Lambda Calculus and Combinatory Logic — WS 2017/18

Exercise sheet 10 — due 12.1. — T. Piecha

Exercise 1 (3 points)

Provide a type for each of the following terms:

(a) $\lambda xy.y$ (1 point)

(b) $\lambda xz.zx$ (1 point)

(c) $\lambda xy.z(xy)$ (1 point)

Remark: You do *not* have to present a derivation in $\lambda\!\to$.

Exercise 2 (2 points)

Decorate the following tree:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash x : \rho \to \sigma}(\mathrm{Id}) \quad \dfrac{\dfrac{}{\vdash y : \tau \to \rho}(\mathrm{Id}) \quad \dfrac{}{\vdash z : \tau}(\mathrm{Id})}{x : \rho \to \sigma, y : \tau \to \rho, z : \tau \vdash}(\quad)}{\vdash x(yz) :}(\quad)}{x : \rho \to \sigma, y : \tau \to \rho \vdash}(\quad)}{x : \rho \to \sigma \vdash}(\quad)}{\vdash \quad : (\rho \to \sigma) \to (\tau \to \rho) \to \tau \to \sigma}(\quad)$$

Exercise 3 (7 points)

Show by giving derivations in $\lambda\!\to$:

(a) $\vdash \mathbf{KI} : \tau \to (\sigma \to \sigma)$ (3 points)

(b) $\vdash \mathbf{SK} : (\sigma \to \tau) \to (\sigma \to \sigma)$ (4 points)

Why is it impossible to show $\vdash \mathbf{SK} : \tau \to (\sigma \to \sigma)$?

Exercise 4 (8 points)

Prove:

(a) If $\Gamma, x : \sigma \vdash M : \tau$ and $\Gamma \vdash N : \sigma$, then $\Gamma \vdash M[N/x] : \tau$. (4 points)

(b) If $\Gamma \vdash M : \sigma$ and $M \rhd_\beta M'$, then $\Gamma \vdash M' : \sigma$. (4 points)