The SURE_REACH Model for Motor Learning and Control of a Redundant Arm: From Modeling Human Behavior to Applications in Robots

Oliver Herbort, Martin V. Butz, and Gerulf Pedersen

Abstract The recently introduced neural network SURE_REACH (sensorimotor unsupervised redundancy resolving control architecture) models motor cortical learning and control of human reaching movements. The model learns redundant, internal body models that are highly suitable to flexibly invoke effective motor commands. The encoded redundancy is used to adapt behavior flexible to situational constraints without the need for further learning. These adaptations to specific tasks or situations are realized by a neurally generated movement plan that adheres to various end-state or trajectory-related constraints. The movement plan can be implemented by proprioceptive or visual closed-loop control. This chapter briefly reviews the literature on computational models of motor learning and control and gives a description of SURE_REACH and its neural network implementation. Furthermore, we relate the model to human motor learning and performance and discuss its neural foundations. Finally, we apply the model to the control of a dynamic robot platform. In sum, SURE_REACH grounds highly flexible task-dependent behavior on a neural network framework for unsupervised learning. It accounts for the neural processes that underlie fundamental aspects of human behavior and is well applicable to the control of robots.

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1 Introduction

Virtually all of the brain's capabilities, from the simplest automatic mechanisms to the most complex cognitive operations, are mediated by the human motor system. Only movements of our bodies can cause consistent manipulation of the environment. Thus, understanding the human motor system is of paramount importance to understand human behavioral control and the involved cognitive processes. However, many open questions remain in our understanding of how the brain translates our will into actual body movements.

Despite this lack of explicit knowledge, our brains transform goals into movements exceedingly well and with astonishing ease. For example, movements are generally executed in a fast, accurate, and energy conserving way [23, 71]. Multiple sources of information are integrated when forming goals or during the control of ongoing movements [18, 47]. If obstacles block the way or our own mobility is restricted, the motor system adapts to these task-dependent constraints from one moment to the other and it even aligns the way we carry out current movements to facilitate future actions [17, 22, 61]. On top of these facts, it needs to be remembered that all these capabilities are acquired by unsupervised motor learning. In the CNS, cortical motor areas have been associated with the unsupervised acquisition of motor behavior and new motor skills [19, 25, 32].

The computational principles underlying motor learning and control are not yet very well understood. A review of existing computational models reveals that most models fall in either of two groups. Some models scrutinize how the fully developed motor system might work offering an account for the flexibility of human behavior, but these models do not account for motor learning (e.g. [14, 63, 64]). Other models focus mainly on the acquisition of motor control structures but they cannot explain the flexibility of human behavioral control (e.g. [3, 4, 7, 9, 40, 43, 49]).

The SURE_REACH neural network model of the cortical control of human reaching aims at integrating these aspects [11, 28, 30, 31]: It offers an account for unsupervised motor learning; It explains how humans can flexibly adapt to changing task constraints; And finally, it is based on plausible mechanisms and structures on the neural as well as functional level.

This is achieved by providing neural learning and control mechanisms that extracts as much information as possible about the relationship between motor commands and changes in sensory input. The abundance of information enables to flexibly generate novel movement pattern and thus adjust quickly to changing situations (c.f. [73] in this volume). This approach offers unprecedented flexibility in movement control and offers new perspectives on understanding motor learning, in both humans and robots (c.f. [60]; [66] in this volume).

In the remainder of this article, we first review related models of motor learning and control. Next, SURE_REACH is described, including the mathematical formulations of spatial representations, learning mechanisms, and the motor control networks. After that, several examples show that the model is well able to account for motor learning and flexible behavior. Additionally, the biological and theoretical foundations of the model are discussed. Finally, we review an application of the underlying framework to the flexible control of robots.

2 Theories of Motor Learning, Movement Preparation, and Control

To enable goal-directed action, the brain has to convert the representation of a goal into a sequence of efferent motor commands, which result in muscle activations and finally in overt body movements. A neural structure that encodes such a mapping from a desired goal location into motor commands is termed *inverse model* [42].[1] Inverse models are at the heart of any theory of goal-directed movement and closely relate to the "memory trace" in Adams' closed-loop theory [1] or the "recall schema" in Schmidt's schema theory [67] (for older accounts see: [6, 27, 52]). Each body reacts differently to different motor commands and thus, an inverse model for controlling movements needs to be acquired by body-dependent motor learning. Even more so, the learning mechanisms need to operate unsupervised from the beginning and without the help of an internal or external teacher. Finally, each possible goal might be reached by an abundance of different motor command patterns (due to motor redundancy). Thus, there is no easy way to directly acquire an inverse model as each goal may be reached with various, alternative movement patterns and the involved learning needs to be unsupervised.

Several theories address how such inverse models might be acquired (e.g. [3, 4, 9, 40, 41, 43, 49, 58]). These theories differ broadly regarding representations, controlled parameters, and learning mechanisms. However, they all have one aspect in common: All these theories assume that an *optimal* inverse model is acquired, which stores for each goal a specific movement that optimizes additional criteria. For example, it might encode those motor commands that would reach a goal location with the smoothest possible trajectory [74]. Some of these theories rely on learning mechanisms that require a one-to-one mapping between goals and motor commands (e.g. [3, 49, 58]). They fail to account for the acquisition of inverse models for more complex, redundant bodies the control of which requires the resolution of a one-to-many mapping. Other theories that are mostly addressing the acquisition of cerebellar control structures (e.g. [43, 41] but see [40]) require at least a coarse inverse model that serves as the teacher. Thus, these models fail to account for unsupervised learning. However, under certain conditions, it is possible to acquire an inverse model unsupervisedly [7, 9].

