameter estimates for the first lag. The second k rows contain the matrix of parameter estimates for the second lag. Etc...

To obtain the idiosyncratic shocks (ε_t) from the composite shocks (e_t) we need the structural parameters, the matrix B

covariance matrix of e_t :

$$\mathbb{E}(e_t e_t') = \Omega$$

relation between shocks: $e_t = B^{-1}\varepsilon_t$

$$\begin{aligned}\mathbb{E}(e_t e_t') &= B^{-1}\mathbb{E}(\varepsilon_t \varepsilon_t') [B^{-1}]' \\ &= B^{-1}D [B^{-1}]'\end{aligned}$$

To identify the structural parameters B , we decompose the variance covariance matrix of composite innovations (Cholesky-Decomposition of Ω).

⇒ Compute estimates of B and D .

(PDF-REPORT)

1. Report estimated equations of the VAR in standard form in Equation (2). (PDF-REPORT)
2. Report the matrices A , D and B .(PDF-REPORT)
3. Report estimated equations of the SVAR in primitive form in Equation (1). (PDF-REPORT)

some maths:

For any real symmetric positive definite matrix Ω exists a lower triangular matrix A with ones along the principal diagonal and a unique diagonal matrix D with positive elements on the principal diagonal

$$\Omega = ADA'$$

i.e. $A = B^{-1}$

Ω : real symmetric positive definite matrix

A : lower triangular matrix with ones along the principal diagonal

D : diagonal matrix with positive elements

$$\Omega = ACC'A' \text{ with } C = \begin{bmatrix} \sqrt{d_1} & 0 & 0 & \dots & 0 \\ 0 & \sqrt{d_2} & 0 & \dots & 0 \\ 0 & 0 & \sqrt{d_3} & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & \sqrt{d_k} \end{bmatrix}$$

$$D = CC' = \begin{bmatrix} d_1 & 0 & 0 & \dots & 0 \\ 0 & d_2 & 0 & \dots & 0 \\ 0 & 0 & d_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & d_k \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ a_{21} & 1 & 0 & \dots & 0 \\ a_{31} & a_{32} & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ a_{k1} & a_{k2} & a_{k3} & \dots & 1 \end{bmatrix}$$

GAUSS hint:

```
load omega = varcv;
```

either compute Y by hand (look `chol` up in the reference!):

```
W = chol(omega); /* note: W'*W = omega*/
```

or compute Y via the program code:

```
{y_inv,W} = reca0(omega,ind);
```

Note: `ind` gives the ordering, check the descriptions in the code.

Compute Y :

```
Y = W';
```

```
C = (diag(Y)' .* ones(k,k) .* eye(k));
```


8th GAUSS assignment ATS

Impulse Response and Variance Decomposition

1. Loading Data and Data Description

You are interested to model the effects of Swiss Monetary policy. The data set `data_var.txt` contains the consumer price index, the GDP in 1990 Swiss francs, the money stock M1 variable, and the quarterly average of three month Swiss franc LIBOR rate of interest. (see 7th assignment for loading data.)

2. Estimating Vector Autoregression Models

The GAUSS code for this assignment sheet uses a procedure collection written for VAR estimation (`var_code.src`) by Paul Fackler. We estimate a VAR model with GAUSS. In the 7th assignment you find the model written in primitive and standard form.

1. First, use the procedure `var()` to estimate the VAR model. (This should be already done in the 7th assignment. Read the documentation of the procedure code to apply the correct global settings.) GAUSS hint: e.g. `ar = var(x,lags,z,nameout);`
2. Use the procedure `reca0()` to perform Cholesky decomposition of the innovation covariance matrix. GAUSS hint: e.g. `{y_inv,W} = reca0(omega,ind);`

