Journal of Semantics 4 : 389–390

BOOK NOTE

G.E. Hughes and M.J. Cresswell, *A Companion to Modal Logic*. Methuen, London and New York, Pp. XVIII + 203.

Ulf Friedrichsdorf and Peter Schroeder-Heister

The title of this book might suggest that it is primarily a continuation of the authors' well-known "Introduction to Modal Logic" (1968). However, this characterization would be strongly misleading. The book is a self-contained presentation of modal logic from a different point of view. Whereas in "Introduction to Modal Logic" emphasis was put on syntactical features and decision algorithms, the new book is written from the semantical point of view and works entirely with Kripke-style model theory. More importantly, it tries to treat a variety of modal systems in a uniform manner. Thus the authors place special emphasis on the technique of canonical models for completeness proofs and on the characterization of logical systems by classes of models or frames. Naturally, they confine themselves to normal systems and mainly to the propositional case. Many recent results, in particular those of van Benthem, Fine and Segerberg, are included.

The first two chapters introduce the standard normal propositional systems K, T, B, S4, S5. and their semantics, and prove completeness using canonical models. Chapter 3 presents some recent results about the characterization of systems with classes of models whose accessibility relation fulfills certain conditions. Although the authors' treatment does not incorporate the most advanced results that can be found in the literature, the presentation of what is also called the "correspondence theory" is entirely satisfactory. Chapter 4 introduces a notion of completeness in the absolute sense as opposed to the completeness with respect to a class of models. A system is called complete if it can be characterized by a class of frames (that each normal system can be characterized by a class of models is trivial – simply take the class which only contains the canonical model). As an example of a theory incomplete in this sense, the authors present the extension of K by the axiom VB: $MLp \vee L(L(Lq \supset q) \supset q)$ ("VB" for "van Benthem").

In Chapter 5 pseudo-epimorphisms between models and frames are defined which, roughly speaking, represent the accessibility relation of one model or frame in another. In addition, frames generated by one world are introduced. This leads to the result that certain (classes of) models or frames can play the role of others, e.g. that if a formula is valid on a frame it is valid on every generated subframe of that frame, or that any complete modal system is characterized by any class of frames for this system that contains all the generated frames for it.

Chapter 6 is devoted to frames of canonical systems. The rule of disjunction is established as sufficient for the frame of the canonical model of S being strongly generated, and a certain amalgamation property is shown to be sufficient for the rule of disjunction. It is proved that the system KW (also known as G, resulting from K by addition of $L(Lp \supset p) \supset Lp$), is complete but not canonical in the sense that the frame of its canonical model is a frame for it.

Chapter 7 discusses the use of subordination frames to obtain completeness results for standard systems without using canonical models and, in particular, without being forced to work with non-countable sets of worlds (remember that with canonical models, worlds are construed as sets of formulas). The method is first used for systems containing D (i.e. systems having $Lp \supset Mp$ or $M(p \supset p)$ as a theorem) and then extended to other systems.

Chapter 8 investigates finite models and proves the finite model property for various systems using the method of filtrations. This method is used to prove the completeness of KW(=G). Systems lacking the finite model property are also discussed in passing. Finally, Chapter 9 gives an outlook on modal predicate logic.

The book contains a bibliography of relevant literature, a glossary of technical terms, a list of axioms for propositional systems and an index. It is also attractively printed and bound.

Unfortunately, as the authors realized themselves, not all of the proofs in the book are correct. For this reason they wrote "A Companion to Modal Logic – Some Corrections" (*Logique et Analyse* 29 (1986), 41-51.). The corrections concern primarily the compactness of K4.3W and of S4.3.1., and, more importantly, the completeness of KW. Although the theorems in the book are right, this article shows that repairing the proofs will require a quite substantial rewriting.

In spite of this shortcoming, our judgment of the book is generally positive. It is very well written, and experts in modal logic as well as students with some general background in logic can profit from it. We used this text in a course on modal logic and found it excellent for this purpose. We strongly recommend "A Companion to Modal Logic" as an introduction to the subject.

Universität Konstanz
Fakultät für Mathematik
Postfach 5560
D-7750 Konstanz
Fed. Rep. Germany