The very notion that inverse models directly map goals onto optimal (but fixed) sequences of motor commands is problematic. Such a model only encodes one possible way to reach a goal and neglects alternatives—particularly those that yielded worse performance at the time of motor learning but may actually prove advanta-

[1] Please note, the term "model" may refer to a scientific theory or the above-mentioned mapping in this text.

geous later on. Since the environment, the controlled body, and tasks change all the time, goals need to be reachable in different ways, depending on the current circumstances—sometimes we only need to reach a certain point in space, sometimes we also need to fulfill proprioceptive constraints, for example, when aligning hand and forearm to a pointing gesture. Likewise, we carry out one action in a way that facilitates the next one [22, 78] or we bypass obstacles with ease [17]. If only one way was represented to attain a certain goal, it is impossible to model the flexibility with which we adjust our movements to novel situations so rapidly. Thus, inverse models cannot implement a direct mapping from goals to (optimal) actions, but have to provide a set of useful action alternatives. A selection amongst those alternatives is then necessary to generate actual behavior.

Furthermore, from a computational point of view, a direct mapping from goals to motor commands implies that the process of mapping goals onto actions is rather simple and somewhat akin to the read-out of a neural look-up table. However, movement preparation in the brain seems to be a rather involved process. This is evident as the time to prepare a movement depends on many features of the goal, the response, and the context of the movement [45, 48, 56, 62]. Hence, theories that predicate a direct mapping from goals to motor commands fail to assign a meaningful role to the obvious complexity of movement preparation.

On the other hand, theory of task-dependent movement preparation exist. Most notably, the posture-based motion planning theory [63, 64, 65] details how different movement alternatives may be evaluated, selected, and refined according to the constraints of the situation. The theory accounts for a wide range of behaviors, including reaching, grasping, and the avoidance of obstacles. It presumes that neural mechanisms link sensory (goal) representations and motor commands but it is mute regarding their neural structure and their acquisition. Besides such abstract approaches, also neural network models for flexible motor behavior have not yet offered an explanation for motor learning [14, 15].

SURE_REACH integrates theories of motor learning and theories of task-dependent movement preparation into a biologically plausible neural network framework. It thereby extents related approaches by accounting for the unsupervised acquisition of internal sensorimotor models and flexible movement planning (e.g. [55]). The next section outlines the general approach of the model and details its structure.

3 Description of the model

Before the model is formulated in detail, an overview over the principles underlying the model is given. The discussion of the literature revealed that many conceptual problems of recent neural network models arise because the proposed learning mechanisms strive to only encode optimal motor commands. This causes some network approaches to fail at controlling a redundant body, others require supervised teaching mechanisms, and again others are left with a highly restrained behavioral repertoire. In contrast, SURE_REACH encodes *all* motor commands relevant for behavioral control. This can be quite easily achieved by ideomotor learning mechanisms. Such mechanisms encode contingent sensorimotor relationships, that is, they store which sensory effects result from which motor commands, depending on the initial state [21, 33, 34, 37]. Together, these sensorimotor contingencies represent how the body may be controlled and thus constitute a task-independent internal body model. However, they may not be directly used to generate movements. On the one side, for certain goals no sensorimotor contingencies may be stored that provide a single motor command to move from the initial state to the goal. This problem is solved by the combination of multiple sensorimotor contingencies. On the other side, many sensorimotor contingencies may provide useful motor commands in the current context, but, of course, only one motor command can be executed at a time. Thus, specific motor commands have to be selected. According to the model, these processes of combining and task-dependently selecting the encoded sensorimotor information is what happens during movement preparation: A task-independent, general body model is used to tailor a task-specific inverse model to the unique demands and constraints of the current situation. Thus, the model links behavioral flexibility, movement preparation processes, and the way humans learn to control their bodies in a meaningful, interdependent manner.

3.1 Neural Network Implementation

In the following, a more detailed account for the specific computational stages and processes is given. First, the simulated controlled body is briefly described. The neural networks have to control a planar kinematic arm with three joints. Each joint is attached to two “muscles” which rotate the joint proportional to the excitation level of the innervating motor neuron. Each muscle is activated by motor commands ranging between $0.0 \leq mc_i \leq 1.0$. To compute the final movement of a joint ϕ_i, activations of antagonistic motor commands are subtracted and the result is multiplied by a gain factor g which scales movement velocity (see appendix for parameter values):

$$\phi_i(t+1) = \phi_i(t) + g(mc_{2i-1} - mc_{2i}), i = 1,2,3$$

This body is surely rather simple but it incorporates two important features. First, in most cases a sequence of motor commands is necessary to reach a goal. Second, the arm is redundant on the kinematic (multiple postures may realize a hand position) and sensorimotor (multiple trajectories may realize transitions between postures) level.

Figure 1 shows the staged structure of the model, which reflects the common notion that goals, such as specific hand locations, are transformed stepwise into a sequence of motor commands [12, 26, 35, 39]. By including multiple layers of representations and different nested transformation processes, it is possible to account for the flexible incorporation of constraints of different modalities during movement preparation.

