

Substructural Logics

Edited by

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Preface

The term *substructural logics*, a designation for logics with restricted structural rules, made its first public appearance at the conference dedicated to such logics that we organized in the *Seminar für natürlich-sprachliche Systeme* of the University of Tübingen on 7–8 October, 1990. The program of this conference was the following:

The Lambek Calculus

Joachim Lambek: Some remarks about residuated monoids

Johan van Benthem: Categorical structure and dynamic logic

Sergei Soloviev: Categories and logics with restricted structural rules

Relevant Logic

Michael Dunn: Gaggle theory — A common abstraction of Galois connections and residuation with applications to substructural logics

Kosta Došen: The first substructural logic

Robert Meyer: The semantics of entailment 0

Linear Logic

Jean-Yves Girard: Proof-nets — A new approach to syntax

Giovanni Sambin: The semantics of pretopologies

Andre Scedrov: Decision problems for propositional linear logic

BCK Logic

Viacheslav Grishin: Logic without contraction

Peter Schroeder-Heister: Proofs and assumptions

Hiroakira Ono: Phase structures and quantales

After the conference we invited our speakers and other experts to prepare papers for a collection of readings on substructural logics. The papers we received make this book.

Substructural logics belong to different traditions, they have various motivations, and the style of the authors writing about them varies considerably, though the results they have reached overlap to a great extent. Our ambition with the book of readings was, as with the conference, to bring together papers from these different traditions. We supposed that if they appeared within the covers of a single volume, it should become clear that they are dealing with the same subject.

Some of the papers in this book belong more or less clearly to one of the traditions delimiting the fields covered by our conference. Lambek's papers are about logics obtained by extending his syntactic calculus with other connectives. Their inspiration is in category theory and universal algebra, but they may also be taken as papers in proof theory.

The papers by Belnap, Kron, Meyer and Mares are in the tradition of relevant logic. Belnap's is a polemical piece defending the distributivity of the lattice of conjunction and disjunction in an area where Gentzen's rules suggest distribution should fail. This distribution is present in the relevant systems without contraction investigated proof-theoretically by Kron. His paper continues work on systems, not far removed from linear logic, that have been investigated in the relevant logic community since the early eighties. Meyer's and Mares' paper is the last (though it bears the least number) in a series of papers on Kripke-style models for relevant logic. It concentrates on systems with Boolean negation and modal operators.

Troelstra's paper is an introduction to linear logic in the recent tradition started by Girard.

The remaining papers are more difficult to assign to a single field.

Bunder's paper is about proof-theoretical matters related to linear implication, but it is inspired by the older tradition of BCK logic. Soloviev's paper is also related to linear logic, which it approaches from the point of view of category theory with a proof-theoretical flavour.

Sambin's paper investigates a semantics for linear and other substructural logics motivated by a constructivist approach to point-free topology. Related to that is Ono's study of algebraic and Kripke-style models for substructural logics, which covers logics in all the four fields. Dunn's paper is part of his project to generalize algebraic ideas that underlie substructural logics.

Gabbay's contribution illustrates how investigations in non-monotonic logics may profit from an association with substructural logics. Finally, van Benthem is concerned with the interplay of ideas between substructural logics and related fields like theoretical linguistics.

We wish to thank first of all the authors that accepted our invitation to speak at the Tübingen conference or send us a paper for this collection. We are especially grateful to those of them who by their comments and suggestions helped us in editing this collection. We have received

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