3. Produce impulse response functions

1. To compute impulse response functions you need the coefficient matrices of the VMA representation. Use the procedure `vma()` to back out the VMA parameters. Report the MA coefficient matrix for the first lag. (PDF-REPORT) GAUSS hint: e.g. `ma = vma(ar,lags,malags);` Denote `malags` as the number of lags used in the VMA. `malags` is 10. The last row of the `ma` matrix contains the constant. The first k rows contain the matrix of parameter estimates for the first lag. The second k rows contain the matrix of parameter estimates for the second lag. Etc...
2. In order to examine the effect of a shock in one variable on all variables in the VAR compute impulse response functions with the procedure `impulse()`. Plot your results in a comprehensive way, i.e. plot the response of one variable to its own shock and the shock in the other three variables. (PDF-REPORT)
GAUSS hint: e.g. `irf = impulse(ma,y_inv,outcode);`
Note: The `irf` collects in the rows the time and in the columns the IR function values. The first column corresponds to the responds of the first variable to a shock in the first variable. The second column corresponds to the responds in the second variable to a shock in the first variable. etc... The fifth column corresponds to the responds in the first variable to a shock in the second variable. The sixth column corresponds to the responds in the second variable to a shock in the second variable. etc...
3. Describe and compare patterns of impulse response functions by answering following questions: How big/small are the responses to shocks, i.e. the response of a variable to

its 'own' shock and to the shock of other variables? How long do shocks persist in the series? (PDF-REPORT)

4. Conduct a variance decomposition

1. Conduct a variance decomposition with the procedure `fedecom` for each economic variable and plot your results in a comprehensive way. (PDF-REPORT)
GAUSS hint: e.g. `varcom = fedecom(irf,outcode);`
Note: The `varcom` collects in the rows the time and in the columns the Variance Decomposition values. The first column corresponds to percent of the variance in the first variable due to the first variable. The second column corresponds to percent of the variance in the first variable due to the second variable. etc... The fifth column corresponds to percent of the variance in the second variable due to the first variable. The sixth column corresponds to percent of the variance in the second variable due to the second variable.
2. Discuss the results and draw a conclusion from the plots of the variance decomposition. In particular analyze how a shock in the GDP affects the other variables e.g. How is the proportion of the movements in a sequence due to its 'own' shocks versus shocks to the other variables? (PDF-REPORT)

9th GAUSS assignment ATS: Cointegration - Engle and Granger Method

1. Loading Data and Data Description

Load the fmt-File data_abx.fmt. The file contains data on a Canadian Stock (ABX- Barrick Gold Corporation), which is traded on the Toronto Stock Exchange and the New York Stock Exchange simultaneously. The data includes the first two hours of trading, ranging from 1st January 2004 to 31st March 2004, sampled at 5 minute intervals.

The first column of the data matrix consist of the time series of the log home market (TSX) midquotes $((bid+ask)/2)$, the second contains the log foreign market (NYSE) midquotes , the log CAD/USD exchange rate is included in the third column, and in the last column you find a dummy variable indicating the first observation of a day (i.e., indicator = 1, if it is the first observation of the day, otherwise zero)

2. Preliminary Analysis

1. After loading the data into Gauss, convert the NYSE midquote series into Canadian dollars (note: we are talking about log series here!).
2. Plot the home market and the converted foreign market midquote series into one graph. Interpret the graph. Which cointegration vector would you suggest? (PDF-REPORT)
3. Conduct a Dickey Fuller unit root test for both series. Include a constant but no time trend into the estimated regression. Interpret and report the result. (PDF-REPORT)

3. Engle-Granger Method, Step 1: Auxiliary Regression

1. In order to estimate the normalized cointegration vector, run a regression of the TSX midquotes (y_{1t}) on the NYSE midquotes (y_{2t}) , back out the estimated parameter and the residual series. (If you are unsure whether or not to include a constant in the regression, have look again at the graph you've made above).
2. What is the estimated cointegration vector? What does *normalized* mean in this context? (PDF-REPORT)
3. Test the residual series for stationarity. Interpret and report the result. (PDF-REPORT)

4. Engle-Granger Method, Step 2: VECM Estimation

1. Create return series for the midquotes (i.e. first differences). Since we want to exclude overnight returns from our estimation, the first observation of each day has to be deleted. To do so, make use of the first day indicator and the *delif* comand (Look it up in the command reference). Note: You have to apply the same selection to the residual series to keep both series at the same dimension!
2. Estimate the VECM using the residual series as the error correction term. (i.e. the matrix of the two return series is the dependent variable matrix, the regressor matrix consists of ones (constant), the residual series and the first lag of the return series):

$$\begin{aligned}\Delta y_{1t} &= a_{10} + \gamma_1 \hat{\varepsilon}_{t-1} + a_{11} \Delta y_{1t-1} + a_{12} \Delta y_{2t-1} + u_{1t} \\ \Delta y_{2t} &= a_{20} + \gamma_2 \hat{\varepsilon}_{t-1} + a_{21} \Delta y_{1t-1} + a_{22} \Delta y_{2t-1} + u_{2t}\end{aligned}\tag{1}$$