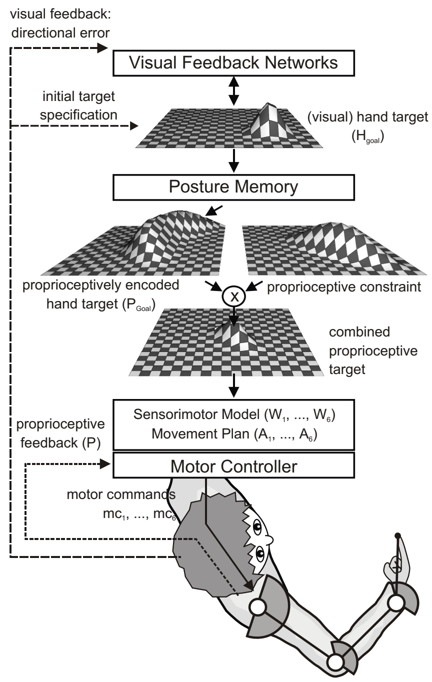

Fig. 1 The cartoon of the model reflects multiple stages of processing from the goal to motor commands. Intermediate representation and transformation processes may be adjusted to incorporate task-demands into the movement preparation process.

In the current implementation, SURE_REACH transforms a desired hand location into a sequence of motor commands, given certain constraints.[2] The individual transformations are realized by interacting, adaptive neural network modules. First, the *posture memory* module encodes a mapping from visual hand space to proprioceptive posture space. Second, the *sensorimotor module* encodes sensorimotor contingencies in posture space. Based on the sensorimotor model, movements are prepared and controlled in posture space.

3.1.1 Body Spaces Representation, Internal models, and Motor Learning

Before movements can be executed, the neural networks have to be learned. The posture memory has to encode the relationship between visually encoded hand positions and proprioceptively encoded arm postures. It stores the kinematic relationship between the two sensory modalities. The sensorimotor model has to encode the relationship between issued motor commands and state transitions in proprioceptive

[2] The origin of these goals or constraints is not part of the model.

posture space. It stores the relationship between action and perception. To simulate early infant movements, random motor commands are executed and related to the consequent sensory input.[3] Throughout the model, goals, sensory input, and motor output are represented by populations of neurons (population codes), in which each neuron represents a certain stimulus, such as a hand position or joint configuration [8, 13, 24, 69]. Hand coordinates are encoded by a population of neurons H. Each neuron h_i of H fires, if the hand coordinates (x,y) are close enough to the neuron's preferred hand location (h_i^x,h_i^y)

$$h_i = \max(1.0 - \frac{|x-h_i^x|}{d_{hand}};0) \cdot \max(1.0 - \frac{|y-h_i^y|}{d_{hand}};0),$$

where d_{hand} is the distance between the preferred values of neurons with adjacent receptive fields. The preferred hand locations are arranged in a grid and cover the arm's workspace. Arm postures are coded in a population of neurons P, where each neuron p_i is activated according to the equation

$$p_i = \prod_{j=1}^{3} \max(1.0 - \frac{|\phi_j - p_i^{\phi_j}|}{d_{posture,j}};0),$$

where $p_i^{\phi_j}$ are the preferred joint angles of each neuron p_i and $d_{posture,j}$ is the distance in the j-th dimension of joint space between the preferred values of neurons with adjacent receptive fields. During the initial learning movements, hand positions and postures are represented by different populations of neurons that constitute neural hand space and posture space. An associative learning rule strengthens the connections between simultaneously activated neurons in hand and posture space, thus creating a mapping between both representations. This mapping is termed posture memory. In each time step of the simulation the current hand (H) and arm state (P) are associated by Hebbian learning:

$$W_{PM}(t) = W_{PM}(t-1) + \varepsilon PH^T,$$

where ε is the learning rate and W_{PM} is the weight matrix that constitutes the posture memory.

The sensorimotor model consists of several neural representations of the posture space, each of which is associated to a specific motor neuron. If the motor neuron associated to an individual neural network is active, connections between the neuron representing the current posture and the neurons representing just visited postures are formed in the respective network. Together, these connections encode sensorimotor contingencies: They represent which transitions in posture space may occur, if the associated motor neuron is activated.

[3] This does not imply that infant movements are merely random, as is clearly not the case [76]. However, the neural networks are learned sufficiently well based on simple random movements. Learning speed and accuracy might be further improved if more structured exploratory movements were employed.

As the simulated arm is controlled by six different motor commands (two for each joint), there are six recurrent neural networks A_i each of which consists of a layer of mutually interconnected neurons. The neural layers A_i have the same size as the posture representation P and their interconnections are encoded by weight matrices W_i. During learning, the neural layers A_i have the following dynamics.

$$A_i(t) = \rho A_i(t-1) + mc_i(t-1)P(t-1),$$

where P is a representation of the current arm posture, ρ is a decay coefficient that enables the learning of temporally extended posture transitions by maintaining a trace of past posture representations, and mc_i is the activation of the i-th motor command during learning. Neural network weights are updated according to the following associative learning rule:

$$w_i^{jk}(t) = w_i^{jk}(t-1) + \delta a_i^j(t) p^k(t)(\theta - w_i^{jk}(t-1)),$$

where w_i^{jk}, a_i^j, and p^k are single values of the weight matrices, neuron layers, and the representation of the current posture, respectively, δ is the learning rate, and $\theta = 0.1$ is a ceiling value that prevent weights from increasing indefinitely.

