Lambda Calculus and Combinatory Logic SS 2016

Exercise sheet 4 due 13.5. T. Piecha

Exercise 1 (4 + 1 points)

A term M is called *minimal* with respect to β-reduction iff for all terms N: If $M \rhd_\beta N$, then $M \equiv_\alpha N$.

Show that all β-normal forms are minimal, but that not all minimal terms are β-normal forms.

Exercise 2 (2 + 2 + 2 + 3 points)

Find λ-terms **B**, **W**, **X** and **Z** such that the following equalities hold:

(a) $\mathbf{B}xyz =_\beta x(yz)$

(b) $\mathbf{W}xy =_\beta xyy$

(c) $\mathbf{X}xy =_\beta \mathbf{X}yx$

(d) $\mathbf{Z}x =_\beta y\mathbf{Z}$

Show that your λ-terms have the desired behaviour by reducing the following terms:

- $\mathbf{B}MNO$
- $\mathbf{W}MN$
- $\mathbf{X}MN$
- $\mathbf{Z}MNO$

Could **Z** be a combinator?

Exercise 3 (3 + 3 points)

Reduce the following terms to β-normal form:

(a) $(\lambda u.\mathbf{R}\underline{0}(\lambda uv.(\lambda xy.x)uv)u)\underline{1}$

(b) $(\lambda u.\mathbf{R}\underline{0}(\lambda uv.(\lambda xy.y)uv)u)\underline{1}$

where $\mathbf{R} :\simeq \mathbf{\Theta}(\lambda uxyz.\mathbf{D}x(y(\mathbf{V}z)(uxy(\mathbf{V}z)))z)$.

Hint: Reduce applications of **D** and **V** according to Lemma 1.29, cases 2 and 3.