LARS HALLNÄS and PETER SCHROEDER-HEISTER, *Local reflection in inductive definitions: the* D-*rule*.
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If we think of an inductive definition D as a set of clauses of the form $A_1, \ldots, A_n \Rightarrow A$, where the A_i are atoms in a given universe, the standard interpretation of D as defining Def(D) is given in terms of a sort of *global reflection principle*: The only objects in Def(D) are those that can be constructed by the clauses in D in a finite number of steps. This global reflection principle gives rise to a principle of *induction on* D. The *local reflection principle* that we propose concerns reasoning from assumptions based on an inductive definition D. It explains what it means to assume an atom A with respect to D. We introduce a notion $X \vdash_{\mathrm{D}} A$ of derivability with respect to D that is based on a certain sequent calculus. The D-rule, which expresses local reflection, is formulated as follows:

$$\frac{X, Y \vdash C \qquad (Y \in \mathrm{D}(A))}{X, A \vdash C},$$

where $\mathrm{D}(A) = \{Y \mid Y \Rightarrow A \text{ is in } \mathrm{D}\}$.

$\mathrm{D}(A)$ represents the definientia of A. So the D-rule intuitively states that everything that follows from the definientia of A follows from A itself.

For the case where definitional clauses are ordinary Post productions the D-rule is closely related to Lorenzen's inversion principle [3]. However, in general we can handle a larger class of definitions, e.g. with implication occurring in definitional clauses. In particular, the D-rule gives us the possibility of handling negation in definitions.

If the D-rule is eliminable at an atom A, then A is somehow redundant as an assumption with respect to D. For example in the case of Post productions this means that either A is in Def(D) or that the definition of A in D is cyclic at A. Results on the eliminability of the D-rule demonstrate its deductive strength.

Using the D-rule also leads to powerful extensions of logic programming [2].

REFERENCES

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We consider only countable models and, without loss of generality, assume that their domains are subsets of $\omega = \{0, 1, 2, \ldots\}$. Such a model $\mathscr{A}$ is called *recursive* if its domain A is recursive and its relations and functions are uniformly recursive. Let R be an additional relation on A, and let $\mathrm{Im}_{\mathscr{A}}(R)$ be the set of images of R under all isomorphisms from $\mathscr{A}$ to recursive models. R is called *intrinsically r.e. on* $\mathscr{A}$ if every set in $\mathrm{Im}_{\mathscr{A}}(R)$ is r.e. We define the *(Turing) degree spectrum of* R *on* $\mathscr{A}$ to be the set of Turing degrees of sets in $\mathrm{Im}_{\mathscr{A}}(R)$. Ash and Nerode proved that, under an additional decidability condition (C) on $\mathscr{A}$, satisfied in many natural examples, the semantic property "intrinsically r.e." is equivalent to a certain syntactic property called "formally r.e." We proved that if (C) holds and R is not intrinsically r.e. on $\mathscr{A}$, then the degree spectrum of R must be infinite. On the other hand, using the results of Goncharov, we obtain a two-element degree spectrum $\{\mathbf{0}, \mathbf{x}\}$, where $\mathbf{x} \leq \mathbf{0}''$ but not $\mathbf{x} \leq \mathbf{0}'$. We also construct a recursive model $\mathscr{A}$ and a recursive relation R on its domain such that the degree spectrum of R on $\mathscr{A}$ is a two-element set $\{\mathbf{0}, \mathbf{x}\}$, where $\mathbf{x} \leq \mathbf{0}'$ and no set in $\mathrm{Im}_{\mathscr{A}}(R)$ is r.e. The model $\mathscr{A}$ codes an enumeration ν of a recursive family $\mathscr{S}$ of r.e. sets, and R codes a recursive set D, where $\mathscr{S}$ has, up to recursive equivalence, exactly two recursive injective enumerations ν and μ and the set $\{n \in \omega : (\exists m \in D)[\nu(m) = \mu(n)]\}$ is a Δ^0_2 set which is not r.e.