3.1.2 Movement Preparation

The previous paragraphs revealed the model's space representations, internal models, and learning mechanisms. The following paragraphs answer how the acquired internal models may be used to plan movements. SURE_REACH accounts for movements to desired hand locations. The input to the model is thus a population encoded hand position. The desired hand position (H_{goal}) is transformed by the *posture memory* into a likewise encoded representation of *all* those arm postures that realize the respective hand location (P_{goal}; *proprioceptively encoded hand target* in Figure 1). It thus transforms the goal from a visual, hand-based into a proprioceptive, posture-based frame of reference. This is modeled by the equation

$$P_{goal} = W_{PM} \times H_{goal}.$$

The redundant representations of postures may be further constrained, for example, by inhibiting neurons which represent undesired final joint angles (see *proprioceptive constraint* in Fig 1). Movement planning is based on this redundant representation. The neural representation of acceptable end-postures is fed into the different neural networks of the motor controller whose connections constitute the sensorimotor model. The activity is propagated through these connections, the dynamics of which are modeled by the following equation:

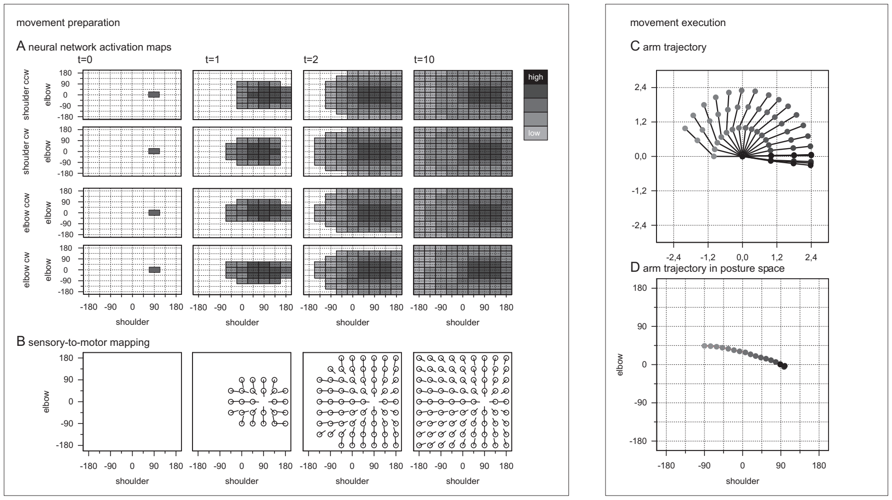

Fig. 2 A) The maps display the activation pattern of neurons of the sensorimotor model networks, which are associated to different motor neurons (rows), at various time points (columns, cw = clockwise, ccw = counterclockwise). White areas are not activated at all, dark areas are highly activated. The activation pattern constitute the neural basis of the movement plan or sensory-to-motor mapping. Please note, for illustration purposes, only those parts of the networks that are necessary to control shoulder and elbow are shown. The networks for control of a three joint arm are more complex. B) Motor commands may be derived from weighting the activations of neurons representing the same arm posture but in different networks (associated to different motor neurons). The arrows show the effects that the motor commands generated by the motor controller would have, depending on the actual arm posture. C) The chart shows the resulting arm movement, which starts from the left (light grey) and terminates at the rightmost posture (black). D) The trajectory in posture space leads quickly from the start posture to the target.

$$
\begin{aligned}
A_i^* &\leftarrow \max\{\beta(\gamma \frac{\sum_j^{j \neq i} A_j}{n-1} + (1-\gamma)A_i), P_{goal}\} \\
A_i &\leftarrow A_i^* + W_i \times A_i^*,
\end{aligned}
$$

where n is the number of neural networks, *max* returns the entry-wise maximum of two vectors, β reduces neural activity, γ specifies the intensity of crosstalk between networks, and P_{goal} is the representation of suitable goal postures normalized so that single values add up to 1.0.

Due to the learning scheme, activity is propagated to neurons that represent postures from which the goal can be reached by executing the motor command associated with the neuron's network. In each network, activity is propagated somewhat differently due to differing synaptic connectivity after learning. Thus, different patterns of activity emerge in the different networks. When the activities of neurons representing identical postures between the individual networks are compared, neurons of those networks are activated strongest whose associated motor neuron is most suitable to reach the goal. Thus, the relationship between the activities of neurons in the different networks defines the movement plan.

Figure 2A illustrates the movement preparation process. The network in the first row is associated to the motor neuron that causes counterclockwise shoulder rotations, the network in the second row is associated to the antagonistic motor neuron. In the first row, activity, which originates from the goal posture, is propagated stronger to the right, that is, to neurons that represent postures from which the goal can be reached by a counterclockwise shoulder rotation. In the second row, activity is propagated mostly to the left and hence to neurons that represent postures from which the goal can be reached by a clockwise shoulder rotation. Thus, after movement preparation, the neural activity pattern of the networks constitute the sensorimotor model for the neural representation of the movement plan (or sensory-to-motor mapping) because they encode which motor neurons should be activated to reach the goal given any possible posture. Note that both movement plan and sensorimotor model are represented in the same networks: the current movement plan is encoded in the activity of the neurons whereas the sensorimotor model is encoded in the neural connections.

The movement plan can be considered as an inverse model that is generated online for the forthcoming target. Furthermore, the movement preparation process can be adapted to different situational constraints. For example, neurons representing postures that would collide with an obstacle can be inhibited, resulting in different activation pattern and consequently a movement that bypasses the obstacle. Likewise, the contribution of connections that are associated with motor commands that cause undesirable movements can be limited and thus, for example, decrease or even prevent movements of certain joints.