3. Back out the estimated parameters. Interpret and report estimated equations of the VECM in Equation (1). (PDF-REPORT)
4. What could be a sensible interpretation of the correction terms coefficients (γ)? (PDF-REPORT)

10th GAUSS assignment ATS

1. Theoretical Background

Using US data on consumption C_t , investment I_t , and output, Y_t , we want to estimate the parameters of a Vector Error Correction model by Maximum Likelihood. Assuming that three variables follow a VECM of order one that can be written as:

$$\Delta y_t = \zeta_1 \Delta y_{t-1} + \alpha + \zeta_0 y_{t-1} + \varepsilon_t,$$

where $y_t = \begin{pmatrix} C_t \\ I_t \\ Y_t \end{pmatrix}$, ζ_1 is a (3×3) parameter matrix, α a (3×1) vector of constants and

$$\zeta_0 = -BA'.$$

We know that there exist two cointegration relations between those three variables, and therefore B and A are (3×2) matrices, where B contains the adjustment coefficients and A denotes the cointegration matrix, which if normalized as proposed by Phillips (see Hamilton page 576) is of the form :

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ a_{31} & a_{32} \end{pmatrix}$$

The disturbances ε_t are Gaussian and the conditional log-likelihood function is:

$$\begin{aligned} \log L(\Omega, \zeta_1, \alpha, \zeta_0) &= \left(-\frac{Tn}{2}\right) \ln(2\pi) - \left(\frac{T}{2}\right) \ln |\Omega| \\ &- \frac{1}{2} \sum_{t=1}^T [(\Delta y_t - \zeta_1 \Delta y_{t-1} - \alpha - \zeta_0 y_{t-1})' \Omega^{-1} (\Delta y_t - \zeta_1 \Delta y_{t-1} - \alpha - \zeta_0 y_{t-1})] \end{aligned}$$

where Ω denotes the covariance matrix of the error terms, n is the number of variables and T the number of observations.

2. Data Processing

1. Load the data contained in `ciy.txt` into Gauss. The file contains quarterly US data for the period 1947Q1 - 1988Q4 for the three variables log consumption (column one), log investment (column two), and log output (column three), the number of observations for each variable is 168.
2. Generate the following four matrices: the first contains the three variables as they have been read in from the data file, the second includes the lag of the three variables, the third the first differences, and the fourth the first lag of the differences. Make sure that there are no missings in any of these matrices and that all have the same dimensions.

3. Computing the log-likelihood Function

We will estimate the system by the Gauss procedure `cml` and therefore we need a procedure that returns the log-likelihood contribution of each observation. Before you start, please read all points listed below and work through them step by step.

1. Read in the vector contained in x1.fmt. It contains the starting values for $x0$.
2. All parameters needed for the log-likelihood will be passed into the procedure by a column vector. Consequently, we have to read out the respective elements from this column vector and reshape them into their former dimensions. The vector of parameters is assumed to be structured as follows:

$$x0 = \begin{pmatrix} \zeta_{11} \\ \zeta_{12} \\ \zeta_{13} \\ \zeta_{21} \\ \zeta_{22} \\ \zeta_{23} \\ \zeta_{31} \\ \zeta_{32} \\ \zeta_{33} \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \omega_{11} \\ \omega_{22} \\ \omega_{33} \\ \omega_{21} \\ \omega_{31} \\ \omega_{32} \\ a_{31} \\ a_{32} \\ b_{11} \\ b_{21} \\ b_{31} \\ b_{12} \\ b_{22} \\ b_{32} \end{pmatrix}$$

Read out the elements of ζ_1 and reshape them into a (3×3) matrix.

