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Recent Frege Studies

MICHAEL D. RESNIK, **Frege and the philosophy of mathematics.** Ithaca and London: Cornell University Press, 1980. 244 pp. $16.50.

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In this book Michael D. Resnik presents a discussion of Frege's philosophy of mathematics which takes into account not only the discussions Frege himself was engaged in but also more recent discussions which are systematically related to Frege's positions. So it is both a book on Frege's philosophy of mathematics and on the philosophy of mathematics in general. Thus, before turning to Frege's philosophy of mathematics in the narrower (i.e. more technical) sense (chapter V), Resnik treats the debates on psychologism (chapter I), formalism (chapter II), deductivism (chapter III) and empiricism (chapter IV) with respect to the philosophy of mathematics.

Chapter I discusses the attempt to found mathematics on psychology—a position which is today often regarded as definitely refuted by Frege's and Husserl's (in his *Logische Untersuchungen* of 1900) rigorous criticism. So the reader is all the more surprised that this position in Resnik's careful presentation is not as unreasonable as he or she would expect (having read for example the preface to Frege's *Grundgesetze*) —at least not in all its aspects. In particular, two of the four senses of 'psychologism' which Resnik distinguishes give rise to interesting reflections: the substitution of mental entities for abstract ones and the description of the genesis of mathematical notions instead of their definition. Resnik sees a possibility that the first version of psychologism can be maintained within a kind of constructive mathematics according to which the mathematical entities are mental ones (and therefore subjective ideas or intuitions) but are in a sense isomorphic—with the result that mathematical validity is intersubjective. Concerning the second version of psychologism, Resnik emphasizes some points in Husserl's *Philosophie der Arithmetik* of 1891 (which he takes to be representative for that view) which are not completely out of date: in particular how basic notions which cannot be reduced to other notions may be introduced and justified (how, for instance, one form of introducing numbers as logical objects can be distinguished from another form which cannot be shown to be extensionally equivalent with the former one). Resnik shows that psychologism is a position which is worthy of further discussion, and that Frege (in spite of his strong arguments) did not refute it once and for all.

After a discussion of the 'game formalism' of Heine and Thomae and Frege's criticism of it, chapter II gives a detailed exposition of the formalistic positions of Curry and Hilbert which may be considered as more recent views escaping some of Frege's objections against formalism. Curry's theory is described on the basis of his *Outlines of a formalist philosophy of mathematics* (1951). Resnik's main objection to Curry is that mathematics as a theory of formal systems is not free from ontological assumptions, contrary to Curry's supposition. Resnik's discussion of Hilbert's program is comparatively extensive. He formulates it in a way similar to Kreisel,[1] as the attempt to prove that the provable nonfinitary ('ideal') formulas are a conservative extension of the true finitary ('real') ones. (This is under certain conditions equivalent to the consistency claim Hilbert himself has always stressed, as Kreisel has pointed out.) The problem, then, arises when we consider a true finitary schema (i.e. one all of whose instances are finitarily true) which is itself not provable by finitary means (such a schema is e.g. the codification of the assertion that arithmetic is consistent, as Gödel has shown). This is for Resnik the crucial point at which Hilbert's program fails: in Resnik's opinion, Hilbert can give no finitary sense to true but not finitarily provable finitary schemata. Thus from the strictly finitary standpoint there is no need to take into consideration 'ideal' extensions of 'real' reasoning other than the search for proof procedures which abbreviate 'real' reasoning.

