Assertion and *Denial* in Proof-Theoretic Semantics

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1. The asymmetry between proofs and refutations

Proof-theoretic semantics is an attempt to define logical consequence and, more generally, analytic reasoning in terms of proof rather than truth (Schroeder-Heister 2006). By its very nature – in emphasizing *proof* rather than *refutation* – it is assertion-driven. It defines what counts as a valid proof of an *assertion*, and even when it deals with *assumptions*, it considers them to be placeholders for valid proofs. Alternative versions of proof-theoretic semantics give the notion of an *assumption* a stronger stance, considering assumption inferences to be on the same level as assertion inferences (Schroeder-Heister 2004). However, even then there remains an asymmetry between proofs and refutations or between assertions and denials. This is reflected by the fact that in such frameworks negation is defined indirectly by reduction to absurdity rather than by a notion in its own right.

2. Constructive duality

We argue that this asymmetry should be removed. Actually, duality arguments show that there is no proper advantage of assertion over denial. In classical truth-condition semantics such duality arguments are well known: There truth with respect to the standard connectives under a valuation *v* is the same as falsity with respect to the dual connectives under the complementary valuation *v'* (which interchanges truth and falsity), and *vice versa*. This fact can be used as an argument that it is not possible to fix both the meaning of truth and falsity and the meaning of the logical connectives at the same time by means of truth conditions. What is not so well known is the fact that even for proof-theoretic semantics, which is a constructive approach leading to intuitionistic logic, some related indeterminacy of meaning can be demonstrated. If one changes the basic concepts of proof-theoretic semantics such as "canonical proof", "proof depending on open (not yet proved) assumptions" etc. into refutation concepts such as "canonical refutation", "refutation leading to open (not yet refuted) conclusions", the meaning of the standard connectives is turned in that of connectives dual to them. In this sense duality is not lost when passing from truth-condition semantics to proof-theoretic semantics. This means that in proof-theoretic semantics, as in truth-condition semantics, there is no fundamental semantic principle available which favours assertion. In any case it is interesting to see which conceptual insight we gain from considering a more symmetric system.

3. Clausal logic of assertions and denials

Therefore it is only natural to consider the possibility of incorporating proofs and refutations, or assertions and denials in a single framework. If one bases such a framework on systems of clauses ('programs', 'definitions'), one should consider assertion and denial clauses depending on assertions and denials. One approach is to consider a special negation '~' as a denial operator which can only occur in outermost position, and allow for both unnegated and negated atoms in the heads and bodies of clauses. This means that clauses have the form

$$(\sim)A \Leftarrow (\sim)B_1,\ldots, (\sim)B_n$$

where the parentheses indicate that the rejection operator may be either present or missing. Certain aspects of dealing with such clauses can be handled according to the model of extended logic programming, where heads and bodies of clauses may contain negations (see Damásio & Pereira 1998).

4. Balanced sets of clauses

Dealing with such generalized reasoning systems leads to novel *symmetry or harmony principles* which go beyond the harmony between assertions and assumptions in sequent systems or between introduction and elimination rules in natural deduction. By means of dualization, assertion rules lead to associated denial rules, whereas rejection rules lead to associated assertion rules. This means that we can distinguish between assertion and denial principles just laid down by definition (*primary* assertion and denial), and those obtained from these principles by dualization (*secondary* assertion and denial). Now we may ask whether the primary principles are such that they cover the secondary principles, i.e. contain their own dual. Reasoning systems based on sets of clauses with this property are called *balanced*. Balanced sets of clauses exhibit a maximum degree of explicitness in the sense that reasoning with the primary clauses suffice to obtain everything that can be extracted from these clauses. When investigating balanced reasoning systems, we discuss the following questions:

(1) Are balanced systems monotone with respect to balanced extensions?
(2) Does the property of being balanced imply that the system is total, which technically means that cut elimination holds?
(3) Does the converse hold, or are there total systems which are not balanced?

Question (1) receives a negative answer, at least when implication is admitted as a connective in the bodies of clauses; hence we remain in the realm of nonmonotonic reasoning. The relationship between totality and being balanced ((2) and (3)) turns out to be more intricate. A positive answer would show that being balanced is a strong indicator for a reasoning system to be 'well-behaved'. In particular, non-wellfounded phenomena such as paradoxes would be excluded. We also relate the systems proposed to reasoning systems with strong negation (see Schroeder-Heister 2005a).

The problems discussed are naturally relevant to the relationship between foundational reasoning in constructivist epistemologies and Popper's refutation-based approach (Schroeder-Heister 2005b). On the basis of the logical and semantic arguments given, there is no reason to prioritise one of the two approaches. On the

contrary, the results support the idea of a uniform framework of proofs and refutations, at least when viewed from a semantic perspective.

References:

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