Next build a column vector that contains the constants $(\alpha_1, \alpha_2, \alpha_3)'$

Then rebuild the normalized cointegration matrix A and the matrix B by addressing the according elements in the vector.

As far as the covariance matrix Ω is concerned, we will use a little trick to make sure that the covariance matrix will always stay positive definite. We define: $\Omega = PP'$ (Cholesky decomposition). Build a matrix P as given below:

$$P = \begin{pmatrix} \omega_{11} & 0 & 0 \\ \omega_{21} & \omega_{22} & 0 \\ \omega_{31} & \omega_{32} & \omega_{33} \end{pmatrix}$$

and get Ω .

3. Compute a column vector that contains the log-likelihood contribution of each observation, i.e. compute the log-likelihood function.
4. Write a procedure around the reshaping commands and the log-likelihood. Output argument is the value of the log-likelihood function. Input arguments are the parameter vector $x0$ and the data set (in our case we pass the data into the procedure via globals, but cml demands those two input arguments). Do not forget to define local variables in the likelihood procedure.

4. CML Estimation

1. Use for starting values x_0 the vector contained in `x1.fmt`.
2. Further use the following options in `cml`:

```
cmlset;
_cml_GridSearch=1;
_cml_algorithm=3;
_cml_lineSearch=2;
_cml_covpar=2;
_cml_GradMethod=1;
_cml_DirTol=1e-4;
__output=1;
```

Since we estimate 26 parameters it might be of help to define labels for the estimated coefficients. This can be done by:

```
_cml_parnames=
"zeta11"|"zeta12"|"zeta13"|
"zeta21"|"zeta22"|"zeta23"|
"zeta31"|"zeta32"|"zeta33"|
"const1"|"const2"|"const3"|
"omega11"|"omega22"|"omega33"|
"omega12"|"omega31"|"omega32"|
"a31"|"a32"|
"b11"|"b21"|"b31"|
"b12"|"b22"|"b32";
```

3. Call `cml` and estimate the parameters. Report the estimated parameters and interpret your results. (PDF-REPORT)

5. Likelihood Ratio Test

1. Estimate the unrestricted model, i.e. ζ_0 is no more equal $-BA'$, but simply a (3×3) parameter matrix. Report the estimated parameters. (PDF-REPORT) The starting values are contained in x2.fmt in the following order:

$$x0 = \begin{pmatrix} \zeta_{11}^1 \\ \zeta_{12}^1 \\ \zeta_{13}^1 \\ \zeta_{21}^1 \\ \zeta_{22}^1 \\ \zeta_{23}^1 \\ \zeta_{31}^1 \\ \zeta_{32}^1 \\ \zeta_{33}^1 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \omega_{11} \\ \omega_{22} \\ \omega_{33} \\ \omega_{21} \\ \omega_{31} \\ \omega_{32} \\ \zeta_{11}^0 \\ \zeta_{12}^0 \\ \zeta_{13}^0 \\ \zeta_{21}^0 \\ \zeta_{22}^0 \\ \zeta_{23}^0 \\ \zeta_{31}^0 \\ \zeta_{32}^0 \\ \zeta_{33}^0 \end{pmatrix}$$

Use the following cml settings:

```
cmlset;
_cml_GridSearch=1;
_cml_algorithm=1;
_cml_linesearch=2;
_cml_covpar=2;
_cml_GradMethod=1;
_cml_DirTol=1e-4;
__output=1;
```

2. Conduct a Likelihood Ratio Test: $LR = 2(L(\hat{\theta}) - L(\tilde{\theta}))$

where $L(\hat{\theta})$ denotes the the value of the log-likelihood function at the unrestricted estimates and $L(\tilde{\theta})$ the value of the log-likelihood function at the restricted estimates (Note: cml returns the mean log-likelihood, so it has to be multiplied by T to obtain $L(\hat{\theta})$ and $L(\tilde{\theta})$.) The test statistic's distribution does not follow a standard distribution, but requires simulation. Hamilton's Table B.10 on page 767 contains the quantiles of the simulated distribution. (Here we need a 5% level, case 2, $g = n - h = 3 - 2 = 1$, critical value = 8.083) Can we reject the restrictions? Report and interpret your result. (PDF-REPORT)

General Motors return and realized volatility series