3.1.3 Movement Execution

To execute the movement, the movement plan is read out in a closed-loop fashion. This is realized by generating motor neuron activations dependent on the relative activations of those neurons in the different sensorimotor model networks (that is, the movement plan) that represent the current arm posture. The read-out of the movement plan is modeled by the equation

$$\begin{aligned} mc_i^* &= P^T A_i \\ mc_i &= \frac{\max(mc_i^* - mc^*_{anta(i)};0)}{\sum_{i=1\ldots6}\max(mc_i^* - mc^*_{anta(i)};0)}, \end{aligned}$$

where P is the current posture, and $mc_{anta(i)}$ is the antagonistic motor command to mc_i. The equation computes a normalized (1-norm) set of motor commands that moves the arm towards the goal. Figure 2B shows the movements in joint space that would result from reading out the movement plan at different postures and at different time points during movement preparation. Figure 2C and D show an exemplar movement which implements the prepared movement plan. The exemplar movement starts in an area of posture space (light Grey dots in Figure 2D), where the activation of the network that is associated with the clockwise shoulder rotation is higher than the activations in other networks. Thus, a movement that is mostly based on a clockwise shoulder rotation is generated (Figure 2D). However, also the somewhat higher activation of networks associated to a counterclockwise elbow rotation contribute to the movement.

4 Simulation of Reaching Movements

The previous section describes the basic components and connectivity of the model. In this section, we review how the model accounts for highly flexible behavior. However, before any goal directed movements can be performed, the neural networks need to learn the relationships between hand positions and arm postures and the relationship between arm movements and motor commands. Figure 3A–G shows how the neural network controller performs when trying to execute goal directed movements after an increasing number of unsupervised learning iterations. Initially, the controller is not yet able to reach the target (Fig 3A–B). However, as more and more experience with the simulated arm is gathered, movements get increasingly accurate and efficient (Fig 3C–I). Finally, the arm can be moved accurately to all goals within the arm's reach (Fig 3H,J).

The evaluation of the model's learning performance shows not only that movement accuracy increases but also that movement preparation time (time needed until the movement plan is sufficiently prepared to generate movements) decreases. This reproduces a salient aspect of the interaction between movement preparation and motor learning [51, 53]. A detailed analysis of the general performance of the model during learning can be found elsewhere [11]. To summarize, the proposed neural networks and learning mechanisms are well able to learn to control a redundant arm.

4.1 Posture and Trajectory Redundancy

Next, we review how SURE_REACH can account for highly adaptive, flexible behavior. First the representation of redundant goal postures allows SURE_REACH to terminate a movement to a single hand target with different arm postures. Figure 4A–B show that that the final posture partially depends on the starting posture. A systematic analysis of such movements reveals that SURE_REACH exploits arm posture redundancy, as do humans [71], to reduce the overall trajectory length. Second, by simply inhibiting certain areas of the posture-based goal representations, movements to hand goals may be constrained to finally acquire a specific angle in a certain joint, for example, when aligning wrist and forearm to make a pointing movement (Fig 4C). Thus, it is possible to integrate visual (hand target) and proprioceptive constraints in the goal representation (Figure 1 exemplifies this process). Third, representing postural redundancy not only enables to adhere to constraints

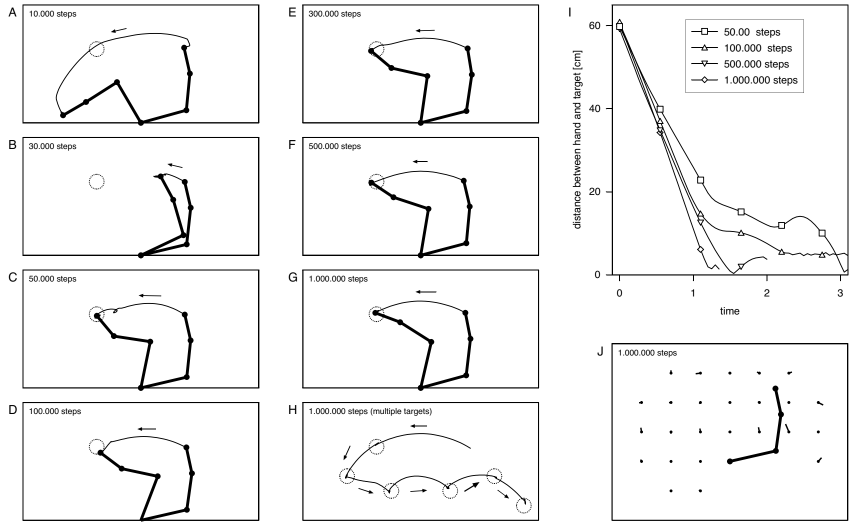

Fig. 3 A–G) The charts show the hand trajectory, initial and final arm posture of movements to an exemplar target (dotted circle) after different durations of motor learning. Movement accuracy and efficiency increases during learning. H) The chart shows the hand trajectories of movements to a number of targets after motor learning. I) The chart shows the distance between hand and target over the duration of the movements that are charted in C, D, F, G. With increasing durations of motor learning, movements get more efficient (faster) and more accurate. J) After learning, movements to reachable targets are quite accurate. Dots display targets, lines point to the actually reached hand position. If lines are not visible, the final hand position is within the respective dot. For the shown target, the average distance between hand and target is 1.80 cm (SD 1.31cm).

posed by the past (as the starting posture of a movement) or by the present (as constraints on a final joint configuration as in pointing) but also to constraints posed by future actions. This is especially important as movements are usually embedded in a larger sequence of actions and most movements can be carried out in ways that facilitate subsequent movements. Indeed, several experimental studies reveal how future goals of humans influence their present movements [46, 22, 38, 57, 70, 78]. In SURE_REACH, the representation of the redundant postures can be overlaid with a movement plan to a future goal to select those postures among the possible ones for an immediate movement that are also good starting postures for the subsequently planned movement, minimizing subsequent movement paths (Fig 5, see [28] for details). The resulting movements are then aligned to the overarching goals of a movement sequence. Recently, supporting evidence for this mode of movement planning has been provided [29]. Thus, representing redundant postures enables the anticipatory adjustment of movements to the demands of future goals (see [22] for a comparable account).