Chapter III deals with the deductivistic attempt to treat mathematics as a science which logically deduces statements from the characteristic axioms of a mathematical theory and does not raise the question whether those axioms are true or not. In the first part Resnik investigates the Frege-Hilbert controversy. He does not simply adopt the attitude that Frege refuted an untenable theory of (implicit) definitions and that the controversy would be ended if one read Hilbert's theory in the light of Frege's improvements (implicit definitions as explicit definitions of second-order predicates etc.) Indeed, he stresses the point that in spite of his correct clarifications of Hilbert's view, Frege (mainly because of his traditional attitude to the status of axioms) was not able to appreciate Hilbert's important contribution to the philosophy of logic and mathematics. This was that logical deduction *from assumptions* (which in addition may contain free variables) is a meaningful concept (which is different from the deduction of conditional sentences) and that it is therefore possible to reduce the truth of a theorem in an axiomatic theory to the purely logical problem of its deducibility from the axioms. The truth of the axioms (within a given model) is then quite another question, if one wants to ask it at all. The second part of the chapter considers the positions of Putnam's 'The thesis that mathematics is logic' of 1967.[2] The main point of Resnik's argument against deductivistic approaches of this kind is that they exist more in a programmatic than an elaborated form and that there a lot of difficulties will probably arise if one wants to apply complicated mathematical theories like the theory of real numbers to empirical sciences.

Chapter IV on Mill's empiricism gives a detailed account of Mill's theory of

1 G. Kreisel, 'Hilbert's programme', *Dialectica*, **12** (1958), 346–372; repr. in P. Benecerraf and H. Putnam (eds.), *Philosophy of mathematics. Selected readings* (1964, Oxford), 157–180.

2 In R. Schoenman (ed.), *Bertrand Russell; philosopher of the century* (1967, London and Boston), 273–303; repr. in Putnam's *Mathematics, matter and method* (1975, Cambridge), 12–42.

arithmetic as based on empirical evidence. Resnik agrees with Frege's arguments against this theory (which form a large part of Frege's criticisms of concurrent positions). Nevertheless, he discusses some aspects of Mill's view which may possibly be re-established; the main problem thereby is the unavoidable presupposition of (at least potentially) infinitely many concrete individuals which are considered to be numbers.

Chapter V is the central chapter of the book. After (1) general considerations of Frege's ontological and epistemological views and (2) reflections on Frege's epistemology of mathematics, Resnik treats (3) Frege's foundation of arithmetic, especially his analysis of the concept of number and (4) important criticisms of Frege's philosophy of mathematics. With respect to (1) Resnik draws a distinction between methodological, ontological, epistemological Platonism and realism in order to clarify recent discussions on Frege's position—in particular that between Dummett and Sluga. Resnik comes to the conclusion that in the *Grundlagen* Frege was both an ontological Platonist and an objective idealist, whereas in his later writings he abandoned the context principle (or at least gave contradictory formulations of it) thus being inclined to a more realistic position. Concerning Frege's special epistemology of mathematics (2) Resnik discusses Frege's reduction of arithmetic to logic and his thesis that arithmetical sentences be analytic, which depends on his definition of analyticity as logical deducibility from axioms and definitions alone. This leads to the question of how definitions and axioms are justified. Resnik points out that according to Frege the definition of a concept that is already in use but without a clear meaning (like the concept of number) is the result of what Carnap later calls explication. In his treatment of Frege's foundation of arithmetic (3), Resnik presents Frege's reasons why numbers as objects cannot be introduced satisfactorily by contextual definitions. He gives a clear description of Frege's definition of the ancestral of a relation and shows that Frege did not attempt to give a contextual definition of class abstraction, even if he seems to do so in introducing 'Wertverläufe' by means of the 'Grundgesetz V'. Of the criticisms of Frege's philosophy of mathematics (4), Resnik gives first a very clear presentation of the contradiction in Frege's system including the contradiction that still arises after Frege's correction in the postscript to the *Grundgesetze* (clearer than e.g. Quine's presentation [3]). He argues that there is no correction of Frege's system which retains its main characteristics. Furthermore, Resnik considers Frege's program to fail because of Gödel's first underivability theorem: Frege's plan was to reduce *all* arithmetical truths to logic by effective proofs from a finite number of axioms, and that is proved impossible by Gödel.

