Lambda Calculus and Combinatory Logic — WS 2017/18

Exercise sheet 3 — due 10.11. — T. Piecha

Exercise 1 (8 points)
Show that

(a) $(\lambda z.(\lambda yx.xy)z)y =_\beta (\lambda xy.xy)(\lambda z.z)((\lambda xz.zy)v)$ (4 points)

(b) $((\lambda xy.y)v)(x((\lambda uv.v)(\lambda uv.vu)x)) =_\beta ((\lambda xyz.zx)xx(\lambda u.uu))$ (4 points)

Exercise 2 (4 points)
Prove the following: If $M \rhd_\beta N$ and $P \rhd_\beta Q$, then $P[M/x] \rhd_\beta Q[N/x]$.

Exercise 3 (6 points)
Find two pairs of terms M_1, N_1 and M_2, N_2 such that, for $i \in \{1,2\}$, $M_i =_\beta N_i$, but neither $M_i \rhd_\beta N_i$ nor $N_i \rhd_\beta M_i$. (Show this.)

Exercise 4 (2 points)
We extend $=_\beta$ to a relation $=_{\beta\varphi}$ by allowing for steps of the form $P\big[\lambda xy.x\big] =_{1\varphi} P\big[\lambda xy.y\big]$.
Prove that this extension leads to inconsistency in the sense that for all λ-terms M,N: $M =_{\beta\varphi} N$.

Hint: The proof can be given by simply showing $=_{\beta\varphi}$-equality for arbitrary λ-terms M and N. It is not necessary to provide a proof by induction.