

THOMAS PIECHA **General Proof Theory:**
PETER SCHROEDER-HEISTER **Introduction**

Abstract. This special issue on general proof theory collects papers resulting from the conference on general proof theory held in November 2015 in Tübingen.

Keywords: General proof theory, Proof-theoretic semantics, Natural deduction.

The term ‘general proof theory’ was coined by Prawitz in the late 1960s. In general proof theory, “proofs are studied in their own right in the hope of understanding their nature”, in contradistinction to Hilbert-style ‘reductive proof theory’, which is the “attempt to analyze the proofs of mathematical theories with the intention of reducing them to some more elementary part of mathematics such as finitistic or constructive mathematics” (Prawitz [17, p. 123]). In a similar way, Kreisel [10] asked for a re-orientation of proof theory. Proofs should be “treated as principal objects of study, not as mere tools for analyzing the consequence relation” (Kreisel [10, p. 109]).

In general proof theory we are primarily interested in how a statement can be *proved*, and not only in its *provability*. This shift in focus shaped a considerable part of subsequent proof theory. It led in particular to proof-theoretic semantics, which, in its widest sense, includes the development of type theory as a novel foundation of mathematics (cf. [11, 22]), categorial proof theory (cf. [2]), and the philosophical investigation of meaning in terms of proof (see [20]).

The idea of general proof theory was clearly already present in Gentzen’s work, in particular in his *Investigations into Logical Deduction* (1935, [4]). As a field in its own right (though not yet under this name) it was introduced by Prawitz’s monograph *Natural Deduction* [16] in 1965. In this work, Prawitz studied the conceptual systematics of natural deduction, to which Gentzen had given the most credit from a philosophical point of view. It seemed appropriate, therefore, to take the 50th anniversary of the appearance of *Natural Deduction* as an opportunity to organize a conference on general proof theory in order to evaluate the status and future prospects of the

Special Issue: General Proof Theory
Edited by **Thomas Piecha and Peter Schroeder-Heister**

field. This conference, which took place in Tübingen in November 2015, is another addition to a series of relevant conferences in Tübingen, including those on proof-theoretic semantics in 1999 (see [9]), 2013 (see [13]) and 2019 (see <http://ls.informatik.uni-tuebingen.de/PTS3/>).

The invited talks were as follows:

- Kosta Došen: Adjunction and normalization in categories of logic
- Per Martin-Löf: The two interpretations of natural deduction: how do they fit together?
- Luiz Carlos Pereira: The Russell–Prawitz translation and schematic rules: a view from proof-theory
- Dag Prawitz: Gentzen’s justification of inferences
- Helmut Schwichtenberg: Decorating natural deduction
- Heinrich Wansing: A more general general proof theory

In addition, there were contributions by Miloš Adžić, Federico Aschieri, Roy Dyckhoff and Sara Negri, Nissim Francez, Lev Gordeev and E. Hermann Haeusler, Norbert Gratzl and Eugenio Orlandelli, Giulio Guerrieri and Alberto Naibo, Danko Ilik, Reinhard Kahle, Angeliki Koutsoukou-Argyrazi, Clayton Peterson, Zoran Petrić, Mario Piazza and Gabriele Pulcini.

Preliminary proceedings which include the program, the slides of most talks, and also the paper by Prawitz on *Gentzen’s justification of inferences* [18] have been published on the internet shortly after the conference (see [14]).

This special issue collects papers by participants of the conference, including the invited speakers. The accepted papers illuminate various aspects of general proof theory. Došen and Adžić [3] examine links between general proof theory and some philosophical views ascribed to Gödel. Gordeev and Haeusler [5] show that arbitrary tautologies of Johansson’s minimal propositional logic are provable by ‘small’ polynomial-size *dag*-like natural deductions in Prawitz’s system. Gratzl and Orlandelli [6] deal with the question of the logicity of modal logics from a proof-theoretic perspective. Guerrieri and Naibo [7] study how to postpone the application of the *reductio ad absurdum* rule in classical natural deduction derivations. Kahle [8] reviews the discussion of ‘Hilbert’s Thesis’, and considers the question of whether one could provide for proofs an analogue to the concept of partial recursive function. Olkhovikov and Wansing [12] consider logical inference as an activity that results in proofs and hence produces knowledge. They suggest to merge the semantical analysis of deliberately ‘seeing-to-it-that’ from *stit*

theory and the semantics of the logic of justification of Artemov and Nogina. Prawitz [19] investigates the question of what it is that makes an inference valid and thereby gives a proof its epistemic power. He considers this question to be the most fundamental problem of general proof theory. Tranchini, Pistone and Petrolo [23] show that the Russell–Prawitz translation of first-into second-order logic preserves identity of proof with respect to a certain enriched system. Piecha and Schroeder-Heister [15] show that intuitionistic propositional logic is incomplete with respect to standard notions of proof-theoretic validity.

Sadly, Kosta Došen, who himself had given a talk at the conference, died shortly before this special issue was ready for publication. As he was a key figure in the field of general proof theory, it is most appropriate to include a short academic obituary [21] in this special issue. A detailed *Curriculum Vitae* provided by Kosta Došen himself can be found in [1].

We thank the editors of *Studia Logica*, and in particular Jacek Malinowski for supporting this special issue. We are very grateful to the reviewers of the contributions published here for the work they have undertaken. Moreover, we are indebted to Marine Gaudefroy-Bergmann for her administrative assistance in preparing and organizing the conference. Without her skills and effort it would not have taken place.

Acknowledgements. This work was carried out within the French–German ANR-DFG Project “Beyond Logic: Hypothetical Reasoning in Philosophy of Science, Informatics, and Law”, DFG Grant Schr 275/17-1.

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