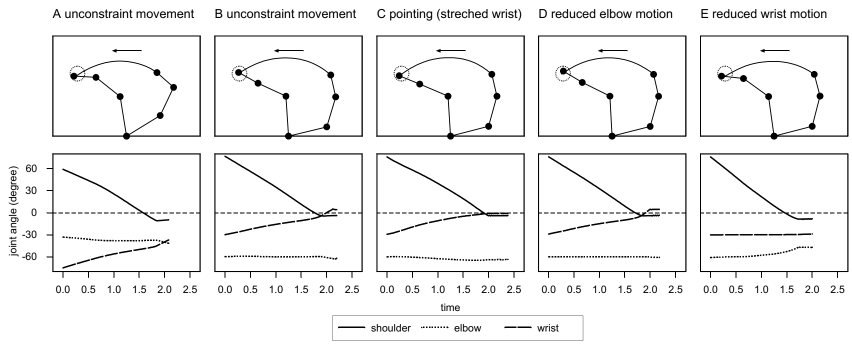

Fig. 4 The top charts show hand trajectories, initial and final arm postures. The dotted circles show the target of the hand. The bottom charts show the trajectory of the angle of shoulder, elbow, and wrist during those movements. A–B) Movements to identical hand targets may end in different arm postures, depending on the starting position. C) Targets can be reached while adhering to postural constraints, e.g. aligning forearm and wrist to make a pointing gesture. D–E) Movement planning may be adjusted to avoid moving specific joints, e.g. if these are in a cast or althralgic. The bottom charts show how such constraints influence the trajectories of individual joints.

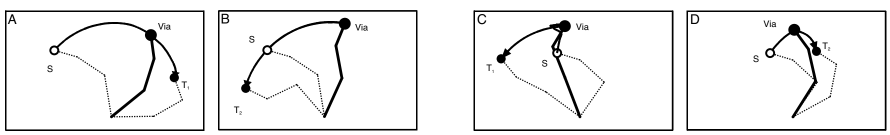

Fig. 5 The hand had to move sequentially from a start position (S) to a first target (Via) and then to one of two possible second targets (T_1 or T_2). The movements in A and B start from identical start postures and proceed to identical first targets (Via) but the second target differs (T_1 in A and T_2 in B). SURE_REACH can anticipate requirements of upcoming goals and thus choose different postures at the first target (compare black posture in A and B), dependent on the subsequent targets. The assumed posture at the first target facilitated the movement to the second target. Charts C and D show another example.

To summarize, the representation of posture redundancies enables the model to account for behavior in which the final state of the arm is not only defined by a desired hand location and a fixed optimality criterion, but also by other implicitly or explicitly defined constraints.

Behavior may get more adaptive by not only making use of the end-posture redundancy but also by adjusting the movement trajectory. As mentioned above, during movement preparation, small pieces of sensorimotor information are put together. Until now, no motor or trajectory constraints were imposed and the shortest movement from start to goal was prepared. This is not always desirable and thus it is possible to adjust the movement preparation process in different ways. To bypass an obstacle, for example, it is sufficient to simply inhibit neurons in the movement plan that represent postures that collide with an obstacle. Another example is reduced joint mobility. In some situations we might want to reduce the motion of an arthralgic joint or a joint that is immobilized by a cast. Movement preparation may be adapted to such inconveniences by trying to prepare a movement plan that relies mainly on executable motor commands and thus minimizes the motion of impaired joints (Figure 4D–E).

4.2 Multimodal Feedback Control

The described neural networks are able to use a visual goal representation to plan a movement, but so far visual feedback of the relationship between a goal and and the hand is not used during movement execution. However, human movement accuracy depends considerably on visual feedback [20, 44, 54, 79]. Due to the hierarchical structure of SURE_REACH, visual feedback can be integrated into the model without compromising its ability to account for flexible behavior [30]. In this case, *visual feedback networks* (Fig 1) decouple the hand-based goal representation from direct visual input and adjust an internal hand target according to a visual error signal (Fig 6A). For example, if the hand is currently slightly left of the target, the internal hand target may be shifted to the right. The hand-based internal target is then transformed into a posture-based representation that can be combined with kinematic constraints as described above. This enables a higher final goal reaching accuracy while keeping behavior flexible. Moreover, on the behavioral side this model of visual feedback generates human-like corrective movements. Figure 6 shows that the model reproduces movements with a fast initial approach component and slow final corrective movements, mimicking human reaching movements even closer [20, 79]. On the neurophysiological side, the model reflects the notion that movements are controlled by a cascade of nested control loops [12, 35].

5 Theoretical, Biological and Psychological Implications

The various simulation experiments have demonstrated three important claims. First, SURE_REACH is able to learn and control a redundant body with an unsupervised learning scheme. Its functionality thus exceeds other neural network models of unsupervised motor learning. Second, the encoded motor redundancy enables to account for behavioral flexibility to a high degree. Third, the simulation of specific features of human movement preparation and execution show that the implemented system has strong correlations with actual psychological processes.