The strongest philosophical argument against Frege's program is what Resnik calls the problem of multiple reduction: even in the logicist account there is some arbitrariness in the definition of the numbers as logical objects. So, for example, different definitions of the successor function are available. But if this is so, we cannot say that numbers *are* certain objects, or that mathematical terms *have* fixed references.[4] All together, Resnik comes to the end that 'the Fregean attempt to found a

3 Compare W.V.O. Quine, 'On Frege's way out', *Mind*, **64** (1955), 145–159; repr. in *Selected logic papers* (1966, New York), 146–158.

4 A closely related argument was given by P. Benacerraf to whom Resnik refers. See Benacerraf's 'What numbers could not be', *Philosophical review*, **74** (1965), 47–73.

(partial) epistemology for mathematics through logic seems destined to fail' (p.234).

To give a general evaluation of this book, we may say that it is certainly not suitable for someone looking for a first introduction to Frege's or to the general philosophy of mathematics. However, for the reader who already has some knowledge of these topics it presents a number of stimulating reflections. Its main advantage, as I see it, is that it is extremely fair also to the positions often very rigorously criticized by Frege.

HANS D. SLUGA, **Gottlob Frege**. London, Boston and Henley: Routledge & Kegan Paul, 1980. xi + 203 pp. £12.95.

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Slugas Frege-Buch ist nicht, wie man aufgrund des Titels der Reihe ('The arguments of the philosophers'), innerhalb der es erschienen ist, erwartet, eine kritische Analyse der Argumente Freges, allenfalls ist es dies am Rande. Nach des Autors Absicht soll vielmehr (die Linie früherer Aufsätze fortsetzend) ein neues Frege-Bild dadurch gezeichnet werden, daß Frege in seinen ursprünglichen historischen Kontext zurückversetzt wird, dessen Berücksichtigung allererst seine Lehren durchsichtig machen könne. Die Stoßrichtung dieser historischen Betrachtungsweise ist nicht nur gegen das Frege-Bild gerichtet, das die analytische Philosophie (vor allem in der maßgeblichen Darstellung von M. Dummett) von Frege entwirft, sondern auch gegen die geläufige Selbstinterpretation dieser Tradition, ohne ein historisches Verständnis der Philosophie auskommen zu können. Letztlich scheint das eigentliche Anliegen Slugas zu sein, gegenüber der (angelsächsischen) analytischen Tradition die Berechtigung der (kontinentalen) hermeneutischen Tradition geltend zu machen.

Das sorgfältig gedruckte Buch schließt mit einem ausführlichen Register ab. Druckfehler (?): Der Vorname von Lasswitz ist 'Kurd', nicht 'Kurt' (p.68); 'Weierstrass', nicht 'Weyerstrass' (p.162, so auch im Register). Freges Zeichen für die Wertverlaufsfunktion, der Spiritus lenis, ist durchgehend mit dem Spiritus asper verwechselt worden. Sluga zitiert Frege nach eigenen Übersetzungen. Seine Stellenangaben beziehen sich deshalb auf die deutschen Ausgaben. Freundlicher gegenüber seinen englischen Lesern wäre es gewesen, die Seitenangaben der englischen Ausgaben mitzuführen.

In seiner Darstellung setzt Sluga eine Bekanntschaft mit Frege voraus. Er selbst versucht dessen Lehren zu einer von der üblichen abweichenden Lesart neu zusammenzufügen. Hauptpunkt dieser neuen Lesart ist, daß Frege keineswegs eine antiidealistische Position vertreten habe, wie Dummett gemeint hat, sondern im Gegenteil idealistische Konzepte in Verbindung mit rationalistischen aufgegriffen habe (vor allem von Leibniz, Kant und Lotze), um gegen die zu seiner Zeit mächtigen Strömungen des Empirismus, Naturalismus und physiologischen Materialismus aufzutreten. Indem Sluga den Psychologismus als Produkt dieser Strömungen nachweist, kann er Freges Kampf gegen den Psychologismus gewissermaßen als Beitrag des Mathematikers und Logikers zu dem Hauptkampf seiner Zeit verstehen. Die