More specifically, SURE_REACH contributes to the debate of the computational bases of movement control in several ways (for an overview see [10]). Neural network theories of motor learning typically assume that an inverse model that encodes one single "best" way to reach a goal is acquired during motor learning and invariably used later on. In contrast, SURE_REACH learns a task-independent body model, which encodes very general properties of the relationships between motor commands and body movements. These general properties may be easily acquired

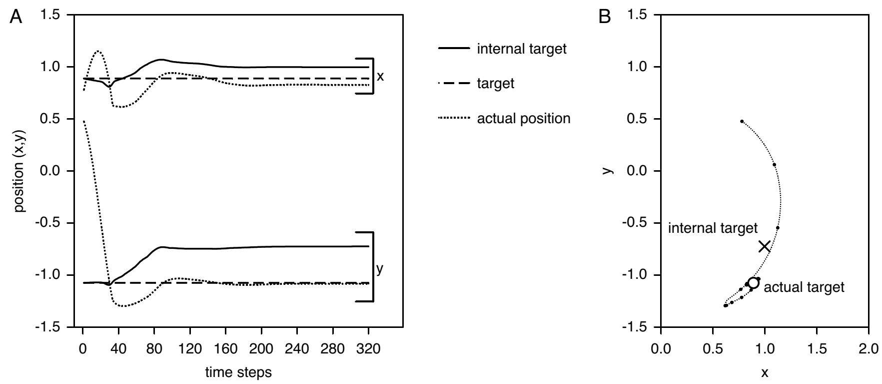

Fig. 6 A) The left chart shows the x- (top three lines) and y-coordinates (lower three lines) of the actual target (dashed), the internal target (black), and the hand location (dotted) of an exemplar movement. B) The right chart displays the hand trajectory, the actual target (circle), and the final position of the internal target (cross). The internal target shifts considerably to compensate for the initial overshoot. The charted location of the internal target is the mean of the preferred values of the dynamic target representation weighted by their activations.

unsupervised and may encode redundant motor control patterns. However, to make use of the body model, movement preparation processes are needed that extract a task-specific inverse model from the task-independent body model. Thus, while movement preparation is necessary to be able to use representations that can be learned unsupervised, the same movement preparation also enables the adjustment of motor plans to the demands and constraints of the current situation. Considering abstract theories of movement preparation, SURE_REACH offers a way to link such models to neural representations and sensorimotor learning mechanisms. In fact, as discussed elsewhere [11], many aspects of the posture-based motion planning theory [64] are realized with the proposed neural network architecture.

The model does not only offer an interesting computational account but it is also supported by neurophysiological and psychological findings. In SURE_REACH, sensory information or goals are represented by populations of many neurons in population codes. This property lays the foundation for unsupervised learning and the representation of motor redundancy but also reflects properties of cortical representations. Single-cell recording studies revealed that sensory, motor, and sensorimotor representations are shaped likewise in the motor cortex and parietal cortex of monkeys [13, 24, 69] and encoded in different coordinate systems, including posture based ones [2, 68].

SURE_REACH also fits in macroscopic theories of the brain, which consider the cerebral motor cortex — which is the site the model relates to — an area in which learning is unsupervised (as opposed to, e.g. the basal ganglia or the cerebellum, see e.g. [19, 36]). One of the key aspects of the model is its ability to account for the representation of redundant goal representations (that is, more than a single possible goal state). While corresponding representations have been recorded from motor areas during movement preparation [5, 13], behavioral studies and theoretical arguments suggest redundant goal representations during movement control [16, 50, 72]. Finally, as in humans or monkeys, movement preparation and movement control is more or less decoupled [5, 75], which enables the adjustment of ongoing movements or the preplanning but withholding of an upcoming movement. In conclusion, the model offers a comprehensive account of how humans learn to control their bodies and how their motor control mechanisms adapt flexibly to ever-changing situational requirements and constraints.

6 Application to Robots

While the previous sections have focused on SURE_REACH as a model for human behavior, this section addresses how the derived principles may be used to make robots more flexible. To this aim, the general idea of SURE_REACH needs to be extended. To control a dynamic robot arm dynamic control components need to be added and need to efficiently interact with the SURE_REACH-based control components.

A simulation of the KUKA KR16 arm was controlled by SURE_REACH enhanced with adaptive PD-control mechanisms [59]. Several modifications were necessary to make the application to the actual physical robot arm possible in the future. First, the learning mechanisms of SURE_REACH were replaced by hard-coded connections. This is possible due to the exact knowledge of the kinematics of the KUKA KR16 arm. Thus, while SURE_REACH shows that unsupervised learning of the neural network structures is possible, the KUKA KR16 arm application shows that it is not necessary given perfect prior knowledge of the targeted arm. Second, the muscle-based sensorimotor models were collapsed into one joint-motor-based sensorimotor model. This allows a more compact representation and thus a speed-up of the activity propagation in posture space—an important step to make real-time control possible. Third, an adaptive PD controller was added to translate the movement commands from SURE_REACH into actual motor control commands. Adding adaptivity to the controller helped to counteract increasing momentum in the KUKA KR16 arm. Fourth, the activity propagation signal was used to estimate the distance to the goal in order to provide the PD controller with movement direction and distance estimates. In effect, the resulting system was able to control the KUKA KR16 arm in simulation, which was realized with the commercially available simulation platform Webots [77].

Figure 7 shows two typical control sequences of the KUKA KR16 arm. In the upper panel, the arm has to move from a fully right-extended posture to a fully left-extended posture. Since the easiest way to achieve this is to only rotate the base joint, the main movements are observable in this joint. In the lower panel, the presence of a “ceiling” obstacle was suggested to the system. Thus, in order to reach the goal posture form the same starting posture, the arm flexes its upper two joints much more to avoid the obstacle and finally re-extends those joints to reach the goal posture.

To conclude, while the adapted SURE_REACH takes care of the kinematics enabling flexible movement planning and behavioral adjustments, the adaptive PD controller invokes suitable motor commands to maintain dynamic system stability while executing the suggested movement plan. Further evaluations of the system confirm the general robustness of the approach within other dynamic arm and with other added constraints and start-goal location or posture combinations [59].

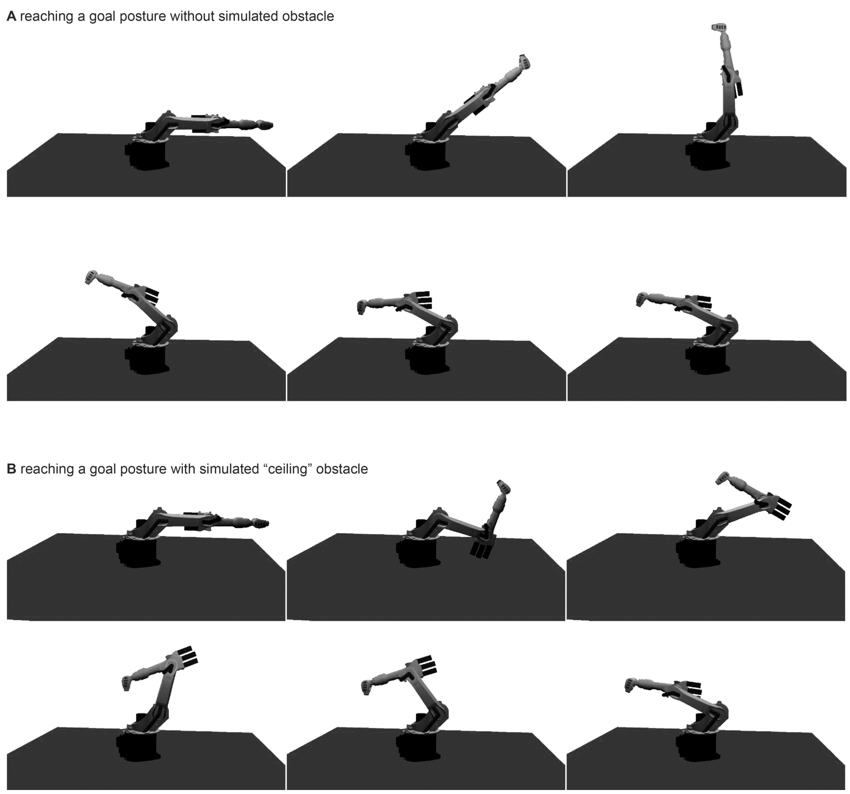

Fig. 7 The figures show the simulated KUKA KR 16 arm that is controlled by a dynamic SURE_REACH system with adaptive PD controller. A shows the straight transition from a start posture to a goal posture in an unconstrained environment, B shows that a more complex movement is exerted if the presence of a "ceiling" obstacle is suggested to the controller.

7 Conclusion

To conclude, the described model offers an interesting perspective on the interplay between movement planning, unsupervised motor learning, and flexible task-dependent motor control in humans. Furthermore, the model's underlying principles are well-suited to control robots in a dynamic environment. In turn, the application of the model (and computational models in general) to real-world robots may hint at critical aspects that have yet to be considered in the models and contribute to a deeper understanding of neural processes. Thus, the application of biological theories to robotics may result in more flexible, robust, and adaptive machines. On the other hand, these applications can be expected to also enhance our knowledge about the intricate neural mechanisms that enable animals and humans to move their bodies seemingly effortlessly and with unmatched sophistication.

8 Appendix

The parameter settings for the simulations depicted in Figures 2 can be found in [11]. The parameter settings for the simulations depicted in Figures 5 can be found in [28]. The simulation results depicted in Figures 3, 4, and 6 are as follows. The lengths of the upper arm, forearm, and hand are $l_1 = 32cm$, $l_2 = 25cm$ and $l_3 = 18cm$, respectively; The shoulder, elbow and wrist joints can assume any angle within $-60° \leq \phi_1 \leq 115°$, $-160° \leq \phi_2 \leq 0°$, and $-75° \leq \phi_3 \leq 50°$, respectively; The gain factor was set to $g = 0.9°$; The preferred hand locations are distributed in a fixed 31×25 grid with $d_{hand} = 5cm$ distance. The grid covers a $150cm \times 120cm$ rectangle, which covers the the arm's work space. Posture neurons are arranged in a $8 \times 7 \times 6$ grid covering the entire posture space. The distance between two adjacent neurons is approximately $d_{posture,1} = d_{posture,3} = 25°$ and $d_{posture,2} = 26.7°$. The neural networks are trained by moving the arm randomly for 1.000.000 time steps. During learning, new sets of motor command are randomly generated and executed for a random duration between 1 to 8 time steps. Sets of motor commands are generated by setting each individual motor neuron to 1.0 with a probability of $p = 0.3$ and to zero otherwise. The learning rate of the posture memory is set to $\varepsilon = 0.001$. The learning rate for the motor controller decays exponentially from $\delta_0 = 0.1$ to $\delta_{1,000,000} = 0.01$ during learning. The upper weight threshold is set to $\theta = 0.1$. The parameters of the equations modeling movement preparation are set to $\beta = 0.17$ and $\gamma = 0.43$. In the charts, the duration unit included 50 time steps. That is, each joint can move 45° in each duration unit.

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